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MEASURING THE PERFORMANCE OF AIRPORT GATES USING DATA ENVELOPMENT ANALYSIS

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Abstract

Airports all over the world are becoming busier and many of them are facing capacity problems. The actual airport capacity strongly depends on the efficiency of the resource utilization. This paper focuses on analysis the performance of gates by data envelopment analysis method to determine the less efficiency gates, and introduce simple heuristic to improve the efficiency of gates.

Keywords: Airport Gate Assignment, Data envelopment analysis, Heuristic.

1- Introduction

The airport gate assignment problem (AGAP) is a critical and essential issue for daily airport operations planning. Airports today become much busier and more complicated than previous days. Aircraft on the ground requires all kinds of diverse services, like the aircraft need to be refueled, new passengers need to board; new supplies have to be put on board. The aircraft has to get cleaned. All the actions take place while the aircraft is standing at a gate (Lim &Wang 2005).

Growing flights congestion makes it necessary and compulsory to find ways to increase the airport operation efficiency. Research on airport gate assignment problem (AGAP) appears extremely significant on facilitate airlines to assess how many gates they should rent from airports to serve their own aircrafts.

Recently AGAP becomes one of core components in the field of airport resource management and naturally appeals the close concentration of current researchers. AGAP can be described as follows: Suppose an airline company owns the business of hosting a certain number of flights every day and to run the business smoothly it must lease a certain number of gates from an airport (Chun etc. 1999).

The main mechanism of flight-to-gate assignment is to assign aircrafts to suitable gates so that not only passengers can conveniently embark or disembark but also the airline companies can minimize the cost in the whole operational process. Efficient airport operation largely depends on how to gate aircrafts in a smooth flow of arriving and departing flights (Chendong Li 2005).

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In this paper, we use Data Envelopment Analysis (DEA) to measure the performance of gates in Cairo airport and then, a simple heuristic algorithm is suggested to raise the performance of airport gates. We present a real case problem about "Cairo airport gates".

2- Problem Definition

Before presenting our case study, we have to explain some definitions as follows:

Aircraft stand: A designated area on an apron intended to be used for parking an aircraft.

Runway: A defined rectangular area on a land aerodrome prepared for the landing and take-off of aircraft.

Taxiway: A defined path on a land aerodrome established for the taxiing of aircraft and intended to provide a link between one part of the aerodrome and another.

Movement area: That part of an aerodrome to be used for the take-off, landing and taxiing of aircraft, consisting of the maneuvering area and the apron(s).

Ground control: is responsible for the airport "movement" areas, as well as areas not released to the airlines or other users. This generally includes all taxiways, inactive runways, holding areas, and some transitional aprons or intersections where aircraft arrive.

After the aircraft is landed on the runway, the pilot tunes ground control for clearance to taxi to the gate. The ground control tells him the taxiway and the suitable gate according to the aircraft type. The International Civil Aviation Organization (ICAO) classified aircrafts A, B, C, D, E and F according to wingspan.

Table 2.1 classification of aircraft wingspan

Code Letter	Wing Span
A	Up to 15 m
B	15 m up to 24 m
C	24 m up to 36 m
D	36 m up to 52 m
E	52 m up to 65 m
F	65 m up to 80 m

The common aircraft types in the Cairo international airport terminal 3 are classified to narrow and wide wingspan aircraft. The narrow wingspan aircraft are A320, A321, B738 the wide wingspan aircraft are A333, A332, B772, B773 (A is the first letter of Airbus and B is first letter of Boeing).

Table2.2 aircraft wingspan

Aircraft Type	Wingspan (M)	Aircraft Type	Wingspan (M)
A320	34.09 m	A332	60.30 m
A321	34.09 m	A333	60.30 m
B738	35.79 m	B772	60.93 m
B773	60.93 m		

The Cairo international airport terminal No. 3 has 15 gates called: (F1, F2, F3, F5, F7, F8, F9, G1, G2, G3, G4, G5, G7, G8, G9). Every gate has specifications. Some gates can take narrow wingspan aircraft and another gate gets wide wingspan aircraft. For example: F1 can assign one aircraft from type C and F2 can assign one aircraft from type E…and so on.

Gates (F5, F7, G5, G8) can assign wide body aircraft or two narrow body aircraft to become (F5A, F5B, F7A, F7B, G5A, G5B, G8A, G8B).

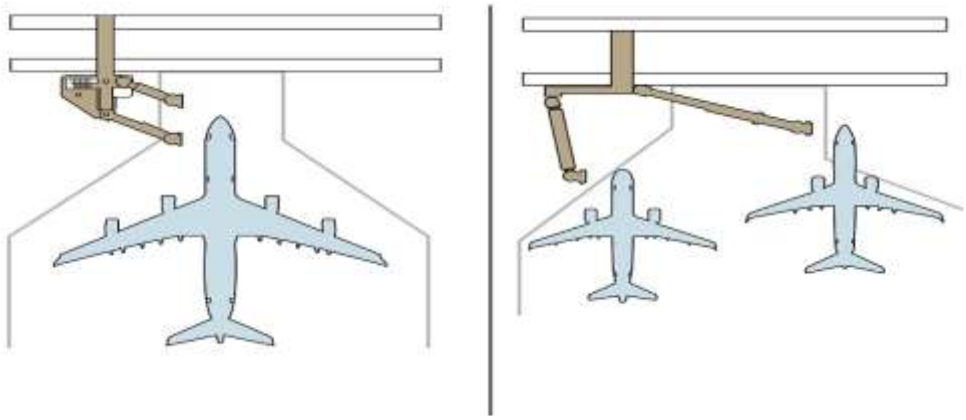

Figure2.1 Large gates can be used as two small gates by small aircraft

Applying DEA to measure performance of each gate aiming to improve its capability, starting from analyzing the process of the gate assignment and, Based on the real data, the gates assignment of Cairo International Airport (CIA) is analyzed to know: which gates have a maximum efficiencies and what are the efficiencies of other gates relative to the maximum efficiencies gates and a heuristic algorithm of AGAP is built to maximize the number of flights assigned to the gates.

2-1- The Objectives

The main objective is to measure the performance of airport gate through applying DEA. We also suggest a heuristic approach to raise the performance and the capability of these gates to improve the efficiency.

2-2- Methodology

The DEA is comparative approach for identify the performance. DEA is commonly used to evaluate the efficiency of a number of producers. A typical statistical approach is characterized as a central tendency approach and it evaluates producers relative to an average producer In contrast, DEA compares each producer with only the "best" producers. By the way, in the DEA literature, a producer is usually referred to as a decision making unit or DMU (c.Ray, 2004).

DEA models will have two orientations: input-oriented and output-oriented, Input-oriented models are used to test if a DMU under evaluation can reduce its inputs while keeping the outputs at their current levels. Output-oriented models are used to test if a DMU under evaluation can increase its outputs while keeping the inputs at their current levels (Milan M, 2009).

In this paper, CRS (constant-returns to scale) output-oriented model is used because the output of the process increases or decreases simultaneously and in step with increase or decrease in the inputs.

Let x_{ij} - denote the observed magnitude of i - type input for gate j ($x_{ij} > 0$, $i = 1, 2, ..., m$, $j = 1, 2, ..., n$) and yrj - the observed magnitude of r-type output for gate j ($y_{rj} > 0$, $r = 1, 2, ..., s$, $j = 1, 2, ..., n$). Then, the Charnes-Cooper-Rhodes (CCR) model is formulated in the following form for the selected gate *k:*

$$Min\ q = \sum_{i=1}^{m} v_i\ x_{ik}$$

subject to

$$\sum_{r=1}^{s} u_r\, y_{rk} = 1$$

$$\sum_{i=1}^{m} v_i\, x_{ij} - \sum_{r=1}^{s} u_r\, y_{rj} \geq 0, \quad (j = 1, 2, \dots, n)$$

$$u_r \geq \mathcal{E}, \quad (r = 1, 2, \dots, s)$$

$$v_i \geq \mathcal{E}, \quad (i = 1, 2, \dots, m)$$

Where:

- v_i is the weights to be determined for input *i*;
- m is the number of inputs;
- u_r is the weights to be determined for output *r*;
- s is the number of outputs;
- q is the relative efficiency of DMUk;
- n is the number of entities;
- ε is a small positive value.

The variables selected for this research are most commonly used for input and output variables affecting airport gate efficiency as found in the published research. In our case we have one input and two outputs. We consider the design of each gate as input variable, the number of aircrafts and the time as output variables.

3- Literature Review

The multi-objective nature of the problem has been gradually acknowledged by researchers working on this problem. Initially research focused mainly on various passenger comfort objectives. For example, (Haghani, 1998) and (Xu, 2001)minimized the distance a passenger walks inside a terminal to reach a departure gate. (Yan, 2001) Introduced an integer programming model to minimize the total passenger walking distance and the total passenger waiting time. (Ding, 2004)Shifted the research interest slightly from passenger comfort to airport operations and solved a multi-objective IP formulation of the GAP with an additional objective: minimization of the number of ungated flights. (Dorndorf, 2007) Went further and claimed that not enough attention has been given to the airport side when solving the GAP. They focused on three objectives: maximization of the total flight to gate preferences, minimization of the number of towing moves and minimization of the absolute deviation from the original schedule. Perhaps surprisingly at the time, in the context of many previous publications, they omitted the walking distance objective, arguing that the airport managers do not consider it an important aspect of the GAP. Our experience with airports indicates that this is probably correct, which is why the objective function presented in Section 4 does not consider passenger walking distance but aims instead to reduce the conflicts by the gates, to allocate gates so that the airline and the size preferences are maximized and to ensure that the time gaps between two adjacent allocations are large enough. The constraints that we have modelled in Section 4 include other aspects of the problem which have already been discussed by other researchers.

It is discussed relative sizes of gates and aircraft, airline preferences and shadowing restrictions. Similarly the importance of the maximization of time gaps between allocations and the robustness of solutions was discussed by (Bolat, 2000). This is also included in our objective function. We aim to avoid small gaps by maximizing the time gaps within a defined time window. (Kumar, 2011)Included towing in their models. They modelled the towing using three flights: (arrival) + (tow away); (tow away) + (tow back); and (tow back) + (departure).

4- Experiment Analysis

Data about performance in Cairo airport was collected to analyze the efficiency and determine the improvement opportunity, our study involved 13 working days at Cairo international airport, terminal 3 that contains 14 gates.

We calculated the number of aircrafts (A/Cs) arriving in this particular period of time, we found them to be 1304 A/Cs, 974 A/Cs of them could be served at terminal 3 because there are types that can't be served in terminal 3 like E170.

Table 4.1 shows the number of planes served at gates in the study time.

Table 4.1 A/C serviced by all Gates

Total number of A/C	A/C serviced by Gates	Percentage
974	668	68.58%

The percentage of A/Cs that was served was 68.58% from the total number of A/Cs that can be served in the terminal, they were distributed over gates as follows in table 4.2

Table 4.2 A/C serviced by each Gate

Gate		Type		Total
		Dom	Int	
F1	Count	6	58	64
	%	9.4%	90.6%	100.0%
F2	Count	0	29	29
	%	0.0%	100.0%	100.0%
F3	Count	3	50	53
	%	5.7%	94.3%	100.0%
F5	Count	4	41	45
	%	8.9%	91.1%	100.0%
F7	Count	0	41	41
	%	0.0%	100.0%	100.0%
F8	Count	8	48	56
	%	14.3%	85.7%	100.0%
G1	Count	9	60	69
	%	13.0%	87.0%	100.0%
G2	Count	0	28	28
	%	0.0%	100.0%	100.0%
G3	Count	11	50	61
	%	18.0%	82.0%	100.0%
G4	Count	0	33	33
	%	0.0%	100.0%	100.0%
G5	Count	1	34	35
	%	2.9%	97.1%	100.0%
G7	Count	6	46	52
	%	11.5%	88.5%	100.0%
G8	Count	9	44	53
	%	17.0%	83.0%	100.0%
G9	Count	5	44	49
	%	10.2%	89.8%	100.0%
Total	Count	62	606	668
	%	9.3%	90.7%	100.0%

The hours in which each gate was in use during the period covered by the study in figure 4.1

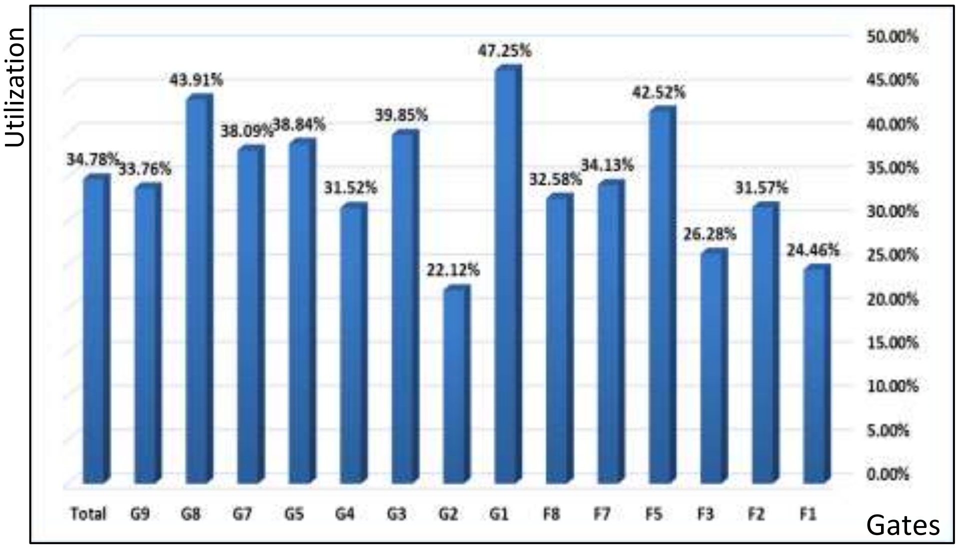

Figure 4.1 Time of use each gate

Utilization of gates was 22.12%-47.25% of Study time which are poor percentages indicating defects in management and operating the gates which requires improving their efficiency.

5- Calculate Efficiency of Gates Using DEA

Output-oriented DEA using CRS model was used for determining the overall technical efficiency for each gate in terminal 3. The model has the following components:

Input

- Width of gate which determine the allowable wingspan of aircraft.

Outputs

- Number of served A/Cs.
- Time in use for each gate.

Table 5.1 shows the values of input and outputs variables for each gate in terminal 3 within the time of study.

Table 5.1 the values of variables for each gate

Gates	Input	Output	Output
	Width	Number of A/C	Time in Use (hrs.)
F1	36	64	76.33
F2	65	29	98.50
F3	52	53	82.00
F5	80	45	132.67
F7	65	41	106.50
F8	52	56	101.66
G1	36	69	147.42
G2	65	28	87.58
G3	52	61	124.33
G4	65	33	98.93
G5	80	35	121.17
G7	52	52	118.83
G8	65	53	137.00
G9	36	49	105.33

Using WinDeap program and analyzing this data, the results of technical efficiency for each gate in terminal 3 were as follows in figure 5.1

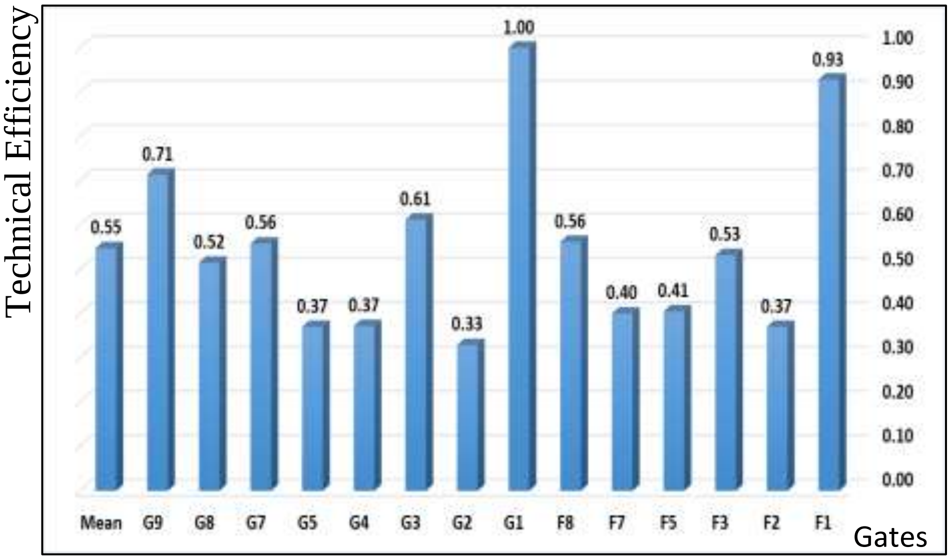

Figure 5.1 Technical efficiency of each gate

Figure 5.1 shows that technical efficiency of gates begins with 0.33, and shows that the mean of efficiency of terminal was 0.55.

It can be deduced that gate G1 is the most efficient, it can be considered as a "peer" to every other gate in the terminal, the Peers weights of each gate was demonstrated in table 5.2.

Table 5.2 Peers weights of each gate

Gate	Peers	peer weights
F1	G1	1.00
F2	G1	1.81
F3	G1	1.44
F5	G1	2.22
F7	G1	1.81
F8	G1	1.44
G1	G1	1.00
G2	G1	1.81
G3	G1	1.44
G4	G1	1.81
G5	G1	2.22
G7	G1	1.44
G8	G1	1.81
G9	G1	1.00

The improvement opportunity for each gate can be determined using DEA, the study choose the outputs orientation to improve the efficiency because it is available to change the values of outputs, and there is not available to change the value of input of any gate, figure 5.2 shows the target number of A/Cs and the total time in use for each gate according to output-oriented DEA.

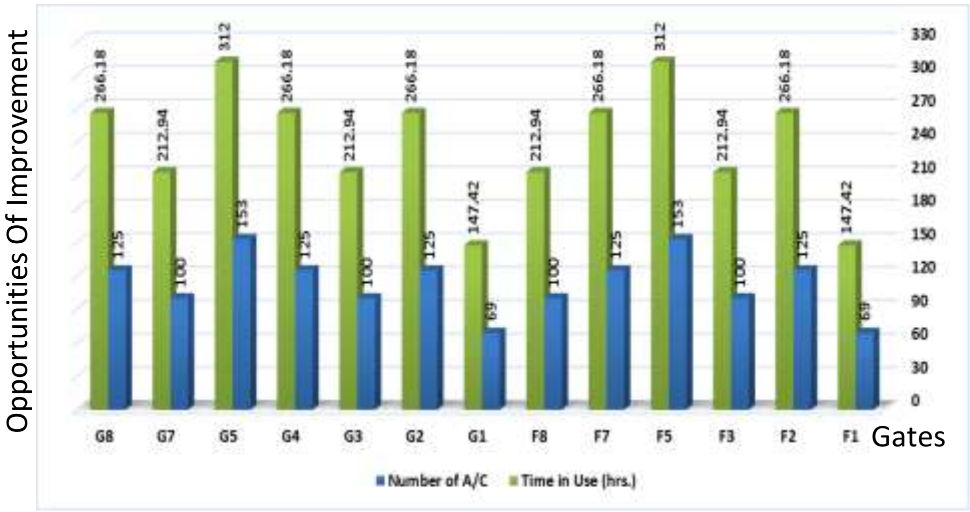

Figure 5.2 Summary of outputs target

Using DEA Shows that it is possible to improve the efficiency of the terminal and each gate, but DEA does not provide a way to achieve the target outputs, so the study proposes a simple heuristic algorithm for improve the efficiency of gate assignment.

6- Suggested heuristic algorithm

The word “heuristic” comes from the Greek word “eurisko”, which means “I find”. It has been used in the last century or so, to indicate a practical decision rule or a practical way to find a solution to a problem, relying upon experience and common sense.

These rules do not aim at satisfying any formal or theoretical property; sometimes “heuristics” have been defined just as practical alternatives in contrast to formal mathematical techniques.

Heuristic algorithms are used to solve large instances of computationally difficult problems, because the computation of an exact solution would require an excessive amount of computing time.

These algorithms can be roughly classified into two types:

- Specific algorithms for specific problems **Heuristics**
- General ideas for almost any problem **Meta-heuristics** (Pearl, 1984)

6-1- Prioritization of gates

As a rule of thumb the gates will be divided into three types, first one for narrow wing span aircraft only, second for wide wing span aircraft only, and third is multi for both narrow and wide wing span aircraft.

1. If the narrow wing span aircraft arrive and all multi gates are not assigned; the priority for narrow wing span gate.
2. If the narrow wing span aircraft arrive and there is multi gate are assigned with narrow wing span aircraft; the priority for this gate.
3. If the wide wing span aircraft arrive; the priority for wide wing span gate only

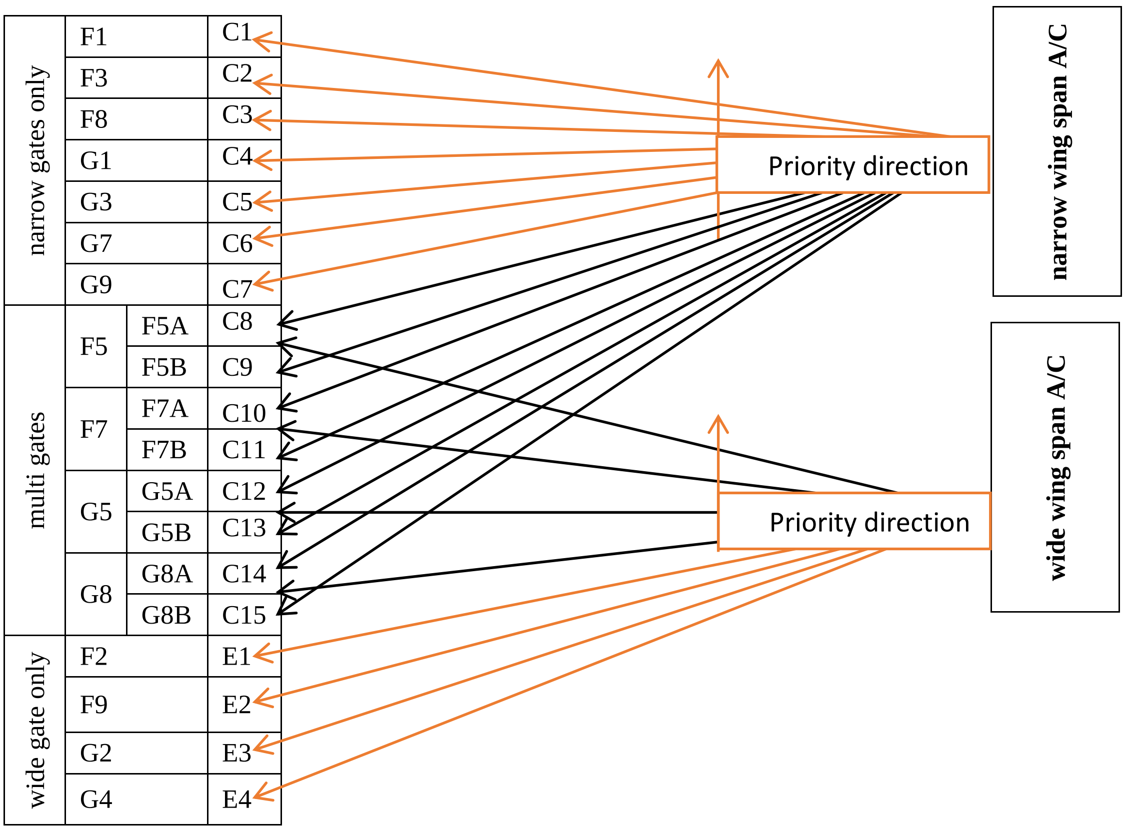

Figure 6-1-1 Prioritization of gates

6-2- Flow chart

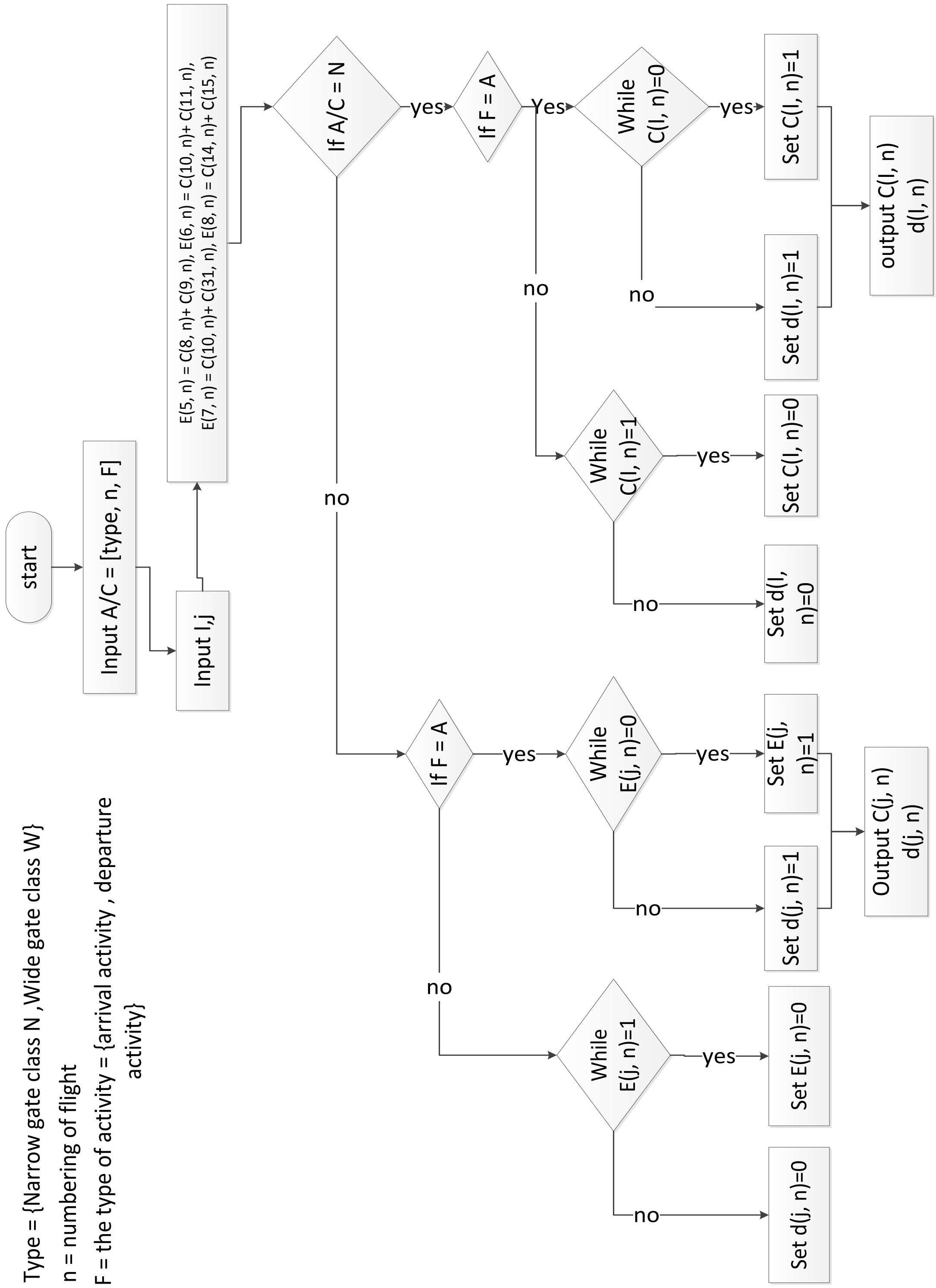

Figure 6-2 showing the flow chart of simple heuristic algorithm for assigning A/C in gates

6-3- Results of applying heuristic algorithm

This part will present the results for three randomly day from historical data. For each day the results of applying simple heuristic algorithm will be compared to the original gate assignment (before).

In figure 6-3-1 showing the served time in hour for each gate in first day before and after applying simple heuristic algorithm, the total served time increased 1.6 from the original assignment.

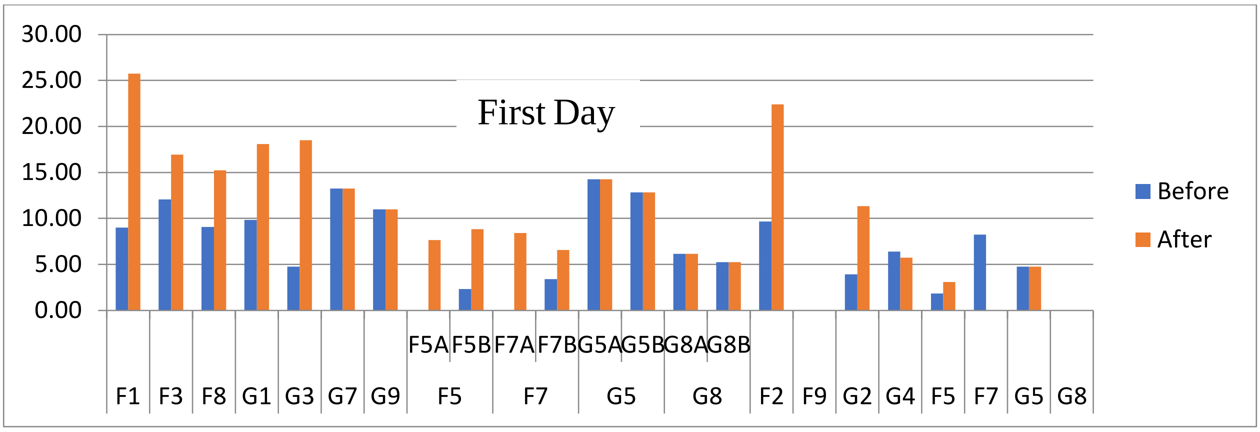

Figure 6-3-1 comparisons between served time in hour before and after applying algorithm in first day

In figure 6-3-2 showing the served time in hour for each gate in second day before and after applying simple heuristic algorithm, the total served time increased 1.97 from the original assignment

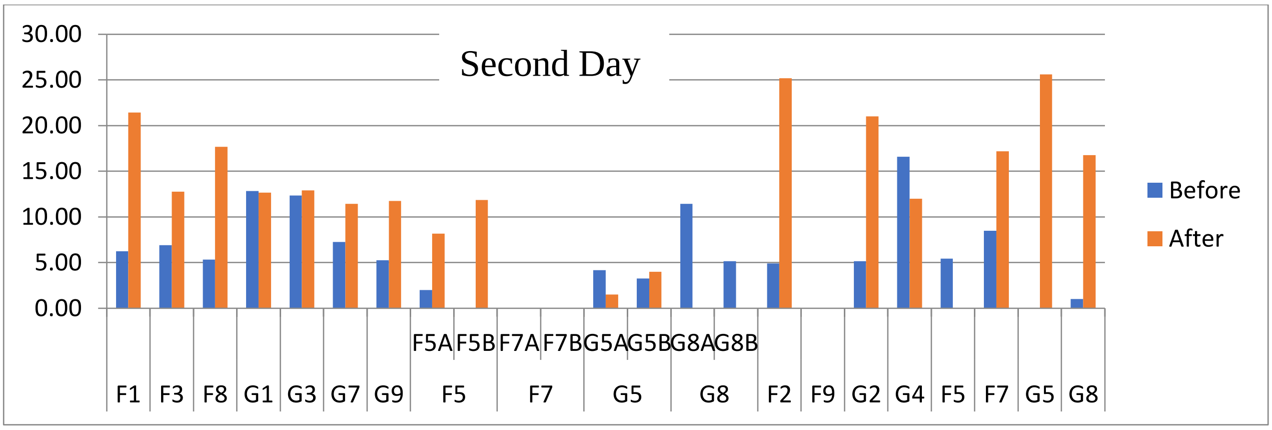

Figure 6-3-2 comparisons between served time in hour before and after applying algorithm in second day

In figure 6-3-3 showing the served time in hour for each gate in third day before and after applying simple heuristic algorithm, the total served time increased 1.96 from the original assignment

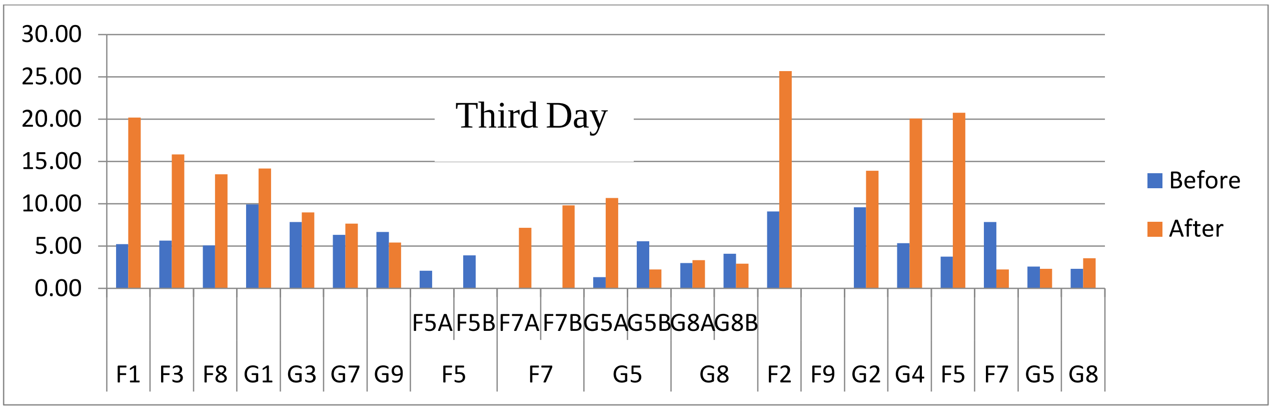

Figure 6-3-3 comparisons between served time in hour before and after applying algorithm in third day

7- Conclusion

This paper attempts to identify the key performance indicators to assess the efficiency of Cairo airport gates. Given sample of preschedule information about airport gates, in our case we have one input and two outputs, we consider the design of each gate as input variable, and the number of aircrafts and the using time of each gate as output variables (most commonly used in the published researches).

The case study involved 13 working days at Cairo international airport, terminal 3 that contains 14 gates.

We calculated the number of aircrafts (A/Cs) arriving in this particular period of time, we found them 1304 A/Cs, and 974 A/Cs of them could be served at terminal 3.

Utilization of gates located in the interval 22.12%-47.25% of study time, which are poor percentages, indicating defects in management and operating the gates, which requires improving their efficiency.

Using data envelopment analysis shows that the mean of efficiency of terminal was 0.55.

Finally, we suggested a simple heuristic algorithm to improve the efficiency of gate and increasing utilization, the total utilization of all gates in terminal 3 increased in three day of applying algorithm are 1.6, 1.97, and 1.96 of the actual assignment.

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A Distance to the Ideal Alternative Approach for Group Decision Making

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Abstract

In Group Decision Making, a compromise strategy is presented to compile the conflicting opinions of decision makers. Many parameters are relevant in solving this category of problems. The Distance to Ideal Alternative (DIA) technique is an efficient method to tackle such category because of its mechanism and robust performance. In this paper, a DIA approach is proposed to solve the Group Decision Making problems and to deal with the ranking abnormalities existed in the ranking orders produced by the Technique for Order Preference by Similarity to an Ideal Solution (TOPSIS) and Multi-Objective Optimization on the basis of Ratio Analysis (MOORA) methods. A Numerical example is adopted for testing the proposed approach.

Keywords: Distance to the Ideal Alternative; Group Decision Making; Multi-Criteria Decision Making; MOORA; TOPSIS.

1. Introduction

A group is defined by set of members gathered for an aim or specific goal. Daily there are a lot groups constructed for certain purposes whether the members of the group are collected in a specific place or time. In management, many examples can be mentioned; a company board of directors having set of actions to choose from, hiring a new employee, facility location to be determined, expansion in markets, and other examples beyond listing here. Many other areas like politics, economics, engineering, and other disciplines are applications for this type. The members of the group might be humans or others races but still there is an ultimate goal to be reached. Birds' swarms seeking their food in groups, fish flocks searching destinations, and insects going shelters are various examples for groups with ultimate collective goals. Group members are sharing information and discussions to reach this ultimate goal. Despite of the might be existence of conflicting interests among the members but still there must be a compromise to be done. Discussions are conformed by various types like voting, brainstorming, analysis to construct unique parameters for the whole group. This process of narrowing the conflicting points of view among group members is studied by many researchers. Some researchers involve some managerial techniques like Delphi technique and SWOT analysis for this aim.

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Every member of the group should present his interests, priorities, hazards, and conflicts clearly in advance before the group goal's construction. After, a procedure is done for reaching a decision or set of decisions to reach the ultimate group goal under the group individuals' parameters mentioned. Problems with many decision makers are complicated because it involves many opposing factors, such as: conflicting individual goals, inefficient knowledge, individual motivation, personal opinion, and power. In brief, MCDM is the art of compromising conflicting criteria for a single decision maker, while GDM is the art of compromising different opinions of group members [10]. In both Multi-Criteria Decision Making (MCDM) and Group Decision Making (GDM), there are two major steps: aggregation and exploitation [4]. Both steps are defined in a different manner relative to the method employed and application area [9]. In a sense, GDM can be a multi-dimensional MCDM problem, but more issues should be studied to treat the conflicting factors aforementioned.

The group members have different intentions, aspects, backgrounds, and preferences. Although they can be clustered into specific sets but still there must be a compromise should be done to reach satisfactory decision for all decision makers. In MCDM, many popular techniques were proposed since mid 1960s [5]. Many researchers contribute in MCDM field and after they extend their work to GDM. Over two decades TOPSIS method had explored many applications and problems [6]. TOPSIS suffers from a crucial drawback so-called abnormalities of ranking orders. Distance to Ideal Alternative (DIA) is a modified version stemmed out from TOPSIS method [8]. DIA outperforms TOPSIS concerning robustness and efficiency. Basically, the present paper is to extend DIA to problems with many decision makers and set relevant modifications and enhancements to the original method. Additional to this goal, the introduced approach get benefit from employing DIA to tackle the well known abnormalities problem. The validation tests show how superior is the proposed approach over TOPSIS and MOORA.

This paper is organized as follows: Section 2, "Group Decision Making Problem" which contains techniques that used in comparison to the introduced approach Section 3, "Solution Methods & Techniques" presenting quick review about "MOORA", "TOPSIS" and "DIA**".** Section 4 "The proposed method" is illustrated. In section 5, "a numerical example" is given for validation, and finally section 6 is made for "Conclusion".

2. Group Decision Making Problem

A problem with many decision makers involved can be seen from two major points of view. Firstly, set of independent MCDM models are developed based on individuals' preferences. For each decision maker there will be a separate MCDM problem from his point of view. Each decision maker is responsible to define the relevant parameters according to him and solves the separate problem to get a solution. Then, the independent solutions are compiled using specific operators to get the group solution. This can be briefly described by compromising resulted solutions (ranks). Secondly, the aggregation is done to the individuals' preference, providing finally a set of group parameters.

For this resulted MCDM problem, it can be solved as a unique problem and the solution (rank) produced is for the whole group. Solving a group decision making problem begins with the construction of decision making matrices. Let $D = \{1,2, \ldots,K\}$a set of decision makers or experts. The Group Decision Making problem can be expressed in k-matrix format in the following way

$$\begin{array}{c|cccc} & C_1 & C_2 & \cdots & C_n \\ \hline A_1 & x_{11}^k & x_{12}^k & \cdots & x_{1n}^k \\ A_2 & x_{21}^k & x_{22}^k & \cdots & x_{2n}^k \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ A_m & x_{m1}^k & x_{m2}^k & \cdots & x_{mn}^k \end{array} \tag{1}$$

where:

- $A_1 ,A_2 ,\ldots , A_m$ are alternatives that decision makers have to choose from
- $C_1 ,C_2 ,\ldots ,C_n$ are the criteria for which the alternative performance is measured
- x_{ij}^k is the k-decision maker rating of alternative A_i with respect to the criterion C_j
- for m alternatives and n criteria we have matrix $X^k = (x_{ij}^k)$ where x_{ij}^k is the performance rating of the i^{th} alternative A_i, with respect to the j^{th} criterion C_j for the k^{th} decision maker
- $i = 1,2,\ldots,m,\quad j = 1,2,\ldots n,$ and $k = 1,2,\ldots,K.$

The importance of each criterion is given by $W^k = \left[w_1^k, w_2^k, \ldots, w_n^k\right]$a weight vector for the k^{th} decision maker, $\sum_{j=1}^{n} w_j^k = 1$.

3. Solution Methods & Techniques

This section contains three well known MCDM techniques which will be illustrated for being used in solving GDM in next section. The three chosen methods are MOORA for being a famous aggregating method, TOPSIS, and the proposed DIA approach. Both TOPSIS and DIA are so similar in their early steps but both of them have a distinct philosophy in reaching a solution.

3.1 MOORA

As shown in Eq. (1) for a single decision maker, the procedure of MOORA for ranking alternatives can be described as following [2]:

***Step 1*:** Normalized decision matrix is computed as in Eq. (2)

$$x_{ij}^{*} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^{2}}}, \qquad i = 1,...,m; j = 1,...,n. \tag{2}$$

***Step 2*:** The composite score is calculated as illustrated in Eq. (3)

$$z_i = \sum_{j=1}^{b} x_{ij}^{*} - \sum_{j=b+1}^{n} x_{ij}^{*} \ , \ \ i = 1,...,m. \tag{3}$$

where $\sum_{j=1}^{b} x_{ij}^{*}$ and $\sum_{j=b+1}^{n} x_{ij}^{*}$ are for the Max (benefit) and Min (cost) criteria, respectively. For different weights of criteria, the composite score becomes

$$z_i = \sum_{j=1}^{b} w_j . x_{ij}^{*} - \sum_{j=b+1}^{n} w_j . x_{ij}^{*}, \ \ i = 1,...,m. \tag{4}$$

where W_j is the weight of j^{th} criterion.

***Step 3*:** The alternatives are ranked in descending order.

The simplicity of MOORA is the main reason of its widely application to various disciplines. The philosophy of aggregating score for alternatives makes the method robust and easy to use. Recently, MOORA has been explored many problems of various fields, such as contractor selection [2], and climate [7].

3.2 TOPSIS

TOPIS method is relying on the idea of comparing set of alternatives to an ideal composite vector constructed from the best performance alternatives under all criteria. Being closer to this ideal vector is better alternatives. In the latest decades, TOPSIS have been extended according to the requirements of different real-world decision making problems; it has been extended to various optimization areas like robot selection [3], and operating systems [1]. For one decision maker a decision matrix is to be constructed as shown in Eq. (1), the TOPSIS solution method consists of the following steps [6]:

Step 1: *The decision matrix is normalized*: it can be calculated as in Eq. (2)
Step 2: *The normalized weighted decision is constructed*: The columns of the normalized decision matrix are multiplied by the associated weights as follows:

$$v_{ij} = w_j \, . \, x_{ij}^* \quad , \quad i = 1,\dots,m; \quad j = 1,\dots,n, \quad \text{and} \quad \sum_{j=1}^{n} w_j = 1 \tag{5}$$

Step 3: *The positive and negative ideal solutions are computed:* The positive and negative ideal value sets are determined, respectively, as follows:

$$A^+ = (v_1^+, v_2^+, \dots, v_n^+) = \{\max\ v_{ij}\ \text{ if } j \in \Omega_b\} \quad \text{or } \{\min\ v_{ij}\ \text{ if } j \in \Omega_c\} \tag{6}$$

$$A^- = (v_1^-, v_2^-, \dots, v_n^-) = \{\min\ v_{ij}\ \text{ if } j \in \Omega_b\} \quad \text{or } \{\max\ v_{ij}\ \text{ if } j \in \Omega_c\} \tag{7}$$

Where $\boldsymbol{\Omega}_b$ is the set of benefit criteria and $\boldsymbol{\Omega}_c$ is the set of cost criteria.

Step 4: *The distance from positive and negative ideal solutions are calculated*: Two Euclidean distances for each alternative are calculated as follows:

$$S_i^+ = \sqrt{\sum_{j=1}^{n} (v_{ij} - v_j^+)^2}, \quad i = 1,\dots,m \tag{8}$$

$$S_i^- = \sqrt{\sum_{j=1}^{n} (v_{ij} - v_j^-)^2}, \quad i = 1,\dots,m \tag{9}$$

Where S_i^+ and S_i^- represents the distance of alternative A_i from the positive and negative ideal solutions, respectively.

Step 5: *The relative closeness to the ideal solution is calculated* : The Relative Closeness (RC) of the i^{th} alternative to the ideal solution is defined as follows:

$$RC_i = \frac{S_i^-}{S_i^+ + S_i^-}, \ i = 1, 2, \dots, m. \ \ 0 \le RC_i \le 1 \tag{10}$$

Step 6: *The alternatives are ranked*: Alternatives must be ranked based on RC_i in which the highest score is the best alternative.

3.3 DIA

The distance to the Ideal Alternative (DIA) algorithm belonging to the MCDM was developed [8] to select dynamically the best network inter-face. It can be seen as an edited version from classical TOPSIS. The philosophy behind DIA is ranking the alternatives based on distances in-between them after taking a snap shot reflecting their locations from positive and negative ideal vectors. The main advantage of DIA is the method's employment of the distances between alternatives and capturing a rank based on the alternatives' relative locations after shifting from the distances to ideal vectors like TOPSIS. This crucial modification done gives a comparative advantage for DIA over TOPSIS as being robust and overcoming the abnormalities of TOPSIS. After decision matrices construction for all decision makers as in Eq. (1), the DIA is identical to the TOPSIS method in the first three steps. In a sense the DIA enhances TOPSIS as shown in steps (4-6). DIA method consists of the following steps for a single decision maker problem [8]:

Steps (1-3): *The same as in TOPSIS method mentioned before in previous subsection*

Step 4: *Calculate the distance to the positive and negative criteria*: as shown in the following two equations

$$D_i^+ = \sqrt{\sum_{j=1}^{n}(v_j^+ - v_{ij})^2}, \; i = 1,2,\dots,m \tag{11}$$

$$D_i^- = \sqrt{\sum_{j=1}^{n}(v_{ij} - v_j^-)^2}, \; i = 1,2,\dots,m \tag{12}$$

Step 5: *Determine the Positive Ideal Alternative*: (PIA) has minimum D^+ and maximum D^-.

$$PIA = \{\min(D_i^+), \max(D_i^-)\}, \quad i = 1,2,\dots,m \tag{13}$$

Step 6: *Calculate the distance of an alternative to the PIA*:

$$R_i = \sqrt{(D_i^+ - \min(D_i^+))^2 + (D_i^- - \max(D_i^-))^2} \tag{14}$$

A set of alternatives is ranked according to the increasing order of R_i

4. Proposed Approach

In the previous section DIA was illustrated for solving MCDM problem with single decision maker. However, the extension of DIA to tackle group decision making problems will differ from single decision maker version. Many aspects should be adopted when introducing a group decision making approach like how the approach will compile different opinions of decision makers, how the proposed approach will compromise different ranks originated. The main reason behind the allocation of DIA to be extended to group decision making is its outperformance to TOPSIS as well as its overcoming the drawback of abnormalities found in classical TOPSIS. Construct the decision matrices for the *k*-decision makers and determine their weights which reflect their priorities. $X^k = (x_{ij}^k)$, $W^k = \left[w_1^k, w_2^k, \dots, w_n^k\right]$ a weight vector for the k^{th} decision maker, $\sum_{j=1}^{n} w_j^k = 1$, for k = 1,2,…,K.

Step 1: *Construct the normalized decision matrix for each decision maker*:

$$x_{ij}^{k*} = \frac{x_{ij}^{k}}{\sqrt{\sum_{i=1}^{m}(x_{ij}^{k})^2}}, \qquad i = 1,\dots,m;\ j = 1,\dots,n;\ k = 1,2,\dots,K \tag{15}$$

Step 2: *Construct the weighted normalized decision matrices for all decision makers*:

$$v_{ij}^{k} = w_{j}^{k} \cdot x_{ij}^{k*}, \quad i = 1,\dots,m; \quad j = 1,\dots,n,\ k = 1,2,\dots,K \tag{16}$$

Step 3: *Determine the positive and negative ideal solutions for all decision makers as following*:

$$A^{k+} = (v_1^{k+}, v_2^{k+}, \dots, v_n^{k+}) = \{\max v_{ij}^{k} \text{ if } j \in \Omega_b\} \text{ or } \{\min v_{ij}^{k} \text{ if } j \in \Omega_c\} \tag{17}$$

$$A^{k-} = (v_1^{k-}, v_2^{k-}, \dots, v_n^{k-}) = \{\min v_{ij}^{k} \text{ if } j \in \Omega_b\} \text{ or } \{\max v_{ij}^{k} \text{ if } j \in \Omega_c\} \tag{18}$$

Where Ω_b is the set of benefit criteria and Ω_c is the set of cost criteria.

$i = 1,\dots,m;\ j = 1,\dots,n,$ and $k = 1,2,\dots,K$

Step 4: *Calculate the distance to the positive and negative criteria for all alternatives under all decision makers*: as shown in the Eqs. (19-22)

$$D_i^{k+} = \sqrt{\sum_{j=1}^{n}(v_j^{k+} - v_{ij}^{k})^2},\ i = 1,2,\dots,m \text{ and } k = 1,2,\dots,K \tag{19}$$

$$D_i^{k-} = \sqrt{\sum_{j=1}^{n}(v_{ij}^{k} - v_j^{k-})^2},\ i = 1,2,\dots,m \text{ and } k = 1,2,\dots,K \tag{20}$$

$$D_i^{+} = \frac{\sum_{k=1}^{K} D_i^{k+}}{k},\ i = 1,2,\dots,m \tag{21}$$

$$D_i^{-} = \frac{\sum_{k=1}^{K} D_i^{k-}}{k},\ i = 1,2,\dots,m \tag{22}$$

Step 5: *Determine the Positive Ideal Alternative for the group*:

$$PIA = \{\min(D_i^{+}), \max(D_i^{-})\},\ i = 1,2,\dots,m \tag{23}$$

Step 6: *Calculate the distance of an alternative to the PIA for the group*:

$$R_i = \sqrt{(D_i^+ - \min (D_i^+))^2 + (D_i^- - \max (D_i^-))^2} \quad (24)$$

A set of alternatives is ranked according to the increasing order of R_i

5. Numerical Example

A numerical example is introduced in this section to stand over the proposed approach. The group involved three experts (decision makers) with four criteria to rank five alternatives. C_1 and C_2 are benefit criteria (the maximum is better) while the other two criteria are cost type (the minimum is better). The first three criteria ratings are subjective; the ratings are allocated by experts (decision makers) while the last criterion (C_4) is measured by fixed rating. As shown in Table 1. Three decision matrices are obtained and put into this augmented table. The allocated criteria weights for each decision maker are found in Table 2.

Table 1. Criteria rating values for the three decision makers

Utility type	Max	Max	Min	Min
	C_1	C_2	C_3	C_4
D1				
Alternative 1	1.1	0.2	8	20
Alternative 2	2	0.3	16	50
Alternative 3	2.5	0.3	10	44
Alternative 4	2	0.53	10	29
Alternative 5	1.9	0.6	15	25
D2				
Alternative 1	0.9	0.3	12	20
Alternative 2	3	0.5	12	50
Alternative 3	1	1.7	10	44
Alternative 4	1.5	0.5	15	29
Alternative 5	1.9	0.6	15	25
D3				
Alternative 1	1.3	0.1	4	20
Alternative 2	1	0.1	20	50
Alternative 3	4	0.43	10	44
Alternative 4	2.5	0.56	5	29
Alternative 5	1.9	0.6	15	25

Table 2. Criteria weights for the three decision makers

	C_1	C_2	C_3	C_4
D1	0.1	0.15	0.25	0.5
D2	0.08	0.18	0.24	0.5
D3	0.12	0.12	0.26	0.5

Using the solution methods illustrated before in previous sections, the resulted scores and ranking orders are shown in Table 3. The rank originated from the three methods are identical based on their different scores and techniques (in DIA the less R score the more preferable).

Table 3. Ranking orders for the full problem

	MOORA		TOPSIS		DIA	
	Z	Rank	RC	Rank	R	Rank
Alternative 1	-0.1417	1	0.73739	1	0	1
Alternative 2	-0.3672	5	0.11151	5	0.318835	5
Alternative 3	-0.2630	4	0.32157	4	0.17148	4
Alternative 4	-0.1428	2	0.71422	2	0.001478	2
Alternative 5	-0.1543	3	0.70211	3	0.01785	3

In this simulation, the ranking abnormality problem is illustrated and the “robustness” of the algorithms to removal of an alternative of the worst alternatives. Alternative 2 (the worst) is removed from the alternatives candidate list. Table 4 presents the overall score of MOORA, the relative closeness to the ideal solution of TOPSIS and the distance to PIA of DIA.

Table 4. Ranking orders after removal of the worst alternative

	MOORA		TOPSIS		DIA	
	Z	Rank	RC	Rank	R	Rank
Alternative 1	-0.1901	1	0.73111	1	0	1
----------------	----------	-----	-----------	----	---------	------
Alternative 3	-0.3542	4	0.25466	4	0.23204	4
Alternative 4	-0.2061	2	0.65269	3	0.02267	2
Alternative 5	-0.2209	3	0.65294	2	0.04347	3

In this situation, the results show that a removal of an alternative causes a change in the ranking order of TOPSIS. The ranking order of MOORA, and DIA remains the same. We continue removing an alternative (i.e. alternative 3) from the alternatives candidate list. The results, in Table 5, show that the ranking order in MOORA and DIA is always stable, but the top ranked alternative in TOPSIS has changed again In Table 3, all methods determine that Alternative 1 is the best. When we remove the worst Alternative (i.e. Alternative 1) out of the candidates list, this does not influence the ranking order of other Alternatives of MOORA and DIA. However, the second best Alternative in TOPSIS changes (i.e. from Alternative 4 to Alternative 5 in Table 4). When another worst Alternative (i.e. Alternative 3) is removed, the second best Alternative in TOPSIS also changes (see Table 5).

Table 5. Ranking orders after removal of the worst two alternatives

	MOORA		TOPSIS		DIA	
	Z	Rank	RC	Rank	R	Rank
Alternative 1	-0.2595	1	0.63465	1	0	1
----------------	----------	-----	-----------	----	---------	------
----------------	----------	-----	-----------	----	---------	------
Alternative 4	-0.2987	2	0.46110	2	0.055482	2
Alternative 5	-0.3065	3	0.46007	3	0.066401	3

The tables' results highlight the ranking abnormality problem of TOPSIS and show that DIA provides a more efficient behavior in this situation.

5. Conclusion

In this paper, a proposed approach based on DIA algorithm is allocated to select the best alternative in a group decision making. The proposed approach extended the advantage of DIA algorithm to explore the group decision making. The solution improves the limitations of the MCDM approach, by producing more robust and efficient ranking orders. The experiments results show that the drawback aforementioned is overcome by using the introduced approach.

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Joint Chance Constrained Programming with Dependent Parameters

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Abstract

In this paper, we consider joint chance constrained programming (JCCP) technique, where two probabilistic constraints are required to jointly satisfy at least a tolerance measure α. We introduce a suggested approach to obtain an equivalent deterministic model for probabilistic model with joint chance constraints (JCC's) when the RHS parameters are dependent random variables, and distributed as (i) single-parameter exponential distributions (S-PED), (ii) two-parameter exponential distributions (T-PED), which is more applicable in most real life applications than the S-PED; since it avoids having its mode at the origin. The joint density function of random RHS parameters is assumed to be the Downton bivariate exponential distribution.

It is shown that the suggested equivalent deterministic model under the assumption of S-PED is a special case of the corresponding equivalent model under the assumption of T-PED when the second parameter is zero. Also the equivalent deterministic model under the assumption of independence between random parameters is a special case of the suggested equivalent deterministic model under the assumption of dependence when the correlation coefficient is zero.

Keywords: Stochastic Programming, probabilistic programming (PP), Joint chance constrained programming (JCCP), Downton bivariate exponential distribution, Non-linear programming, convex model.

1. Introduction

Stochastic Programming approach is one of the Mathematical Programming (MP) approaches where some or all of the model parameters are random variables, taking into account the probability distributions of the random parameters in the underlying problem (Prékopa, 1995). Randomness may exist in the RHS parameter or in some (or all) LHS input coefficients, or in some (or all) objective function coefficients.

Chance constrained programming (CCP) technique is one of the stochastic programming techniques developed by Charnes and Cooper (1959, 1962, 1963) which offers powerful means of modeling stochastic decision problems with assumption that the stochastic constraints will hold with probability at least α, where α refers to the confidence level provided as an appropriate safety margin by the decision-maker and is called tolerance measure. CCP aims to transform the stochastic model into a deterministic one, then solving the deterministic model using a suitable mathematical programming technique. For theoretical background see (Prékopa, 2003).

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Several contributions have been suggested by many researchers assuming different probability distributions for random parameters, for example; the Normal distribution (Charnes and Cooper, 1962; Symonds, 1967; Prekopa, 1974; Jagannathan, 1974; Ackooij et al., 2011; Houda and Lisser, 2015), the Chi-square distribution (Sengupta, 1972; El-Dash, 1984), the Gamma distribution (Lingaraj and Wolfe, 19 74; Atalay and Apaydin, 2011), and the Dirichlet distribution (Gouda and Szántai 2010).

The exponential distribution is our concern in this paper, and it is applicable for a wide class of economic, demographic and reliability models where most of the input coefficients and resources have to be nonnegative, which require distributions with nonnegative range such as demand, supply, prices,…etc (Hafez et al., 2017). Some attempts Gamma diswere introduced in literature to deal with exponentially distributed random parameters (Sengupta, 1972; El-Dash, 1984; Biswal et al., 1998; Hafez et al., 2017). However; they assumed Individual chance constraints (ICC's) with independent parameters where the input coefficients are random; and only Hafez et al. (2017) introduced also the case where the input coefficients are dependent exponential random variables based on the Downton bivariate exponential distribution.

In this paper; we propose an equivalent deterministic model of joint chance constrained (JCC) model where JCC's are two constraints with random RHS parameters assuming that RHS parameters are dependent exponential random variables based on the Downton bivariate exponential distribution.

This paper is organized as follows; Section 2 "Problem definition" presenting the original mathematical model. Section 3 "The Suggested approach" which includes the proposed approach for obtaining the equivalent deterministic model of JCCP models with two joint constraints and the RHS parameters are exponentially distributed based on the two-parameter Downton bivariate exponential distribution. Section 4 illustrates some "Special cases" of the proposed equivalent deterministic model. Section 5 shows a "Numerical example", and finally Section 6 "Conclusion"

2. Problem definition:

Consider the following programming model:

$$Max.(Min.)z = f(x_1, x_2, \dots, x_m) \tag{2.1}$$

S.t.; $$\sum_{j=1}^{m} a_{ij}x_j \le \tilde{b}_i \quad ; i = 1,2 \tag{2.2}$$

$$\sum_{j=1}^{m} a_{ij}x_j \le b_i \quad ; i = 3, \dots, n \tag{2.3}$$

$$x_j \ge 0 \tag{2.4}$$

Where a_{ij} $(i = 1, \dots, n; j = 1, \dots, m)$ and b_i $(i = n' + 1, \dots, n)$ are constants, and $\tilde{b}_i$ $(i = 1,2)$ are continuous random variables with a known joint distribution function, and x_j are decision variables.we refer to the stochastic parameters as $\tilde{b}_i$.

There are two major types of chance constraints: individual chance constraints (ICCs) and joint chance constraints (JCCs). In the former case, we put probabilistic requirements on an individual constraint. In the latter case, probabilistic constraints are required to jointly satisfy at least the tolerance measurement α. That is the solution will strongly depend on how the individual random variables depend on each other, and on the tolerance measure α as well (Miller and Wagner, 1965; Liu, 2002; Prékopa, 2003; King, 2012). Constraints (2.5) represent JCC's

$$P_r\left(\sum a_{ij}x_j \le \tilde{b}_i \quad ; i = 1,2, \dots, n\right) \ge \alpha \tag{2.5}$$

While the ICCs are separately considered as in constraints (2.6).

$$P_r\left(\sum a_{ij}x_j \le \tilde{b}_i\right) \ge \alpha_i \qquad ; i = 1,2,\dots,n \tag{2.6}$$

In this paper; we are concerned with JCCP since JCCP is more preferable on ICCP and appropriate in contexts in which;

i) There may be dependence between random parameters in different constraints, for example; demand and supply are both functions of price which means that there is a dependence relation between quantity demanded and quantity supplied of certain product (Luedtke, 2007).

ii) It is important to have all constraints satisfied with single probability in the model. That is; ICCP constrain the individual probability of constraint violations independently; however, considering these constraints separately neglects to explicitly consider simultaneous violation conditions in the overall system. For example, in reliability systems, specifically in a power flow problem; where the system consists of several nodes and we need to maximize the voltage of nodes. If the ICCP approach is considered, it may be independently guaranteed that the voltage at each node in the system is under its maximum with a high probability, but this could result in a situation where multiple nodes simultaneously have overvoltage conditions. Thus, JCCP is more appropriate in this context to restrict the probability of all voltages, line flows, states of charge, etc. being within prescribed limits throughout the system (Kyri and Bridget, 2017).

Miller and Wagner (1965) are the first to consider the mathematical properties of JCC's of the form of (2.5), where RHS parameters are random and independent. They show that under certain restrictions the model can be viewed as a deterministic nonlinear programming problem, and by assuming that RHS parameters are independent with single-parameter exponential marginal; the equivalent deterministic constraint of the JCC (2.5) could be linear after applying a logarithmic transformation. In this paper; we propose an equivalent deterministic model of JCC model where JCC's are two constraints with random RHS parameters assuming that RHS parameters are dependent and distributed with i) single-parameter exponential distributions, and ii) two-parameter exponential distributions, which is more applicable in most real life applications than the one with single-parameter since it avoids having its mode at the origin.

As for the bivariate exponential distribution, many different forms were presented in literature. Gumbel (1960) presented the first type of bivariate exponential distribution, followed by Freund (1961), then many other forms were presented after that, see (Kotz et al., 2000). In this paper; Downton bivariate exponential distribution, introduced by Downton (1970), is used to model the dependence between $\tilde{b}_i, i = 1,2$ with correlation coefficient $\rho\,; 0 \le \rho < 1$ in a similar fashion as presented by Hafez et al. (2017) for random $\tilde{a}_i$ for both single-parameter and two-parameter exponential distribution.

3. The Suggested approach

In this section, we present the equivalent deterministic model of the probabilistic linear programming model with JCC's under the assumption that random RHS parameters are dependent with $\tilde{b}_1$and $\tilde{b}_2$ (i.e only two joint dependent constraints), where $\tilde{b}_1$and $\tilde{b}_2$ are distributed as Downton bivariate exponential distribution with two-parameter. The approximated Pdf of the Downton bivariate exponential distribution introduced by Hafez et al. (2017) is;

$$f\left(\tilde{b}_1,\tilde{b}_2\right)=\frac{\lambda_1\lambda_2}{(1-\rho)^2}\ exp\left\{\frac{\sum_{i=1}^{2}\lambda_i\gamma_i}{1-\rho}\right\}exp\left\{-\frac{\lambda_1\tilde{b}_1+\lambda_2\tilde{b}_2}{1-\rho}\right\};0\le\gamma_i\le\tilde{b}_i\,;\lambda_i>0\,;0\le\rho<1,\quad i=1,2 \qquad (3.1)$$

where ρ is the correlation between $\tilde{b}_1$and $\tilde{b}_2$.

Theorem (3.1): Let the CCP model with JCC's be of the form of

$$Max.(Min.)\ z=f(x_1,x_2,\dots,x_m) \qquad (3.2)$$

S.t.; $\mathrm{P_r}\left(\sum_{j=1}^{m}a_{ij}x_j\le\tilde{b}_i\quad;i=1,2\right)\ge\alpha \qquad (3.3)$

$$\sum_{j=1}^{m}a_{ij}x_j\le b_i\quad;i=3,\dots,n \qquad (3.4)$$

$$x_j\ge0 \qquad (3.5)$$

where a_{ij} and b_i are constants, and $\tilde{b}_1,\tilde{b}_2$ are continuous random variables and distributed as the Downton bivariate two-parameter exponential distribution with joint density function $f\left(\tilde{b}_1,\tilde{b}_2\right)$ defined in (3.1), then the deterministic equivalent model is:

$$Max.(Min.)\ z=f(x_1,x_2,\dots,x_m) \qquad (3.6)$$

S.t.;

$$\sum_{i=1}^{2}\sum_{j=1}^{m}\lambda_i\,a_{ij}x_j\le\sum_{i=1}^{2}\lambda_i\gamma_i-(1-\rho)\ln\alpha \qquad (3.7)$$

$$\sum_{j=1}^{m}a_{ij}x_j\ge\gamma_i\ ;\ i=1,2 \qquad (3.8)$$

$$\sum_{j=1}^{m}a_{ij}x_j\le b_i\quad;i=3,\dots,n \qquad (3.9)$$

$$x_j\ge0 \qquad (3.10)$$

Proof: Since the LHS of JCC's (3.3) is:

$$P_r\left(\tilde{b}_1\ge\sum_{j=1}^{m}a_{1j}x_j\,,\ \tilde{b}_2\ge\sum_{j=1}^{m}a_{2j}x_j\right)=$$

$$\int_{\sum_{j=1}^{m}a_{2j}x_j}^{\infty}\int_{\sum_{j=1}^{m}a_{1j}x_j}^{\infty}\frac{\lambda_1\lambda_2}{(1-\rho)^2}exp\left\{\frac{\sum_{i=1}^{2}\lambda_i\gamma_i}{1-\rho}\right\}exp\left\{-\frac{\lambda_1\widetilde{b_1}+\lambda_2\widetilde{b_2}}{1-\rho}\right\}\ d\widetilde{b_1}\ d\widetilde{b_2}$$

$$=exp\left\{\frac{\sum_{i=1}^{2}\lambda_i\gamma_i}{1-\rho}\right\}.exp\left\{-\ \frac{\lambda_1\sum_{j=1}^{m}a_{1j}x_j+\lambda_2\sum_{j=1}^{m}a_{2j}x_j}{1-\rho}\right\} \qquad (3.11)$$

Then the equivalent deterministic constraint of the JCC's (3.3) is:

$$exp\left\{\frac{\sum_{i=1}^{2}\lambda_i\gamma_i}{1-\rho}\right\}.exp\left\{-\ \frac{\lambda_1\sum_{j=1}^{m}a_{1j}x_j+\lambda_2\sum_{j=1}^{m}a_{2j}x_j}{1-\rho}\right\}\ge\alpha \qquad (3.12)$$

By applying a logarithmic transformation for constraint (3.12), constraint (3.12) becomes equivalent to:

$$\sum_{i=1}^{2}\sum_{j=1}^{m}\lambda_i\,a_{ij}x_j\le\sum_{i=1}^{2}\lambda_i\gamma_i-(1-\rho)\ln\alpha \qquad (3.13)$$

And the equivalent deterministic model of the probabilistic model (3.2)- (3.5*)* using (3.13) is given by (3.6)-(3.10)

Note that constraints (3.7)-(3.10) are linear constraints; then the deterministic model (3.6)-(3.10) is a convex model if the objective function (3.6*)* is convex function. Therefore; a global optimal solution is guaranteed.

4. Special Cases

1) By setting the $\gamma_i=0$ *;* $i=1,2$ constraints (3.7) and (3.8) are reduced to

$$\sum_{i=1}^{2}\sum_{j=1}^{m}\lambda_i\, a_{ij}x_j \leq -(1-\rho)\ln\alpha \quad (4.1)$$

$$\sum_{j=1}^{m} a_{ij}x_j \geq 0\ ;\ i = 1,2 \quad (4.2)$$

which is the case when $\tilde{b}_1$ and $\tilde{b}_2$ are distributed as Downton bivariate single-parameter exponential distribution with pdf defined as (Hafez et al., 2017)

$$f(\widetilde{b_1},\widetilde{b_2}) = \frac{\lambda_1\lambda_2}{(1-\rho)^2}\, exp\left\{-\frac{\lambda_1\widetilde{b_1}+\lambda_2\widetilde{b_2}}{1-\rho}\right\};\ \ \widetilde{b_\iota} \geq 0\ ;\lambda_1,\lambda_2 > 0\ ;$$
$$0 \leq \rho < 1, i = 1,2 \quad (4.3)$$

2) by setting $\boldsymbol{\rho} = \mathbf{0}$, constraint (3.7) is reduced to

$$\sum_{i=1}^{2}\sum_{j=1}^{m}\lambda_i\, a_{ij}x_j \leq \sum_{i=1}^{2}\lambda_i\gamma_i - \ln\alpha \quad (4.4)$$

which is the case under the assumption that $\tilde{b}_1$ and $\tilde{b}_2$ are independent and distributed as two-parameter exponential distributions. Similarly, constraint (4.1) is reduced to constraint

$$\sum_{i=1}^{2}\sum_{j=1}^{m}\lambda_i\, a_{ij}x_j \leq -\ln\alpha \quad (4.5)$$

The above case is under the assumption that $\tilde{b}_1$ and $\tilde{b}_2$ are independent and distributed as single-parameter exponential distributions (Hafez et al., 2017).

5. Numerical Example

In this section, we demonestrate the suggested approach for obtaining an equivalent deterministic model of JCCP models presented in section 3 throught the following example.

Consider the following linear programming model

$$Max\ Z = 5x_1 + 6\,x_2 \quad (5.1)$$

S.t.;

$$P(\ 5x_1 + 4x_2 \leq \tilde{b}_1\ ;\ 10x_1 + 2x_2 \leq \tilde{b}_2) \geq 0.90 \quad (5.2)$$

$$2x_1 + x_2 \leq 5 \quad (5.3)$$

$$x_1, x_2 \geq 0 \quad (5.4)$$

Where $\tilde{b}_1, \tilde{b}_2$ are distributed as the Downton bivariate two-parameter exponential distribution defined in (3.1), where $\lambda_1 = 0.5$, $\lambda_2 = 0.3$, $\gamma_1 = 5, \gamma_2 = 7$ and $\rho = 0.8$. Then the equivalent deterministic model of the JCCP model (5.1)-(5.4) could be obtained using theorem (3.1) as follows:

$$Max.\ Z = 5x_1 + 6\,x_2 \quad (5.5)$$

S.t.;

$$27.5\,x_1 + 13\,x_2 \leq 23.021 \quad (5.6)$$

$$5x_1 + 4x_2 \geq 5 \quad (5.7)$$

$$10x_1 + 2x_2 \geq 7 \quad (5.8)$$

$$2x_1 + x_2 \leq 5 \quad (5.9)$$

$$x_1, x_2 \geq 0 \quad (5.10)$$

Model (5.5)-(5.10) is a linear model and the optimal solution is obtained using the Simplex method as $z^* = 6.013, x_1^* = 0.599, x_2^* = 0.503$.

6 Conclusion

In this paper, we introduced the equivalent deterministic model of JCCP model when the RHS parameters are random variables and dependent. We focused on the exponential distribution since previous researches are limited in case of non-negative random parameters. We assumed that the JCC's consists of two constraints and we introduced a suggested approach to obtain the equivalent deterministic constraint of such joint constraints assuming that $\tilde{b}_1, \tilde{b}_2$ are distributed as bivariate Downton exponential distribution with

i) Two-parameters (through theorem (3.1)) and
ii) Single-parameter (as a special case of theorem (3.1)).

By setting the second parameter $\gamma_i = 0, i = 1,2$; It was shown that the equivalent deterministic model of theorem (3.1) for Downton T-PED could be reduced to the equivalent deterministic model for Downton S-PED. And for both deterministic models we obtain a linear constraint replacing the JCC's, which is convex, so that a global optimal solution could be obtained when the objective function is convex.

By setting the correlation coefficient $\rho = 0$; It was shown that the equivalent deterministic model of theorem (3.1) for dependent Downton T-PED or S-PED could be reduced to the equivalent deterministic model for independent two-parameter or single-parameter exponential random parameters, respectively.

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