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A Linear Programming Method for Solving Rough Interval Transportation Problem

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Abstract:

A rough interval linear programming (RILP) method is developed for transportation model. The concept of ‘‘rough interval’’ is introduced in the modeling framework to represent rough parameters. In transportation problem the cost of transportation from source to destination may depends on many factors which may not be deterministic in nature. In this paper, the unit cost of transportation, source and destination are considered to be rough interval .A numerical examples given for illustration.

Keywords: Transportation problem, Rough interval, Fuzzy programming, rough optimal solution.

1. Introduction:

Transportation problem is considered to be special case of linear programming problems. Many efficient methods have been developed to solve a transportation problem. In many practical statuses, the cost of transportation may not deterministic in nature. The cost depends on many factor such as road condition, traffic load causing delay in delivery, break down of vehicles, labor problem for loading and unloading which may increase the cost. All these factors are uncertain which cannot be predicted beforehand due to insufficient information, unpredictable environment. These types of cost can be represents as a rough interval.

In order to solve such type of transportation problem three types of solution methods are presented in this paper ,such as a linear programming method, rough optimal and fuzzy programming approach [6].

. 2-Preliminaries

2.1-Rough Interval :

Definition 1: The rough interval can be expressed as follows:

$$RI = ([LAI]:[UAI]) = ([a,b]:[c,d]) \quad where \quad (c \le a \le b \le d)$$

A rough interval Y^R is defined as an interval with known lower

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approximation interval [LAI] and upper approximation interval [UAI] bounds but known imprecise for y: where
$Y^{RI} = [Y^L : Y^U]$ where $Y^L = [y^a, y^b]$ and $Y^U = [y^c, y^d]$ are lower and upper approximation intervals of Y^R, respectively . Y^L and Y^U are conventional intervals and $Y^L \subseteq Y^U$

Definition 2: Let $* \in \{+, -, \times, \div\}$ be a binary operation on rough interval Y^R and Z^R when Y^R , $Z^R \geq 0$ we have :

Y^L , Y^U , Z^L and Z^U are conventional intervals, the above operations can be further transferred to the following functions if letting $Y^L = [Y^a, Y^b]$, $Y^U = [Y^c, Y^d]$, $Z^L = [Z^a, Z^b]$, $Z^U = [Z^c, Z^d]$ where Y^a, Y^b, Y^c, Y^d, Z^a, Z^b, Z^c and Z^d are deterministic numbers denoting the lower and upper bounds of Y^L , Y^U , Z^L and Z^U

$$Y^R + Z^R = ([Y^a + Z^a, Y^b + Z^b] : [Y^c + Z^c, Y^d + Z^d])$$
$$Y^R - Z^R = ([Y^a - Z^a, Y^b - Z^b] : [Y^c - Z^c, Y^d - Z^d])$$
$$Y^R \times Z^R = ([Y^a \times Z^a, Y^b \times Z^b] : [Y^c \times Z^c, Y^d \times Z^d])$$
$$Y^R \div Z^R = ([Y^a \div Z^a, Y^b \div Z^b] : [Y^c \div Z^c, Y^d \div Z^d])$$

where Z^a, Z^b, Z^c and $Z^d \neq 0$ in $\{\div\}$binary operation.

3. Rough interval linear programming

Prior to proposing rough interval linear programming, a set of definitions for rough intervals need to be provided. Generally, a rough interval comprises two parts: an upper approximation interval [UAI] and a lower approximation interval [LAI]. The variable is assumed to be inside of its UAI and cannot get the qualitative value outside of this interval. [4].

Definition 3: Let A^{RI} and B^{RI} denote two sets of rough intervals .A rough interval linear programming model can be formulated as follows:

$$\begin{aligned} &\max \quad f^{RI} = C^{RI} \cdot X \\ &subject\ \ to\ \ A^{RI} \cdot X \leq B^{RI} \qquad (1) \\ &\qquad\qquad X \geq 0 \end{aligned}$$

Where $A^{RI} \in R^{RI\,(m \times n)}$, $B^{RI} \in R^{RI(m \times 1)}$, $C \in \{\Pi\}^R$ and Π denotes a set of deterministic parameters , X^{RI} denotes a set decision variables, f^{RI} denotes objective function value.

4. Transportation problem:

4.1. The crisp model of transportation:

Suppose that [8]there are *m* sources and *n* destinations. Let ai be the number of supply units available at sources *i (i=1,2,3….m)* and let *bj* the number of demands units required at destination j *(j=1,2,3….n)*. Let *cij* represents the unit transportation cost for transportation the units from source *i* to destination *j*. The target is to determine the number of units to be transported from source *i* to destination *j*, so that the total transportation cost is minimum. Let x_{ij} be the decision variable which denotes the number of units shipped from sources *i* to destination *j*.

$$
\begin{aligned}
& Minimiza \quad f = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}\, x_{ij} \\
& subject\ to \quad \sum_{j=1}^{n} x_{ij} = a_i \quad , \ i = 1,2,\text{........m} \\
& \qquad \sum_{i=1}^{m} x_{ij} = b_j \quad , \ j = 1,2,\text{........n} \\
& \qquad \sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j \\
& \qquad x_{ij} \ \geq \ 0
\end{aligned} \tag{2}
$$

4.2. The interval model of transportation problem:

The formulation[6] of interval transportation problem of minimizing interval cost of objective function under constrains of interval sources and interval demands:

$$
\begin{aligned}
& Minimiza \quad f = \sum_{i=1}^{m}\sum_{j=1}^{n} [c_{ij}^{L}, c_{ij}^{R}]\, x_{ij} \\
& subject\ to \quad \sum_{j=1}^{n} x_{ij} \in [a_i^{L}\ ,\ a_i^{R}] \ , \ i = 1,2,\text{........m} \\
& \qquad \sum_{i=1}^{m} x_{ij} \in [b_j^{L}\ ,\ b_j^{R}] \ , \ j = 1,2,\text{........n} \\
& \qquad \sum_{i=1}^{m} a_i^{L} = \sum_{j=1}^{n} b_j^{L} \quad and \quad \sum_{i=1}^{m} a_i^{R} = \sum_{j=1}^{n} b_j^{R} \\
& \qquad x_{ij} \ \geq \ 0
\end{aligned} \tag{3}
$$

4.3. The rough interval model of transportation problem:

$$
\begin{aligned}
&Min \quad f^{RI} = \sum_{i=1}^{m}\sum_{j=1}^{n} c^{RI}{}_{ij}\, x_{ij} \quad , \mathrm{RI} = a, \mathrm{b}, \mathrm{c}, \mathrm{d} \\
&subject\ to \qquad \sum_{j=1}^{n} x_{ij} \in a^{RI}{}_{i} \quad , \ i = 1, 2, \ldots\ldots. \mathrm{m} \\
&\qquad\qquad\qquad \sum_{i=1}^{m} x_{ij} \in b^{RI}{}_{j} \quad , \ j = 1, 2, \ldots\ldots. \mathrm{n} \\
&\qquad\qquad\qquad \sum_{i=1}^{m} a_i^{RI} = \sum_{j=1}^{n} b_j^{RI} \\
&\qquad\qquad\qquad x_{ij} \ \geq \ 0
\end{aligned}
\tag{4}
$$

The cost c_{ij} is a rough interval variable of the form let

$$C_{ij}^{RI} = ([c_{ij}^{a}, c_{ij}^{b}]:[c_{ij}^{c}, c_{ij}^{d}]) \text{ where } c_{ij}^{c} \leq c_{ij}^{a} \leq c_{ij}^{b} \leq c_{ij}^{d}$$

The supply a_i is a rough interval $a_i^{RI} = ([a_i^a, a_i^b]:[a_i^c, a_i^d])$

The demand b_j is rough interval $b_j^{RI} = ([b_j^a, b_j^b]:[b_j^c, b_j^d])$ and the objective function becomes a rough variable of the form $f^{RI} = ([f^a, f^b]:[f^c, f^d])$

5. Solving rough interval transportation problem:

5.1 Traditional method

Divide the model to four sub-model *A*, *B*,*C* and *D* and solve them separating of each other. The results corresponding to the model is f^a , f^b, f^c, f^d and X^a , X^b, X^c, and X^d, respectively[8].

5.2 Rough optimality method:

The steps of rough optimality method as follows:

Step 1: Formulate the problem

Step 2: Solve the second -desired upper-bound sub-model UAI[d] with Vogel method and get solutions of X_{opt}^{d} *and* f_{opt}^{d}

Step 3: Solve the first lower sub-model UAI[c], and add constraint $X_{ij} \leq X_{opt}^{d}$ to get the solution of X_{opt}^{c} *and* f_{opt}^{c}

Step 4: Solve the second upper sub-model LAI [b], and add constraint $X_{ij} \geq X_{opt}^{c}$ to get the solution of X_{opt}^{b} *and* f_{opt}^{b}

Step 5: Solve the first lower sub-model LAI[a], and add constraint $X_{ij} \geq X_{opt}^{c}$ and $X_{ij} \leq X_{opt}^{b}$ to get the solution of

X^{a}_{opt} and f^{a}_{opt}

Step 6: Incorporate the solutions of all sub-models to obtain primal solutions as follows[4]:

$$f^{RI} = ([f^{a}_{opt}, f^{b}_{opt}]:[f^{c}_{opt}, f^{d}_{opt}])$$

$$X^{RI} = ([X^{a}_{opt}, X^{b}_{opt}]:[X^{c}_{opt}, X^{d}_{opt}])$$

5.3-Fuzzy programming approach:

In fuzzy programming approach, we convert the interval model to multiobjective model in the first. find the lower bound as LF_R and the upper bound as UF_R for the r th objective function .

$$Min \quad f_R(x) = \sum_{i=1}^{m}\sum_{j=1}^{n} Cc_{ij} x_{ij} + \sum_{i=1}^{m}\sum_{j=1}^{n} Cw_{ij} x_{ij}$$

$$Min \quad f_c(x) = \sum_{i=1}^{m}\sum_{j=1}^{n} Cc_{ij} x_{ij}$$

subject to

$$\sum_{j=1}^{n} X_{ij} \in [a_L, a_U], \quad \mathrm{i} = 1,2,3.....m \tag{5}$$

$$\sum_{i=1}^{m} X_{ij} \in [\mathrm{b}_L, b_U], \quad j = 1,2,3.....\mathrm{n}$$

$$\sum_{i=1}^{m} a_L = \sum_{j=1}^{n} b_L$$

$$\sum_{i=1}^{m} a_R = \sum_{j=1}^{n} b_R$$

$$x_{ij} \geq 0 \quad \forall i, \forall j ,$$

$$Where \; f_R = (w + c)\,\mathrm{X}_{ij} \quad , \quad f_C = (c) X_{ij} \quad , \mathrm{W} = (\mathrm{b} - \mathrm{a}) \div 2 \quad and \;\; C = (\mathrm{b} + \mathrm{a}) \div 2.$$

When the aspiration levels for each of the objective have been specified, a fuzzy model is formed and then the fuzzy model is converted into a crisp model.

The solution of Model 5 can be obtained by the following steps:

step 1. Solve the Model 5 as a single-objective transportation problem 2 times by taking one of the objective at a time.

step 2. From the above results, determine the corresponding values for objective at each solution derived. According to each solution and value for every objective, we can find a pay-off matrix,

step 3. From step 2, find for each objective the Ur and the Lr corresponding to the set of solutions,

step 4. Define a membership function $\psi_1(f_R)$, $\psi_2(f_c)$, for the f_R, f_c respectively.

$$\psi_1(f_R) = \begin{cases} 1 & if \quad f_R \le Lf_R \\ 1 - \dfrac{f_R - Lf_R}{Uf_R - Lf_R} & if \quad Lf_R \le f_R \le Uf_R \\ 0 & if \quad Uf_R \le f_R \end{cases} \tag{6}$$

$$\psi_2(f_C) = \begin{cases} 1 & if \quad f_C \le Lf_C \\ 1 - \dfrac{f_C - Lf_C}{Uf_C - Lf_C} & if \quad Lf_C \le f_C \le Uf_C \\ 0 & if \quad Uf_C \le f_C \end{cases}$$

step 5. Convert the fuzzy model of the problem, obtained in Step 3, into the following crisp model

$$\begin{aligned} & Max \quad \psi; \\ & subject\ to \quad f^a + \psi(\mathrm{U}f^a - Lf^a) \le \mathrm{U}f^a \\ & \qquad f^b + \psi(\mathrm{U}f^b - Lf^b) \le \mathrm{U}f^b \\ & \qquad \sum_{j=1}^{n} X_{ij} = [a_L, a_U], \quad \mathrm{i} = 1, 2, 3.....m \\ & \qquad \sum_{i=1}^{m} X_{ij} = [\mathrm{b}_L, b_U], \quad j = 1, 2, 3.....\mathrm{n} \\ & \qquad \sum_{i=1}^{m} a_L = \sum_{j=1}^{n} b_L \\ & \qquad \sum_{i=1}^{m} a_R = \sum_{j=1}^{n} b_R \\ & \qquad x_{ij} \ge 0 \quad \forall i, \forall j, \psi \ge 0 \end{aligned} \tag{7}$$

step 6. Solve the crisp model by an appropriate mathematical ogramming algorithm.

step 7. The solution obtained in step 6 will be optimal compromise solution of the Model 5

6- Illustration example

Example 1:

	B1	B2	B3	B4	S
A1	([6,7]:[5,9])	([6,7]:[5,9])	([5,7]:[4,8])	([6,7]:[5,9])	([7,9]:[6,10])
A2	([9,11]:[8,12])	([15,17]:[14,18])	([16,17]:[14,19])	([18,21]:[17,22])	([17,21]:[16,22])
A3	([1,13]:[10,14])	([3,5]:[2,6])	([9,11]:[8,12])	([3,4]:[1,5])	([16,18]:[14,20])
D	([10,12]:[9,13])	([2,4]:[1,5])	([13,15]:[12,16])	([15,17]:[14,18])	([40,48]:[36,52])

Consider a problem with three sources (i= 1,2,3) and four destination (j=1,2,3,4). The unit of transportation cost are rough interval, the availability at each sources, demand of each destination are rough interval

The problem is solved by three methods, applying Lingo package reaching to the final results.

6.1-Traditional method:

A					
	B1	B2	B3	B4	S
A1	6	6	5	6	7
A2	9	15	16	18	17
A3	11	3	9	3	16
D	10	2	13	15	40

f^a **= 284**

X^a = (0,0,7,0,10,1,6,0,0,1,0,15)

B					
	B1	B2	B3	B4	S
A1	7	7	7	7	9
A2	11	17	17	21	21
A3	13	5	11	4	18
D	12	4	15	17	48

f^b **= 421**

X^b = (0,3,6,0,12,0,9,0,0,1,0,17)

C					
	B1	B2	B3	B4	S
A1	5	5	4	5	6
A2	8	14	14	17	16
A3	10	2	8	1	14
D	9	1	12	14	36

f^c = 208

X^c = (0,0,6,0,9,1,6,0,0,0,0,14)

D					
	B1	B2	B3	B4	S
A1	9	9	8	9	10
A2	12	18	19	22	22
A3	14	6	12	5	20
D	13	5	16	18	52

f^d = 506

X^d = (0,0,10,0,13,3,6,0,0,2,0,18)

f^{RI}=([284,421]:[208,506]),

X_{12}([0,0]:[0,3]), X_{13}=([6,7]:[6,10]) X_{21}=([10,12]:[9,13]), X_{22}=([1,1]:[0,3]), X_{23}=([6,6]:[6,9]) ,X_{32} =([1,1]:[0,2]), X_{34}=([15,17]:[14,18])

6.2-Rough optimality method:

Step 1: divide the original model into LAI [a ,b] and UAI [c ,d]

LAI[a,b]					
	B1	**B2**	**B3**	**B4**	**S**
A1	([6,7])	([6,7])	([5,7])	([6,7])	([7,9])
A2	([9,11])	([15,17])	([16,17])	([18,21])	([17,21])
A3	([11,13])	([3,5])	([9,11])	([3,4])	([16,18])
D	([10,12])	([2,4])	([13,15])	([15,17])	([40,48])

UAI[c,d]					
	B1	**B2**	**B3**	**B4**	**S**
A1	([5,9])	([5,9])	([4,8])	([5,9])	([6,10])
A2	([8,12])	([14,18])	([14,19])	([17,22])	([16,22])
A3	([10,14])	([2,6])	([8,12])	([1,5])	([14,20])
D	([9,13])	([1,5])	([12,16])	([14,18])	([36,52])

Then, classifying each one into sub-model according to the lower and upper approximation interval, LAI[a] LAI[b],and UAI[c] UAI[d] respectively

LAI

A					
	B1	**B2**	**B3**	**B4**	**S**
A1	6	6	5	6	7
A2	9	15	16	18	17
A3	11	3	9	3	16
D	10	2	13	15	40

B					
	B1	**B2**	**B3**	**B4**	**S**
A1	7	7	7	7	9
A2	11	17	17	21	21
A3	13	5	11	4	18
D	12	4	15	17	48

UAI

C					
	B1	**B2**	**B3**	**B4**	**S**
A1	5	5	4	5	6
A2	8	14	14	17	16
A3	10	2	8	1	14
D	9	1	12	14	36

D					
	B1	**B2**	**B3**	**B4**	**S**
A1	9	9	8	9	10
A2	12	18	19	22	22
A3	14	6	12	5	20
D	13	5	16	18	52

Step 2 : solve UAI[d] then obtaining the solution of X^d_{opt} *and* f^d_{opt}

X^d = (0,0,10,0,13,3,6,0,0,2,0,18) f^d = 506

Step 3 : Solve UAI[c] after adding the following constraints

X_{13} <=10, X_{21} <=13 , X_{22} <=3, X_{23} <=6, X_{32} <=2 , X_{34} <=18

Reaching to the following solution of X^c_{opt} *and* f^c_{opt}

X^c = (0,0,6,0,9,1,6,0,0,0,0,14) f^c = 208

Step 4: Solve LAI[b] add the following constraints

X_{13} >=6 , X_{21} >=9 , X_{22} >=1 , X_{23} >=6 , X_{34} >=14

Reaching to the following solution of X^{b}_{opt} *and* f^{b}_{opt}

X^b = (0,0,9,0,12,3,6,0,0,1,0,17) f^b = 421

Step 5: Solve LAI[a] and adding the following constraints

X_{13} >=6 , X_{21} >=9 , X_{22} >=1 , X_{23} >=6 , X_{34} >=14 and X_{13}<=9,

X_{21} <=12 , X_{22} <=3 , X_{23} <6 , X_{32} <=1, X_{34} <=17

Reaching to the following solution of X^{a}_{opt} *and* f^{a}_{opt},

X^a = (0,0,7,0,10,1,6,0,0,1,0,15) f^a = 284

Step 6 : Incorporate the solution of all sub-models to obtain primal solution as follows

f^{RI}=([284,421]:[208,506]),

X_{13}=([7,9]:[6,10]) X_{21}=([10,12]:[9,13]),

X_{22}=([1,3]:[1,3]) X_{23}=([6,6]:[6,6]), X_{32}=([1,1]:[0,2]),

X_{34}=([15,17]:[14,18])

6.3-Fuzzy programming approach:

6.3.1-Fuzzy programming approach for each interval:

Divide the original model into LAI [a ,b] and UAI [c ,d]. convert the sub-interval model into multiobjective model

$$Min \quad f_R(x) = \sum_{i=1}^{m}\sum_{j=1}^{n} Cc_{ij} x_{ij} + \sum_{i=1}^{m}\sum_{j=1}^{n} Cw_{ij} x_{ij}$$

$$Min \quad f_c(x) = \sum_{i=1}^{m}\sum_{j=1}^{n} Cc_{ij} x_{ij}$$

subject to

$$\sum_{j=1}^{n} X_{ij} \in [a_L, a_U], \quad \mathrm{i} = 1,2,3.....m$$

$$\sum_{i=1}^{m} X_{ij} \in [\mathrm{b}_L, b_U], \quad j = 1,2,3.....\mathrm{n}$$

$$\sum_{i=1}^{m} a_L = \sum_{j=1}^{n} b_L$$

$$\sum_{i=1}^{m} a_R = \sum_{j=1}^{n} b_R$$

$$x_{ij} \geq 0 \quad \forall i, \forall j ,$$

$$Where \; f_R = (w + c)\mathrm{X}_{ij} \quad , \; f_C = (c) X_{ij} \quad , \mathrm{W} = (\mathrm{b} - \mathrm{a}) \div 2 \quad and \; C = (\mathrm{b} + \mathrm{a}) \div 2.$$

$$
\begin{aligned}
&Min \quad f_R(x) = 7x_{11} + 7x_{12} + 7x_{13} + 7x_{14} + 11x_{21} + 17x_{22} \\
&\qquad\qquad + 17x_{23} + 21x_{24} + 13x_{31} + 5x_{32} + 11x_{33} + 4x_{34} \\
&Min \quad f_c(x) = 6.5x_{11} + 6.5x_{12} + 6x_{13} + 6.5x_{14} + 10x_{21} + 16x \\
&\qquad\qquad + 16.5x_{23} + 19.5x_{24} + 12x_{31} + 4x_{32} + 10x_{33} + 3.5
\end{aligned}
$$

$$
\begin{aligned}
&subject\ to \\
&\qquad x_{11} + x_{12} + x_{13} + x_{14} \le 9 \\
&\qquad x_{11} + x_{12} + x_{13} + x_{14} \ge 7 \\
&\qquad x_{21} + x_{22} + x_{23} + x_{24} \le 21 \\
&\qquad x_{21} + x_{22} + x_{23} + x_{24} \ge 17 \\
&\qquad x_{31} + x_{32} + x_{33} + x_{34} \le 18 \\
&\qquad x_{31} + x_{32} + x_{33} + x_{34} \ge 16 \\
&\qquad x_{11} + x_{21} + x_{31} \le 12 \\
&\qquad x_{11} + x_{21} + x_{31} \ge 10 \\
&\qquad x_{12} + x_{22} + x_{32} \le 4 \\
&\qquad x_{12} + x_{22} + x_{32} \ge 2 \\
&\qquad x_{13} + x_{23} + x_{33} \le 15 \\
&\qquad x_{13} + x_{23} + x_{33} \ge 13 \\
&\qquad x_{14} + x_{24} + x_{34} \le 17 \\
&\qquad x_{14} + x_{24} + x_{34} \ge 15 \\
&\qquad x_{ij} \ge 0 \quad \forall i, \forall j \ ,
\end{aligned}
$$

$$
\begin{aligned}
&Max \quad \eta; \\
&subject\ to \ \ f^a + \eta(\mathrm{U}f^a - Lf^a) \le \mathrm{U}f^a \\
&\qquad\qquad f^b + \eta(\mathrm{U}f^b - Lf^b) \le \mathrm{U}f^b \\
&\qquad\qquad \sum_{j=1}^{n} X_{ij} \in [a_L, a_U], \quad \mathrm{i} = 1,2,3.....m \\
&\qquad\qquad \sum_{i=1}^{m} X_{ij} \in [\mathrm{b}_L, b_U], \quad j = 1,2,3.....\mathrm{n} \\
&\qquad\qquad \sum_{i=1}^{m} a_L = \sum_{j=1}^{n} b_L \\
&\qquad\qquad \sum_{i=1}^{m} a_R = \sum_{j=1}^{n} b_R \\
&\qquad\qquad x_{ij} \ge 0 \quad \forall i, \forall j \ , \eta \ge 0
\end{aligned}
$$

$$
\begin{aligned}
&\max \quad \eta \\
&subject\ to \\
&7x_{11} + 7x_{12} + 7x_{13} + 7x_{14} + 11x_{21} + 17x_{22} + 17x_{23} + 21x_{24} \\
&+13x_{31} + 5x_{32} + 11x_{33} + 4x_{34} + 45\eta \le 387 \\
&6.5x_{11} + 6.5x_{12} + 6x_{13} + 6.5x_{14} + 10x_{21} + 16x_{22} + 16.5x_{23} + 19.5x_{24} \\
&+12x_{31} + 4x_{32} + 10x_{33} + 3.5x_{34} + 53.5\eta \le 364.5 \\
&x_{11} + x_{12} + x_{13} + x_{14} \le 9 \\
&x_{11} + x_{12} + x_{13} + x_{14} \ge 7 \\
&x_{21} + x_{22} + x_{23} + x_{24} \le 21 \\
&x_{21} + x_{22} + x_{23} + x_{24} \ge 17 \\
&x_{31} + x_{32} + x_{33} + x_{34} \le 18 \\
&x_{31} + x_{32} + x_{33} + x_{34} \ge 16 \\
&x_{11} + x_{21} + x_{31} \le 12 \\
&x_{11} + x_{21} + x_{31} \ge 10 \\
&x_{12} + x_{22} + x_{32} \le 4 \\
&x_{12} + x_{22} + x_{32} \ge 2 \\
&x_{13} + x_{23} + x_{33} \le 15 \\
&x_{13} + x_{23} + x_{33} \ge 13 \\
&x_{14} + x_{24} + x_{34} \le 17 \\
&x_{14} + x_{24} + x_{34} \ge 15 \\
&x_{ij} \ge 0 \quad \forall i, \forall j \ , and \quad \eta \ge 0
\end{aligned}
$$

[a,b]					
	B1	B2	B3	B4	S
A1	[6,7]	[6,7]	[5,7]	[6,7]	[7,9]
A2	[9,11]	[15,17]	[[16,17]	[18,21]	[17,21]
A3	[11,13]	[3,5]	[9,11]	[3,4]	[16,18]
D	[10,12]	[2,4]	[13,15]	[15,17]	[40,48]

The solution
X= (0,0,3,0,7,0,6,0,0,1,0,14)
f^a =310
f^b =358

[c,d]					
	B1	B2	B3	B4	S
A1	[5,9]	[5,9]	[4,8]	[5,9]	[6,10]
A2	[[8,12]	[14,18]	[14,19]	[17,22]	[16,22]
A3	[10,14]	[2,6]	[8,12]	[1,5]	[14,20]
D	[9,13]	[1,5]	[12,16]	[14,18]	[36,52]

X = (3,1,6,0,9,1,10,0,0,0,0,15)
f^c =208
f^d =361

f^{RI}=([310,358]:[208,361]),

X_{11}=[0,3] , X_{12}=[0,1] X_{13}=[3,6] , X_{21}=[7,9], X_{22}=[0,1], X_{23}=[6,10], X_{32}=[0,1], X_{33}=[0,0], X_{34}=[14,15]

6.3.2-Fuzzy programming approach for rough interval:

Divide the original model into LAI [a ,b] and UAI [c ,d]

convert the 2 sub-interval model into one multiobjective model

$$Min \quad f_{LR}(x) = \sum_{i=1}^{m}\sum_{j=1}^{n} CcL_{ij}x_{ij} + \sum_{i=1}^{m}\sum_{j=1}^{n} CwL_{ij}x_{ij}$$

$$Min \quad f_{Lc}(x) = \sum_{i=1}^{m}\sum_{j=1}^{n} CcL_{ij}x_{ij}$$

$$Min \quad f_{UR}(x) = \sum_{i=1}^{m}\sum_{j=1}^{n} CcU_{ij}x_{ij} + \sum_{i=1}^{m}\sum_{j=1}^{n} CwU_{ij}x_{ij}$$

$$Min \quad f_{Uc}(x) = \sum_{i=1}^{m}\sum_{j=1}^{n} CcU_{ij}x_{ij}$$

subject to

$$\sum_{j=1}^{n} X_{ij} \in ([a_i^a, a_i^b] : [a_i^c, a_i^d]) \ , \quad i = 1,2,3\ldots m$$

$$\sum_{i=1}^{m} X_{ij} \in ([b_j^a, b_j^b] : [b_j^c, b_j^d]) \ , \quad j = 1,2,3\ldots n$$

$$\sum_{i=1}^{m} a_i^{RI} = \sum_{j=1}^{n} b_j^{RI}$$

$$x_{ij} \geq 0 \quad \forall i, \forall j \ ,$$

$$Where \ f_{RL} = (wL + cL)X_{ij} \quad , \ f_{CL} = (cL)X_{ij} \ , WL = (b-a) \div 2 \quad and \ CL = (b+a) \div 2.$$

$$Where \ f_{RU} = (wU + cU)X_{ij} \quad , \ f_{CU} \ (cU)X_{ij} \ , WU = (d-c) \div 2 \quad and \ CU = (d+c) \div 2$$

	B1	B2	B3	B4	S
A1	[[6,7]:[5,9]]	[[6,7]:[5,9]]	[[5,7]:[4,8]]	[[6,7]:[5,9]]	[[7,9]:[6,10]]
A2	[[9,11]:[8,12]]	[[15,17]:[14,18]]	[[16,17]:[14,19]]	[[18,21]:[17,22]]	[[17,21]:[16,22]]
A3	[[11,13]:[10,14]]	[[3,5]:[2,6]]	[[9,11]:[8,12]]	[[3,4]:[1,5]]	[[16,18]:[14,20]]
D	[[10,12]:[9,13]]	[[2,4]:[1,5]]	[[13,15]:[12,16]]	[[15,17]:[14,18]]	[[40,48]:[36,52]]

$$
\begin{aligned}
Min \quad f_{RL}(x) &= 7x_{11} + 7x_{12} + 7x_{13} + 7x_{14} + 11x_{21} + 17x_{22} \\
&\quad + 17x_{23} + 21x_{24} + 13x_{31} + 5x_{32} + 11x_{33} + 4x_{34} \\
Min \quad f_{CL}(x) &= 6.5x_{11} + 6.5x_{12} + 6x_{13} + 6.5x_{14} + 10x_{21} + 16x_{22} \\
&\quad + 16.5x_{23} + 19.5x_{24} + 12x_{31} + 4x_{32} + 10x_{33} + 3.5x_{34} \\
Min \quad f_{RU}(x) &= 9x_{11} + 9x_{12} + 8x_{13} + 9x_{14} + 12x_{21} + 18x_{22} \\
&\quad + 19x_{23} + 22x_{24} + 14x_{31} + 6x_{32} + 12x_{33} + 5x_{34} \\
Min \quad f_{CU}(x) &= 7x_{11} + 7x_{12} + 6x_{13} + 7x_{14} + 10x_{21} + 16x_{22} \\
&\quad + 16.5x_{23} + 19.5x_{24} + 12x_{31} + 4x_{32} + 10x_{33} + 3x_{34}
\end{aligned}
$$

subject to

$$
\begin{aligned}
& x_{11} + x_{12} + x_{13} + x_{14} \le 10 \\
& x_{11} + x_{12} + x_{13} + x_{14} \ge 6 \\
& x_{21} + x_{22} + x_{23} + x_{24} \le 22 \\
& x_{21} + x_{22} + x_{23} + x_{24} \ge 16 \\
& x_{31} + x_{32} + x_{33} + x_{34} \le 20 \\
& x_{31} + x_{32} + x_{33} + x_{34} \ge 14 \\
& x_{11} + x_{21} + x_{31} \le 13 \\
& x_{11} + x_{21} + x_{31} \ge 9 \\
& x_{12} + x_{22} + x_{32} \le 5 \\
& x_{12} + x_{22} + x_{32} \ge 1 \\
& x_{13} + x_{23} + x_{33} \le 16 \\
& x_{13} + x_{23} + x_{33} \ge 12 \\
& x_{14} + x_{24} + x_{34} \le 18 \\
& x_{14} + x_{24} + x_{34} \ge 14 \\
& x_{ij} \ge 0 \quad \forall i, \forall j \ ,
\end{aligned}
$$

$$
\begin{aligned}
&Max \quad \eta ;\\
&subject\ to \quad f_{RL} + \eta(\mathrm{U}f_{RL} - Lf_{RL}) \le \mathrm{U}f_{RL}\\
&\qquad f_{CL} + \eta(\mathrm{U}f_{CL} - Lf_{CL}) \le \mathrm{U}f_{CL}\\
&\qquad f_{RU} + \eta(\mathrm{U}f_{RU} - Lf_{RU}) \le \mathrm{U}f_{RU}\\
&\qquad f_{CU} + \eta(\mathrm{U}f_{CU} - Lf_{CU}) \le \mathrm{U}f_{CU}\\
&\qquad \sum_{j=1}^{n} X_{ij} \in [a_i^a, a_j^b]:[a_i^c, a_i^d], \quad \mathrm{i} = 1,2,3...m\\
&\qquad \sum_{i=1}^{m} X_{ij} \in [b_j^a, b_j^b]:[b_j^c, b_j^d] \quad j = 1,2,3...\mathrm{n}\\
&\qquad \sum_{i=1}^{m} a_i^{RI} = \sum_{j=1}^{n} b_j^{RI}\\
&\qquad x_{ij} \ge 0 \quad \forall i, \forall j \ , \eta \ge 0
\end{aligned}
$$

$$Where\ f_{RL} = (wL + cL)\mathrm{X}_{ij}, f_{CL} = (cL)X_{ij}, \mathrm{W}L = (\mathrm{b} - \mathrm{a}) \div 2\ and\ CL = (\mathrm{b} + \mathrm{a}) \div 2.$$

$$Where\ f_{RU} = (wU + cU)\mathrm{X}_{ij}, f_{CU} = (cU)X_{ij}, \mathrm{W}U = (\mathrm{d} - c) \div 2\ and\ CU = (\mathrm{d} + c) \div 2$$

$$
\begin{aligned}
&Max \quad \eta\\
&subject\ to:\\
&7x_{11} + 7x_{12} + 7x_{13} + 7x_{14} + 11x_{21} + 17x_{22}\\
&\quad + 17x_{23} + 21x_{24} + 13x_{31} + 5x_{32} + 11x_{33} + 4x_{34} + 109\eta \le 424\\
&0.5x_{11} + 0.5x_{12} + 1x_{13} + 0.5x_{14} + 1x_{21} + 1x_{22}\\
&\quad + 0.5x_{23} + 1.5x_{24} + 1x_{31} + 1x_{32} + 1x_{33} + 0.5x_{34} + 15\eta \le 35\\
&9x_{11} + 9x_{12} + 8x_{13} + 9x_{14} + 12x_{21} + 18x_{22}\\
&\quad + 19x_{23} + 22x_{24} + 14x_{31} + 6x_{32} + 12x_{33} + 5x_{34} + 108\eta \le 46\\
&2x_{11} + 2x_{12} + 2x_{13} + 2x_{14} + 2x_{21} + 2x_{22}\\
&\quad + 2.5x_{23} + 2.5x_{24} + 2x_{31} + 2x_{32} + 2x_{33} + 2x_{34} + 10.5\eta \le 85.5
\end{aligned}
$$

$$
\begin{aligned}
&x_{11} + x_{12} + x_{13} + x_{14} \le 10\\
&x_{11} + x_{12} + x_{13} + x_{14} \ge 6\\
&x_{21} + x_{22} + x_{23} + x_{24} \le 22\\
&x_{21} + x_{22} + x_{23} + x_{24} \ge 16\\
&x_{31} + x_{32} + x_{33} + x_{34} \le 20\\
&x_{31} + x_{32} + x_{33} + x_{34} \ge 14\\
&x_{11} + x_{21} + x_{31} \le 13\\
&x_{11} + x_{21} + x_{31} \ge 9\\
&x_{12} + x_{22} + x_{32} \le 5\\
&x_{12} + x_{22} + x_{32} \ge 1\\
&x_{13} + x_{23} + x_{33} \le 16\\
&x_{13} + x_{23} + x_{33} \ge 12\\
&x_{14} + x_{24} + x_{34} \le 18\\
&x_{14} + x_{24} + x_{34} \ge 14\\
&x_{ij} \ge 0 \quad \forall i, \forall j \ , and \quad \eta \ge 0
\end{aligned}
$$

X=(2,1,3,0,7,0,9,0,0,0,0,0,14)
Z=([282,328]:[223.375])

7-Conclusion

This paper considers a formulation of rough interval transportation problem which has been studied. Traditional, rough optimality method and fuzzy programming approach are applied in form of the third case which the objective and constraints are roughness. The types solution of three methods is rough interval, interval number, and crisp number.

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A Hybrid Approach for Solving Course Timetable Problem

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Abstract

Combinations of two metaheuristic algorithms have provided very good results for solving variety scheduling problems. This paper describes the development of such a hybrid approach of Genetic Algorithm and Simulating Annealing Optimization Algorithm for course timetabling. This problem concerned with the assignment of a set of events (courses, subjects and teachers) to specific number of appointments (timeslots, days and rooms). For a solution to be feasible, a set of hard constraints must be satisfied. And a set of soft constraints should be satisfied for the quality of the solution. It evaluated by a fitness function. The hybrid approach tested over established datasets and compared against other techniques from the literature. The experiment results obtained confirm that the approach is able to produce solutions with the highest fitness values than in the literature on the benchmark. It is concluded that the hybrid approach represents a particularly effective method for producing high quality solutions to the course timetabling problem.

Keywords: Course Timetable Problem, Genetic Algorithm, Simulating Annealing Algorithm

1. Introduction

Timetabling is essentially in life activities, it is frequently facing in educations, companies, sports event, and transportation etc. Course Timetable Problem (CTTP) is a one of the most famous education types. CTTP has classified as NP-complete problem as it combining a multi dimensional elements and multi objectives. Constructing a solution for timetabling is very exhausting work. Traditional methods need a very difficult effort and time to satisfy all problem constraints. Generally, the CTTP constraints has classified into two types. The first primary type is the hard constraints, which must be satisfied because it effect on the solution feasibility, and soft constraints which should be satisfied also but it may violate some of its constraints if necessary. The soft constraints mainly effect on the solution quality. CTTP is a process of assignment a set of events (regular meeting between student and teacher for a specific subject) into a limited number of periods in a fit room. The main objective of timetabling problem is to build feasible solution by considering the hard constraints in addition to maximize the number of stratified soft constraints.

In this work, a hybrid approach is used to solve the course timetable problems and compete with the other techniques. The proposed algorithm hybrid the Genetic Algorithm (GA) and Simulating Annealing Optimization (SAO).The GA as a population based type of metaheuristic approaches explores the search space, which guides to find a solution with high quality as can. While, the SAO is used as a global search method to generate new neighbours and select the fittest one.

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This paper is organized as follows. Section 1 is the introduction. In Section 2, literature review of CTTP has described. In Section 3, the CTTP problem definition is defined. In Section 4, the solution structure representation is described. In Section 5, the hybrid approach of Genetic Algorithm and Simulating Annealing Algorithm to solve the CTTP is proposed. In Section 6, the testing and the experimental results of the proposed algorithm for a number of artificial hdtt instances. Finally, conclusions are included in Section 7.

2. Literature Review

In recent years, there has been an increasing interest in course timetable problems. Previous studies have primarily concentrated on solving this type of problems using Meta-heuristic approaches. Basically, there are two main types of meta-heuristic approaches: single based and population based approaches. Single based approaches manipulate with single solution and applying the changes on the current solution until find the local optima solution. This approach has a powerful of exploitation ability. The course timetable problem has been investigated by many researchers using the single-based algorithms. Simulated annealing (Zhang, et al., 2010), tabu search (Zhipeng and Jin-Kao, 2010) and great deluge (Shaker and Abdullah, 2009). While population based approaches depends on number of solution during finding the optimal solution. This type of approaches has an exploring ability. Also many researchers have solved course timetable problems using population based approaches such as Genetic algorithms [(Raghavjee, et al., 2012); (El-Sherbiny, et al., 2015)], artificial bee colony (Bolaji, et al., 2014). Beside Meta-heuristic approaches, there are other types of methodology have implanted to solve course timetable problems for examples Timeslot-Filling Based as heuristic method (Michael, et al., 2013) and Integer programming (Phillips, et al., 2015). In addition to the two types of meta-heuristic approaches, the hybridization of a population based as global search and the local improvement has become one of the most powerful approaches in solving course timetable problem as knows as a hard combinatorial optimization problems which gives balancing between exploration and exploitation of the search space. Hybrid methodology has been studied over the latest few decades because of its effectiveness to obtain fit solutions. (Zhipeng and Jin-Kao, 2008) have solved the course timetable with a hybrid heuristic algorithms and (Al-Betar, et al., 2012) has used the electromagnetic-like mechanism as a population based global optimization algorithm that is based on physics theory of simulating attraction and repulsion of sample points great deluge as a local search method that may accepted worse solutions by given upper boundary. (Abdullah, et al., 2012) has hybridized the harmony search algorithm with hill climbing and particle swarm optimization. While, (Badoni, et al., 2015) has combined the genetic algorithm with iterated local search algorithm and (Fong, et al., 2015) has also implemented the artificial bee colony algorithm with the great deluge algorithm for improve the local search ability.

3. Course Timetable Problem Definition

As described in the introduction, CTTP is known as multi-dimensional assignment problem, which is a combining of a shard objects (teacher, class, subject, room, day, and period). The teacher is assigned to teach a subject for a class in room at a specific time of the day. This assignment has to consider a set of hard and soft constraints.

Let T is a set of teachers,

C is a set of classes,

S is a set of subjects,

R is a set of rooms,

P is a set of periods,

D is a set of days,

$A_{t,s}$ is a set of periods when teacher t of subject s is available, And let also the following decision variables:

$$x_{tcsrpd} = \begin{cases} 1 & if\ teacher\ t\ teaches\ subject\ s\ for\ class\ c\ \ in\ room\ r \\ & at\ \ period\ p\ in\ day\ d \\ 0\ otherwise & \end{cases}$$

3.1 hard constraints

The set of hard constraints are the main element of solving the CTTP. The solution cannot be feasible without satisfy all the hard constraints. The set of CTTP hard constraints follows as (Zhang, et al., 2010):

HC1: A teacher cannot be assigned to more than one class during any time slot.

$$\sum_{c \in C} x_{tcsrpd} \leq 1 \quad \forall\ t \in T_s\ , s \in S, r \in R, p \in P, d \in D, \qquad (\mathbf{1})$$

HC2: A class cannot be assigned more than one teacher for any time slot.

$$\sum_{s \in S} \sum_{t \in T_s} x_{tcsrpd} \leq 1 \quad \forall\ c \in C, r \in R, p \in P, d \in D, \qquad (\mathbf{2})$$

HC3: A room cannot be allocated more than once for any time slot.

$$\sum_{t \in T_s} \sum_{c \in C} \sum_{s \in S} x_{tcsrpd} \leq 1 \quad \forall\ r \in R, p \in P, d \in D, \qquad (\mathbf{3})$$

HC4: A teacher t, teaching subject s, is to be assigned $l_{t,s}$ lectures (or time periods or time slots) per week.

$$\sum_{c \in C} \sum_{r \in R} \sum_{d \in D} \sum_{p \in P} x_{tcsrpd} = l_{t,s} \quad \forall\ s \in S, t \in T_s\ , \qquad (\mathbf{4})$$

HC5: A class c must attend l_c lectures per week.

$$\sum_{s \in S} \sum_{t \in T_s} \sum_{r \in R} \sum_{d \in D} \sum_{p \in P} x_{tcsrpd} = l_c \quad \forall\ c \in C, \qquad (\mathbf{5})$$

3.2 Soft Constraints:

The set of soft constraints are a measure tool of the solution quality. Every problem has its soft constraints upon the enterprise rolls such as teacher preference time, consecutive or isolated of lectures, and room utilization.

3.3 Fitness Function:

Fitness function is used to evaluate the solution fitness. That is differentiating between the solutions. (Doulaty, et al., 2013):

$$eval(f) = \frac{1}{1+\mathrm{cost(f)}} \qquad (\mathbf{6})$$

Since, $\mathrm{cost(f)}$ is the number of violating constraints and it can be calculated as:

$$cost(f) = \sum_{i=1}^{ct} n_i(f) \times w_i \qquad (\mathbf{7})$$

Where, ct is the count of constraints, $n_i(f)$ is the penalty of violating constraint i and w_i is the weight of the constraint i.

The objective of CTTP is to minimize the count of violated soft constraints for the feasible solution which already generated by applying the hard constrains.

4. Solution Structure

A well design of the solution structure is essentially for solving a complicated problem like CTTP. It is important to create a direct and clear relation between solution objects. Since, working with multidimensional is very complicated during constricting the algorithm phases, especially when using a hybrid approach. So, it is useful to group the related object together for converting the objects into two dimensional. Mainly the timetable has two major entities as in Figure 1. The first main entity is the available appointments $\mathrm{A} = \{\mathrm{a}_1, \mathrm{a}_2, \dots, \mathrm{a}_\mathrm{j}\}$, which combining the periods $\mathrm{P} = \{1,2, \dots, \mathrm{p}\}$, rooms $\mathrm{R} = \{1,2, \dots, \mathrm{r}\}$, and days $\mathrm{D} = \{1,2, \dots, \mathrm{d}\}$. Since, the appointments can be represented as a vector with a length $(\mathrm{p} \times \mathrm{r} \times \mathrm{d})$. The other entity is the required events$\mathrm{E} = \{\mathrm{e}_1, \mathrm{e}_2, \dots, \mathrm{e}_\mathrm{i}\}$, which links subjects $\mathrm{S} = \{1,2, \dots, \mathrm{s}\}$, teachers $\mathrm{T} = \{1,2, \dots, \mathrm{t}\}$, and classes (group of students) $\mathrm{C} = \{1,2, \dots, \mathrm{c}\}$.

The appointment contains information about room, day, and period. So, the appointment index can be designed to know the appointment details, which easy to occur by using Index encoding and decoding (El-Sherbiny, et al., 2015).

Events	e_1	e_2	e_3			e_{i-1}	e_i
Appointments	3	4	19	a_j	...		77

Figure 1: Solution Structure

4.1 Index encoding

The equation **(1)** used to encode the appointment index

$$\text{Index} = (\text{r} \times \text{Days\#} \times \text{Periods\#}) + (\text{d} \times \text{Periods\#}) + \text{p} \qquad (8)$$

Figure 1: Solution Structure

Where r, d, and p denotes to room number, day number, and period number respectively. While, Days# is the working days per week, Period# is available periods per day.

Example:

Let's assume that there are 3 rooms (Rooms#), 5 working days (Days#), and 12 periods per day (Periods#). So, the length of the appointment vector will be:

$$\text{length} = [3 \times 5 \times 12]\ (180\ \text{avilible appointments}).$$

To encode the appointment in (r) room number 2 on (d) the day number 2 at (p) period 9. Then the appointment index will be:

$$\text{Index} = (2 \times 5 \times 12) + (2 \times 12) + 9 = 153$$

4.2 Index Decoding

Index decoding is the reverse of index encoding equation, which used to get information about specific appointment such as a room number (r), a day (d), or a period number (p). The below equations are used to index decoding.

$$p = index\ Mod\ Periods\# \qquad (9)$$

$$d = (index\ /\ Periods\#)\ Mod\ Days\# \qquad (10)$$

$$r = \big(index\ /\ (Periods\#\ \times Days\#)\big)\ Mod\ Rooms\# \qquad (11)$$

Mod denotes to modulo function, which used to get the remainder after a number is divided by a divisor.

Example:

If the index =162, then relative information about this appointment Index (period, room and day) can be found by the following:

$$p = \ 162\ Mod\ 12\ = 6$$

$$d = (162/12)\ Mod\ 5 = 3$$

$$r = (162/(12 \times 5)\ Mod\ 5\ = 2$$

So, the appointment will be in room number 2 on day number 3 at period number 6.

5. A Hybrid Genetic and Simulating Annealing Algorithm (HGSAA)

In this section, a HGSAA is built on two stages. The first stage starts with GA platform that used to expand the search space and select the best solution, then transfer to next stage of SA by receiving the output solution from GA phase to improve its quality. HGSAA has developed to reduce the time of GA with the powerful of SA optimizer technique to avoid trapping into local optimal solution. Figure 2 illustrates the HGSAA flowchart. Since, HGSAA depends on the traditional technique of the genetic algorithm to build an initial population by generate a random solutions. Then, reproduction new offspring and mature its fitness. But, the HGSAA has used the simulating annealing technique to generate new offspring (neighbourhoods) instead of GA reproduction technique.

The HGSAA Algorithm

Step 1: Identify the number of generation and the population size.

Step 2: Declare an initial solution to be the best solution.

Step 3: Set g =1.

Step 4: Create initial population of solutions.

Step 5: For each solution in the population.

a. Evaluate the solution.

b. If the fitness of the solution is better than the best solution.

c. Replace the best solution by the current solution.

d. If the best solution is idle solution go to step 10.

Step 6: For population size times do.

a. Select solution from the population.

b. Apply the mutation function on the selected solution using simulating annealing Optimization.

c. Add the mutated solution to the new population.

Step 7: Update the population with the new population.

Step 8: g =g+1.

Step 9: Repeat step 5 to step 8 until g > the number of generation.

Step 10: Return the best solution.

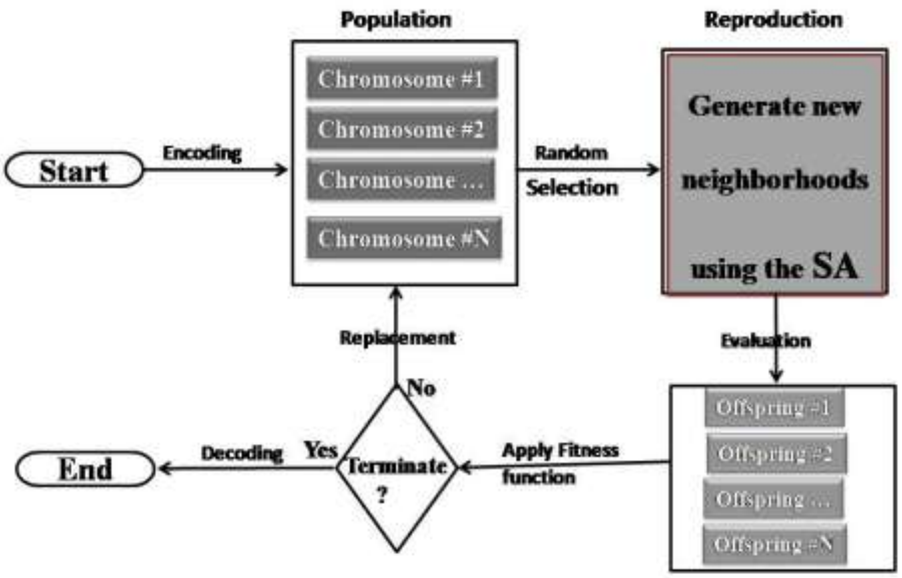

Figure 2: HGSAA Flowchart

5.1. Initial Population

As described in solution structure, there are group of events (Teacher, Class and Subject) need an appointments (Room, Day and Period) to create the solution. For each new solution the proposed algorithm orders the events by its number of lectures, then start with the first one and select randomly an appointment from the free appointments list (dynamic list for only the free appointments) and allocate the index of the appointment to the event in the solution. In Parallel, the proposed algorithm uses a short memory wise to tabu the appointments for each teacher and class to prevent the clashing as can as possible by comparing this list with the free appointments list at the next event to get a fit assignment.

5.2. Neighbourhoods

GA has two operators which are used to reproduction new children population from its parents. Crossover is one of them, which depends on reorder the solution (chromosome) based on taking parts from the parents. But the solution structure mainly builds on ordering the events. So, the HGSAA depends only on the mutation operator, which it is not effect of the solution order. The previous work (El-Sherbiny, et al., 2015) has experimented 16 type of mutation on the hdtt instances. It used the Relative Percentage Deviation (RPD) to determine the most fit mutation type. The mutation function number 5 is providing the minimum RPD value. This mutation is based on generating a random number for $NS \epsilon [1, a]$, where a is the length of the available events. Then, the basic two point swap is performed NS times. The NS is calculated as following:

$$NS = Int\big(Rand(1, a)\big) \tag{12}$$

5.3 Simulating Annealing Optimization

SAO is an optimization global searcher. SAO accept only the solution that is fitter than the best one, but it sometimes accept a worse fit solution by probabilistic probability $Pr = e^{-\Delta/T}$, where Δ is the difference between the current solution and the new generated solution cost and T is the temperature value starts with a big value then decrease slowly by using cooling rate parameter . The acceptance is achieved by generating a random number R between[0,1], if $P < Pr$ the new generated solution is accepted. This concept is used to jump from the local minimum (Rutenbar, 1989).

Simulating Annealing Algorithm

Step 1: Set M = Number of moves to attempt.

Step 2: Set T= temperature value, $CR =$ cooling rate.

Step 3: Set best soluation $= \ currentSol$.

Step 4: For m =1 to M

a. Generate a new neighbour newSol (based on number of swap mutation).

b. Evaluate the change in energy between currentSol and the newSol, ΔE;

c. If ($\Delta E < 0$) then

d. Replace the currentSol by the newSol.

e. Else Set P = Random value between [0.1]

f. If $P \leq e^{-\Delta/T}$

g Replace the currentSol by the newSol.

Step 5: Set T $= T - (T * CR)$. //CR is the cooling rate

6. Experimental results

The datasets bank of OR Library provides timetable datasets called HDTT. All Instances and its description are available and share the official URL website[4]. HDTT denotes to the instances are **hard** timetabling problems. All periods must be used with very little or no options for each allocation. Each week has a five working days and each day has a six periods with 30 timeslots are available per room.

The hybrid proposed approach has been implemented by Visual Studio 2010 in C# language. Output results of all runtimes have measured using the Intel Core 2 Duo with 2.27 GHz, but the program depends on only one core for this experiment with 2GB RAM.

The parameter values that are used for solving the CTTP are (10) population size, (20000) generation number, (0.0003) rate cool (RC), (10000) temperature number (T). The proposed algorithm are compared with other literature methods used to solve the CTTP for the same benchmark instances which described as follows: comparison of the hybrid proposed algorithm (HGSAO) with the previous work results of the Genetic algorithm with hill climbing optimizer (GAHCO) (El-Sherbiny, et al., 2015). In addition to, some of literature methods have solved hdtt instances benchmark.

- **SA1** Simulated Annealing Algorithm (Abramson, et al., 1993).
- **SA2** Simulated Annealing Heuristic (Randall, et al., 2001).
- **TS** stand to Tabu Search, **GS** a Greedy Search, **NN-T2** and **NN-T2** are Hopfield Neural Networks (Smith, et al., 2003).
- **DWTAN** and **CPMF** are neural network approaches (Carrasco, et al., 2004).
- **SA3** is a Simulated Annealing variant (Gendreau, et al., 2005).
- **TFH** Timeslot-filling heuristic and **EAH** event-assignment heuristic. The both of them are a heuristic methods described in (Pimmer and Raid, 2013).
- **GAHCO** denotes to Genetic Algorithm with Hill Clamping Optimization (El-Sherbiny, et al., 2015).
- **HGSAO** denotes to the hybrid proposed approach.

The next figure 3 demonstrates the average number of solutions that the proposed algorithm requires to get the final timetable. Figure 4 shows the average CPU processing time of the proposed algorithm, and how the proposed algorithm is fast to generate the final timetable.

[4] http://people.brunel.ac.uk/mastjjb/jeb/orlib/tableinfo.html.

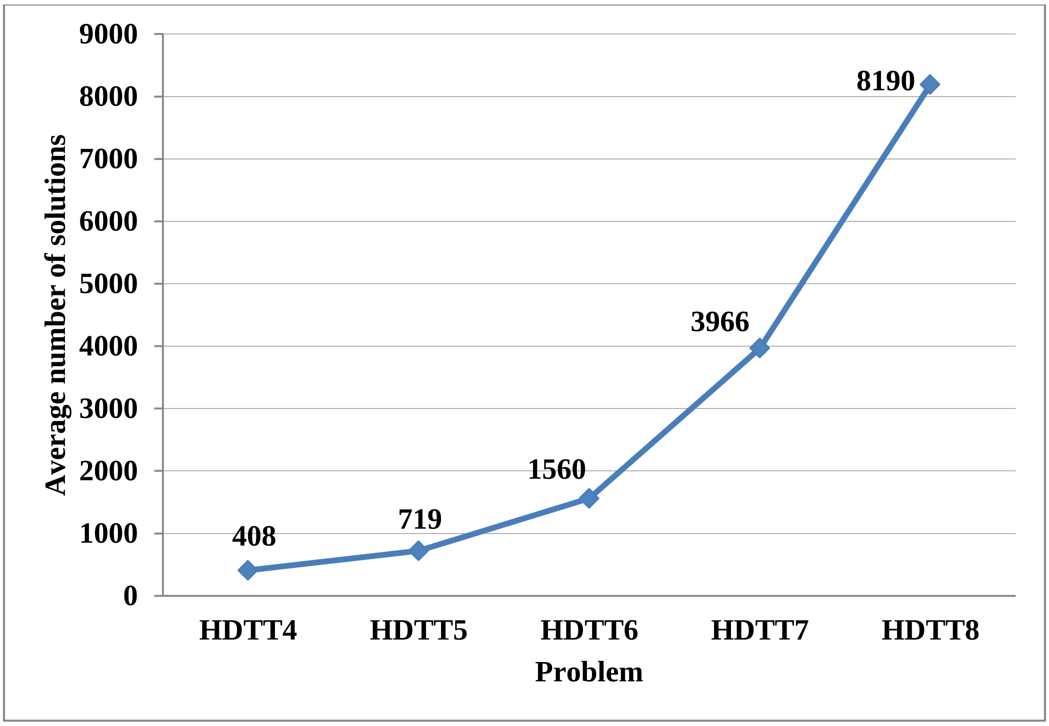

Figure 3: Avreage number of generated soluations over all runs.

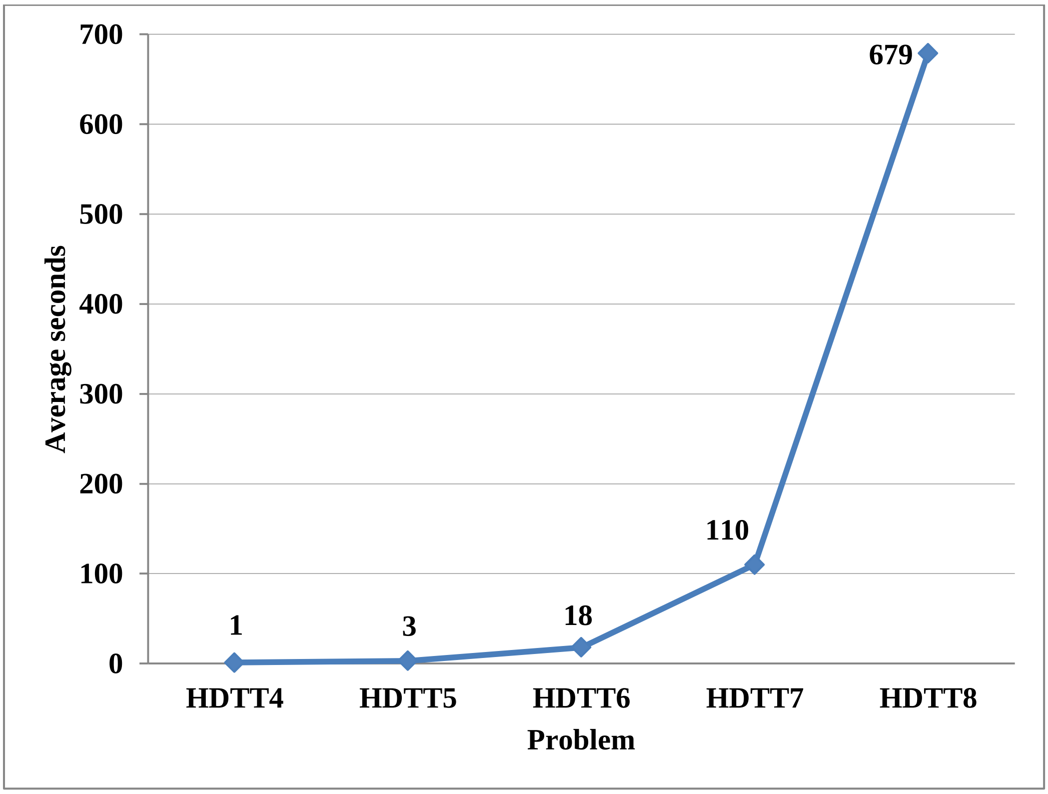

Figure 4: Average CPU seconds over all runs

Table 1 Comparative results of the HGSAO and each method.

Method	HDTT4		HDTT5		HDTT6		HDTT7		HDTT8	
	Best Cost	**Avg Cost**	**Best Cost**	**Avg Cost**	**Best Cost**	**Avg Cost**	**Best Cost**	**Avg Cost**	**Best Cost**	**Avg Cost**
SA1	-	-	0	0.7	0	2.5	2	2.5	2	2.5
SA2	**0**	0	0	0.3	0	0.8	0	1.2	0	1.9
TS	0	0.2	0	2.2	3	5.6	4	10.9	13	17.2
GS	5	8.5	11	16.2	19	22.2	26	30.9	29	35.4
NN-TT2	0	0.1	0	0.5	0	0.8	0	1.1	0	1.4
NN-TT3	0	0.5	0	0.5	0	0.7	0	1	0	1.2
CPMF	5	10.7	8	13.2	11	18.7	18	25.6	15	28.6
DWTAN	**0**	0	0	0.4	0	1.65	0	2.1	0	3.25
SA3	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	0.4
EAH	**0**	0	2	5.4	6	7.9	9	12	13	15
TFH	**0**	0	0	0.6	0	2.1	0	2.5	0	3.1
GAHCO	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	0.6
HGSAO	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0.3**

Table 1 compares the results of HGSAO with other methods. The first column indicates the count of violated constraints for the best result achieved, and the second column for the average of violated constraints of 20 runs. The results show that the HGSAO is the best of all methods, which yields no cost as the best cost value with a fit processing time for all instances. Also, no average cost for the first 4 datasets, and exceed the SA1 method with average cost 0.3 for hdtt8.The performance of the proposed algorithm is competitive when compared to the other methods, and performs the best finding timetables for all 20 runs conducted for each data set.

7. Conclusion

The hybrid proposed algorithm which combined Genetic Algorithm (GA) with Simulating Annealing Optimization (SAO) is a described for solving CTTP. The hybrid proposed algorithm is based only on the mutation operation of GA, since the crossover operator was prevented by the solution structure design. The swap mutation is based on the result of The RPD in the previous work, which is used to discover the fittest mutation function of 16 mutation functions. The hybridization of GA and SAO allow decreasing the generation number of GA which is shown in all artificial hdtt instances. The hybrid proposed algorithm gives optimal solutions with one hundred percentages of fitness function values within a few seconds. The experiment was showed the effectiveness of the proposed algorithm over other methods and approached in literature. So, the proposed approach can be added as a new method for solving CTTPs. The future work will investigate to solve other types of timetable problems such as examinations, transportation and sporting scheduling. The performance of the proposed algorithm can be tuning by using a method to optimize the parameters of the approach.

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An Interval Transportation Problem with N- Vehicle

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Abstract : N-vehicle cost varying interval transportation problem is presented in this article . In N- vehicle transportation problem the vehicles capacity can not carry all units, so we may use each vehicle in several trips until the transportation of all units. The cost will be varies due to the capacity of vehicles. The type of conveyance that will be used to transport the units from each source to each destination will be identified according to the quantities of units. Here we used fuzzy goal programming for multi-objective transportation problems to solve this problem, which gives minimum cost. To illustrate this method we will use a numerical example.

Keywords. Interval Transportation Problem, Cost Varying Transportation Problem, Fuzzy Programming, fuzzy goal programming for multi-objective transportation problems.

1. Introduction

The transportation problem is a special type of linear programming problem, when we shipping single type of commodity from sources to destinations by using single mode of conveyance. Sometimes the decision maker prefers give the data as interval or fuzzy number. S. Chanas et al [14] used parametric programming to solve The transportation problem with fuzzy supply values and with fuzzy demand values. Chanas et al [15] developed an algorithm to solve the transportation problem with integrality condition when the problem has fuzzy supply and fuzzy demand values. M. Zangiabadi et al [9] present a fuzzy goal

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programming approach to get optimal compromise solution for the multi objective transportation problem, by assuming that each objective function has a fuzzy goal, and assign hyperbolic membership function to each objective and to describe each fuzzy goal.

In 1997, R. H. Mohamed [13] introduced similarities between goal Programing and fuzzy programing and explain how each one can lead to the other and present some new forms of fuzzy programs.

K. Venkatasubbaiah et al [8] used Fuzzy Goal Programming Method for Solving multi objective Transportation Problems, they proposed membership functions and goal deviation functions from Pareto solutions for each objective, and added these functions as constraints.

By introducing a max-min operator λ an auxiliary variable, then the equivalent fuzzy interactive goal programming problem is formulated to maximize (λ) and the solution is obtained by using LINGO software.

Waiel et al [16] present an interactive fuzzy goal programming approach to determine the preferred compromise solution for the multi-objective transportation problem, this approach assumes that each objective function has a fuzzy goal and by using the minimum operator considers the imprecise nature of the input data. To reach the efficient solution that is near to the best lower bound of each objective function the researcher focuses on minimizing the worst upper bound, also many researchers as [4,5,7,11,12] present papers to solve Interval Transportation Problem.

In N- vehicle transportation problem [2] the conveyances can't carry all units in one trip so it must return the trip till transport all units, and according to the quantity of units we can identify the type of conveyance that will use. Arpita Panda [3] proposed an algorithm to solve Cost Varying Interval Transportation Problem under Two Vehicle, here in this paper we used Interactive fuzzy goal programming for multi-objective transportation problems [16] to resolve the problem which gives cost less than [3]. To illustrate the method we will use a numerical example.

2. Preliminary

Definition 2.1. (The closed interval; Arpita Panda [3]) is defined by an order pair of brackets as:

$$A = [a_L, a_R] = \{a: a_L \leq a \leq a_R, a \in R\} \tag{1}$$

where a_L is the left limit, and a_R is the right limit of A.

The interval is also denoted by its center and half width as [3]:

$$A = [a_c, a_w] = \{a: a_c - a_w \leq a \leq a_c + a_w, a \in R\} \tag{2}$$

where: $a_c = \frac{a_L + a_R}{2}$ (3) & $a_w = \frac{a_R - a_L}{2}$ (4) are the center and half width of A.

Definition 2.2. If A and B are two closed intervals [3], and be a binary operation on the set of real number then:

$$A * B = \{a * b: a \in A, b \in B\} \tag{5}$$

is defined a binary operation. According to the above definition interval operations are defined as:

as: $A + B = [a_L, a_R + b_L, b_R] = [a_L + b_L, a_R + b_R],$ (6)

as: $A + B = [a_c, a_w + b_c, b_w] = [a_c + b_c, a_w + b_w],$ (7)

$kA = k[a_L, a_R] = [ka_L, ka_R] \quad if \;\; k \geq 0$ (8)

$kA = k[a_L, a_R] = [ka_R, ka_L] \quad if \;\; k < 0$ where k is a real number. (9)

$$A * B = \min\{a_L b_L, a_L b_R, a_R b_L, a_R b_R\} \tag{10}$$

Definition 2.3. The order relation $\leq_{LR}$ between $A = [a_L, a_R]$ and $B = [b_L, b_R]$ is defined as [3] :

$$A \leq_{LR} B \;\; iff \;\; a_L \leq b_L \;\; and \;\; a_R \leq b_R \tag{11}$$

Definition 2.4. The order relation $\leq_{LR}$ the decision maker's preference for the alternative with lower minimum cost and maximum cost [1], that is, if $A \leq_{LR}$ then A is preferred to B.

3. Interval Transportation Problem.

The formulation of Interval Transportation Problem **(ITP)** is the problem of minimizing interval valued objective function with interval costs, interval sources and interval demands parameters, is given in the mathematical model (1):

$$min \quad z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} \tag{12}$$

$$\text{s.to. } c_{ij} \in [D_{L_{ij}}, D_{R_{ij}}] \tag{13}$$

$$\sum_{j=1}^{n} X_{ij} \in [a_{L_i}, a_{R_i}], \qquad i = 1,2, \dots \dots m. \tag{14}$$

$$\sum_{i=1}^{m} X_{ij} \in [b_{L_i}, b_{R_i}], \qquad j = 1,2, \dots \dots n. \tag{15}$$

$$\sum_{i=1}^{m} a_{L_i} = \sum_{j=1}^{n} b_{L_i}, \qquad \sum_{i=1}^{m} a_{R_i} = \sum_{j=1}^{n} b_{R_i} \tag{16}$$

$$and \; x_{ij} \geq 0, \forall i, j. \tag{17}$$

Where $c_{ij} = [D_{L_{ij}}, D_{R_{ij}}]$ is an interval representing the uncertain cost for transportation problem. The sources parameter lies in $[a_{L_i}, a_{R_i}]$ and destination parameter lies in $[b_{L_i}, b_{R_i}]$ depending on $[a_{L_i}, a_{R_i}]$ and $[b_{L_i}, b_{R_i}]$ We determine c_{L_i} and c_{R_i} Then we define :

$$D_{L_{ij}} = \min\left\{c_{L_{ij}}, c_{R_{ij}}\right\} \; and \; D_{R_{ij}} = \max\left\{c_{L_{ij}}, c_{R_{ij}}\right\} \tag{18}$$

Definition 3.1. Optimal Solution: $x^* \in S$ is an optimal solution if there is no other solution $x \in S$ which satisfies $Z(x) \leq_{Rc} Z(x^*)$.

The right limit $Z_R(x)$ of the interval objective function $Z(x)$ in given problem may be elicited as :

$$Z_R(x) = \sum_{i=1}^{m} \sum_{j=1}^{n} D_{c_{ij}} x_{ij} + \sum_{i=1}^{m} \sum_{j=1}^{n} D_{w_{ij}} x_{ij} \tag{19}$$

The center of the objective function $Z_c(x)$ can be elicited as

$$Z_c(x) = \sum_{i=1}^{m} \sum_{j=1}^{n} D_{c_{ij}} x_{ij} \tag{20}$$

The solution of the Model (1) by definition (3.1) can be taken as the optimal solution of following mathematical model (2):

$$\min \quad Z_1(x) = \sum_{i=1}^{m} \sum_{j=1}^{n} D_{c_{ij}} x_{ij} + \sum_{i=1}^{m} \sum_{j=1}^{n} D_{w_{ij}} x_{ij} \tag{21}$$

$$\min \quad Z_2(x) = \sum_{i=1}^{m} \sum_{j=1}^{n} D_{w_{ij}} x_{ij} \tag{22}$$

$$\sum_{j=1}^{n} X_{ij} \in [a_{L_i}, a_{R_i}], \qquad i = 1,2, \dots \dots m. \tag{23}$$

$$\sum_{i=1}^{m} X_{ij} \in [b_{L_i}, b_{R_i}], \qquad j = 1,2, \dots \dots n. \tag{24}$$

$$\sum_{i=1}^{m} a_{L_i} = \sum_{j=1}^{n} b_{L_i}, \quad \sum_{i=1}^{m} a_{R_i} = \sum_{j=1}^{n} b_{R_i} \tag{25}$$

$$and\ x_{ij} \geq 0, \forall i, j. \tag{26}$$

4. Fuzzy Programming Technique

In fuzzy programming technique, we first find the lower bound as LZ_R and the upper bound as $UZ_R$for the r^{th} objective function $UZ_R(x)$. Similarly the lower bound as LZ_c and the upper bound as UZ_c for the

$$dZ_R = UZ_R - LZ_R \tag{27}$$

The degradation allowance for objective $Z_R(x)$.

$$dZ_c = UZ_c - LZ_c \tag{28}$$

The degradation allowance for objective $Z_c(x)$, when the aspiration levels for each of the objective have been specified, a fuzzy model is formed and then the fuzzy model is converted into a crisp model.

The solution of Model (1) can be obtained by the following steps:

Step 1. Solve the Model (1) as a single-objective transportation problem (2) times by taking one of the objective at a time.

Step 2. From the above results, determine the corresponding values for objective at each solution derived.

According to each solution and value for every objective, we can find a pay-off matrix as follows: Pay off table of the solution of (r) single objective TP.

Table 1. represent pay off table

	Z_R	Z_w
X^1	Z_{1R}	Z_{1w}
X^2	Z_{2R}	Z_{2w}

where X^1, X^2 are the isolated optimal solutions of the (r) different transportation problems for 2 different objective functions $Z_{1R}, Z_{1w}, Z_{2R}, Z_{2w}$ are the values of objective functions.

step 3. From Step 2, find for each objective the U_r and the L_r corresponding to the set of solutions, where :

$$UZ_R = \max\{Z_{1R}, Z_{2R}\}, \quad LZ_R = \min\{Z_{1R}, Z_{2R}\} \tag{29}$$

$$UZ_w = \max\{Z_{1w}, Z_{2w}\}, \quad LZ_w = \min\{Z_{1w}, Z_{2w}\} \tag{30}$$

An initial fuzzy model of the problem can be Find

$x_{ij}, \quad i = 1,2, \ldots, m, \quad j = 1,2, \ldots, n.$

step 4. Define a membership function μZ_R, μZ_w for the Z_R, Z_w respectively.

step 5. Convert the fuzzy model of the problem, obtained in Step 3, into the following crisp model (3).

$$\max \lambda \tag{31}$$

$$\text{s.to. } \lambda \leq \mu(Z_R), \qquad \lambda \leq \mu(Z_w) \tag{32}$$

$$\sum_{j=1}^{n} X_{ij} \in [a_{L_i}, a_{R_i}], \qquad i = 1,2, \ldots \ldots . m. \tag{33}$$

$$\sum_{i=1}^{m} X_{ij} \in [b_{L_i}, b_{R_i}], \qquad j = 1,2, \ldots \ldots . n. \tag{34}$$

$$\sum_{i=1}^{m} a_{L_i} = \sum_{j=1}^{n} b_{L_i}, \quad \sum_{i=1}^{m} a_{R_i} = \sum_{j=1}^{n} b_{R_i} \tag{35}$$

$and\ x_{ij} \geq 0, \forall i, j.$

step 6. Solve the crisp model by an appropriate mathematical programming algorithm.

step 7. The solution obtained in step 6 will be optimal compromise solution of the Model (1).

4.1. Fuzzy Programming Technique with Linear Membership Function

A linear membership function is defined as:

$$\mu_1(Z_R) = \begin{cases} 1 & if\ Z_R \leq LZ_R \\ 1 - \frac{Z_R - LZ_R}{UZ_R - LZ_R} & if\ LZ_R \leq Z_R \leq UZ_R \\ 0 & if\ UZ_R \leq Z_R \end{cases} \tag{36}$$

and

$$\mu_1(Z_R) = \begin{cases} 1 & if\ Z_w \leq LZ_w \\ 1 - \frac{Z_w - LZ_w}{UZ_w - LZ_w} & if\ LZ_w \leq Z_w \leq UZ_w \\ 0 & if\ UZ_w \leq Z_w \end{cases} \tag{37}$$

If we use a linear membership function, the crisp model can be simplified in Model (4) as follows:

$$\max \lambda \tag{38}$$

$$\text{s.to. } Z_R + \lambda \leq (UZ_R - LZ_R) \leq UZ_R \tag{39}$$

$$Z_w + \lambda \leq (UZ_w - LZ_w) \leq UZ_w \tag{40}$$

$$\textstyle\sum_{j=1}^{n} X_{ij} \in [a_{L_i}, a_{R_i}], \qquad i = 1,2, \dots \dots m. \tag{41}$$

$$\textstyle\sum_{i=1}^{m} X_{ij} \in [b_{L_i}, b_{R_i}], \qquad j = 1,2, \dots \dots n. \tag{42}$$

$$\textstyle\sum_{i=1}^{m} a_{L_i} = \sum_{j=1}^{n} b_{L_i}, \quad \sum_{i=1}^{m} a_{R_i} = \sum_{j=1}^{n} b_{R_i} \tag{43}$$

$$and\ x_{ij} \geq 0, \forall i, j. \tag{44}$$

4.2. Interactive fuzzy goal programming for multi-objective transportation problems [16].

The steps to solve multi objective transportation problem by interactive fuzzy goal programming problem, which will used to solve N-Vehicle cost varying interval transportation problem, will summarized as follows:

Step1. Develop the MOTP as :

$$mim\ \ z = \textstyle\sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}^{r} x_{ij} \tag{45}$$

s.to.

$$\textstyle\sum_{j=1}^{n} x_{ij} = a_i, \qquad i = 1,2, \dots \dots m. \tag{46}$$

$$\textstyle\sum_{i=1}^{m} x_{ij} = b_j, \qquad j = 1,2, \dots \dots n. \tag{47}$$

$$\textstyle\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j \tag{48}$$

$$r = 1,2, \dots, R.\ \ and\ x_{ij} \geq 0, \forall i, j \tag{49}$$

where

a_i : The units available at source (i).

b_i: The units demanded at destination (j).

x_{ij}: The unknown quantity shipped from source (i) to destination (j).

c_{ij}: The cost of shipping units from source (i) to destination (j).

Step2. Solve the first objective function as a single objective transportation problem. Continue this process (r) times for the (r) objective functions. If all the solutions are the same, $(X^1 = X^2 = \cdots = X^k = (X_{ij}), \quad i = 1,2, \dots, m. \quad j = 1,2, \dots, n.)$

select one of them as an optimal compromise solution and go to Step 8. Otherwise, go to Step 3.

Step3. Evaluate the objective function at the (r^{th}) solution and determine the best lower bound (L_r) and the worst upper bound (U_r).

Step4. Define the MF of each objective function and also the initial aspiration level.

Step5. Then problem is developed as follows:

$$\max \lambda \tag{50}$$

$$\text{s.to. } \lambda \le \mu_r(Z^r(x)), \qquad r = 1,2, \dots, R. \tag{51}$$

$$\sum_{j=1}^{n} X_{ij} = a_i, \qquad i = 1,2, \dots \dots . m. \tag{52}$$

$$\sum_{i=1}^{m} X_{ij} = b_j, \qquad j = 1,2, \dots \dots . n. \tag{53}$$

$$Z^r(x) - d_r^+ + d_r^- = G^r \tag{54}$$

$$0 \le \lambda \le 1, \qquad d_r^+, d_r^-, x_{ij} \ge 0,$$

and x_{ij} integer for i=1,2,….,m, j=1,2,…,n (55)

and solve it as a mixed integer goal programming problem.

Step6. Present the solution to the decision maker. If the DM accepts it, go to Step 8. Otherwise, go to Step 7.

Step7. Evaluate each objective function of the solution. Compare the upper bound of each objective with the new value of the objective function. If the new value is lower than the upper bound, consider this as a new upper bound. Otherwise, keep the old one as is. Repeat this process (r) times and go to Step 4.

Step8. Stop.

5. Numerical Examples, example 1:

Let us consider the following interval transportation problem.

Table 1. Tabulated form represent the cost varying interval transportation [3] problem.

	D_1	D_2	D_3	stock
S_1	5,7	4,6	8,10	[50,70]
S_2	2,3	6,8	7,9	[40,50]
S_3	3,4	10,12	4,6	[30,80]
Demand	[60,90]	[40,80]	[20,30]	

It is given that there are two types of vehicle $V_1$and V_2. The cost of V_1 from source `i' destination `j' is R_{ij}^1 for a single trip. The cost of $V_2$from source `i' destination`j' is $R_{ij}^2$for a single trip.the capacity of V_1 is C_1= 10 and the capacity of $V_2$is C_2= 20

5.1. The solution Methods:

To solve the above problem follws from $c_{L_{ij}}$ [3] as:

Table 2. Tabulated form represent the lower cost.

	D_1	D_2	D_3	stock
S_1	5,7	4,6	8,10	[50]
S_2	2,3	6,8	7,9	[40]
S_3	3,4	10,12	4,6	[30]
Demand	[60]	[40]	[20]	

Then we have

$$c_{L11} = \frac{5}{10}, c_{L12} = \frac{12}{40}, c_{L13} = \frac{8}{10}, c_{L21} = \frac{6}{40}, c_{L22} = \frac{16}{40}, c_{L23} = \frac{9}{20},$$

$$c_{L31} = \frac{3}{10}, c_{L32} = \frac{10}{10}, c_{L33} = \frac{6}{20}, \quad \mathbf{ZL=5+12+6+3+6=32}$$

To determine $c_{R_{ij}}$ we consider the cost varying TP as:

Table 3. Tabulated form represent the upper cost.

	D_1	D_2	D_3	stock
S_1	5,7	4,6	8,10	[70]
S_2	2,3	6,8	7,9	[50]
S_3	3,4	10,12	4,6	[80]
Demand	[90]	[80]	[30]	

Then we have

$c_{R11} = \frac{14}{40}, c_{R12} = \frac{22}{70}, c_{R13} = \frac{18}{30}, c_{R21} = \frac{6}{40}, c_{R22} = \frac{6}{10}, c_{R23} = \frac{16}{30},$
$c_{R31} = \frac{11}{50}, c_{R32} = \frac{10}{10}, c_{R33} = \frac{10}{30},$ **ZR=22+6+6+11+10=55**

So, interval c_{ij} of interval TP determined as $c_{ij} \in [\min\{c_{Lij}, c_{Rij}\}]$, we have

$c_{11} \in [\frac{14}{40}, \frac{5}{10}], c_{12} \in [\frac{12}{40}, \frac{22}{70}], c_{13} \in [\frac{18}{30}, \frac{8}{10}]$

$c_{21} \in [\frac{6}{40}, \frac{6}{70}], c_{22} \in [\frac{16}{40}, \frac{6}{10}], c_{23} \in [\frac{9}{20}, \frac{16}{30}],$

$c_{31} \in [\frac{11}{50}, \frac{3}{10}], c_{32} \in [\frac{10}{10}, \frac{10}{10}], c_{33} \in [\frac{6}{20}, \frac{10}{30}]$

Then we formulate cost varying interval TP

Min $Z_R(x)$=

$(0.388 + 0.1125)*x_{11}+(0.307 + 0.007)*x_{12}+ (0.7 + 0.1) *x_{13}+$

$(0.15 + 0.0)*x_{21}+(0.5 + 0.1)*x_{22}+(0.49 + 0.042)*x_{23} +$

$(0.26 + 0.04)*x_{31}+(1 + 0.0)*x_{32}+(0.3166 + 0.017)*x_{33}$

Min $Z_w(x)$= $(0.1125) *x_{11} +(0.007) *x_{12}+ (0.1) *x_{13}+(0) *x_{21}+ (0.1) *x_{22}+ (0.042) *x_{23} +(0.04) *x_{31}+ (0) *x_{32}+ (0.017) *x_{33}$

now we will use interactive fuzzy goal programming to solve multi-objective transportation problems[16] as:

Step1. Develop the MOTP as

Min Z_R=0.5*x11+0.314*x12+0.8*x13+0.15*x21+0.6*x22+

0.7*x23+0.3*x31+1*x32+0.333*x33;

Min Z_c= 0.1125*x_{11} +0.007 *x_{12}+ 0.1 *x_{13}+0 *x_{21}+ 0.1 *x_{22}+ 0.042*x_{23} +0.04 *x_{31}+ 0 *x_{32}+ 0.017 *x_{33}

subject to supply constraints

x11 + x12 + x13 >= 50;   x11 + x12 + x13 <= 70;   x21 + x22 + x23 >= 40;

x21 + x22 + x23 <= 50;   x31 + x32 + x33 >= 30;   x31 + x32 + x33 <= 80;

demand constraints

x11 + x21 + x31 >= 60;   x11 + x21 + x31 <= 90;   x12 + x22 + x32 >= 40;

x12 + x22 + x32 <= 80;   x13 + x23 + x33 >= 20;

Step2. The solution of each single objective transportation problem is: (X^{1*})=(0,50,0,50,0,0,10,30,20),

(X^{2*})=(0,50,0,50,0,0,10,0,20).

Step3. Evaluate the objective function at the (r^{th}) solution and determine the best lower bound (L_r) and the worst upper bound (U_r).

$1.09 \leq Z_W \leq 1.09$ and $32.86 \leq Z_R \leq 62.86$

Step4. Define the MF of each objective function and also the initial aspiration level.

0.0687*x11+0.0065*x12+0.0917*x13+0*x21+0.0917*x22+0.1146*x23+ 0.0366*x31+0*x32+0.0152*x33+0.5746* λ <=1;

0.0079*x11+0.0049*x12+0.0127*x13+0.0023*x21+0.0095*x22+0.0111 *x23+0.0047*x31+0.0159*x32+0.0053*x33+0.9252* λ <=1;

Step5. Then problem is developed as follows

max=λ;

s.to   x11 + x12 + x13 >= 50; x11 + x12 + x13 <= 70; x21 + x22 + x23 >= 40;   x21 + x22 + x23 <= 50; x31 + x32 + x33 >= 30; x31 + x32 + x33 <= 80;

x11 + x21 + x31 >= 60; x11 + x21 + x31 <= 90; x12 + x22 + x32 >= 40;

x12 + x22 + x32 <= 80; x13 + x23 + x33 >= 20;

0.0687*x11+0.0065*x12+0.0917*x13+0*x21+0.0917*x22+0.1146*x23+ 0.0366*x31+0*x32+0.0152*x33+0.5746* λ <=**1**;

0.0079*x11+0.0049*x12+0.0127*x13+0.0023*x21+0.0095*x22+

0.0111*x23+0.0047*x31+0.0159*x32+0.0053*x33+0.9252* λ <=**1**;

0.075*x11+0.007*x12+0.1*x13+0 *x21+0.1*x22+ 0.125*x23+0.04*x31

+0 *x32+0.017*x33+d_1^--d_1^+=**1.09**;

0.5*x11+0.314*x12+0.8*x13+0.15*x21+0.6*x22+0.7*x23+0.3*x31+

1*x32+0.333*x33+d_2^--d_2^+=**62.86**;

$\leq \lambda \leq 1$, $\quad d_1^+, d_1^-, d_2^+, d_2^-, x_{ij} \geq 0$ and x_{ij} integer for i=1,2,3, j=1,2,3

The problem is solved by using **LINGO** package

$\lambda = 0.8701706\text{E} - 02$, d_1^-=0, d_1^+=0, d_2^-=30, d_2^+=0 $\quad Z_c$=1.09,

Z_R=32.86

[X*]= [0, **50** , 0, **50**, 0, 0, **10**, 0, **20**], x_{12} =16/50, x_{21} =8/50, x_{31} =3/10, x_{33} =6/20.

Total cost =33, Total units =130, **the cost per unit=33/130=0.2538**

6. Conclusion

In this article , we have solved N-vehicle cost varying interval transportation problem[3], by using Interactive fuzzy goal programming for solving multi-objective transportation problems [16]. We consider the upper bound of interval as first objective and the half width of interval as second objective, by that the problem turned to bi objective problem. This method improved the solution better than method used in[3].

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Optimizing the Parameters of an Artificial Immune System Algorithm for Solving the Assembly Line Balancing Problem

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Abstract

The assembly line balancing problem is considered one of the NP-hard problems, which attracted both of practitioners and researchers of operations research for almost a half century. In this paper an artificial immune system algorithm is developed to solve the type-1 of simple assembly line balancing problem, which is concerned with minimizing the number of stations with a given cycle time. The algorithm uses the smoothness index with the solution structure as discrimination criteria that help to increase the search space. In order to get better results, the parameters and their levels of the proposed algorithm have been tuned by using Taguchi's method. To validate the improvement of the outputs of the new proposed algorithm, the worst results of the old algorithm are resolved by the new one.

Keywords: Assembly line balancing problem; Convergence; Artificial immune; Taguchi's method.

1. Introduction

Assembly lines are used widely in the mass production systems to minimize unit cost, to maximize efficiency and speed of production and to make the manufacture processes easier. The assembly line is a collection of stations connected by a material-handling device, where the assembly tasks are allocated to stations according to their precedence relationships and the total processing time of the allocated tasks per station must not exceed the cycle time. The assembly line balancing problem (ALBP) can be defined as optimally assigning the assembly tasks to the stations with respect to some objectives. This paper develops the proposed artificial immune system algorithm (AIS) for solving the simple assembly line balancing problem type 1 (SALBP-1), which was produced in Mohammad et al. (2014). After clarifying the development of the proposed algorithm, its parameters will be optimized by using Taguchi's method.

The paper is organized as follows; Section 2 is assigned to describe and formulate the problem of assembly line balancing type 1. In section 3, we discussed some literature that solve computational problems by using an artificial immune system and demonstrated some literature that present algorithms for AIS. The proposed artificial immune system algorithm is presented in section 4. In section 5, the Taguchi's method will be presented to optimize the parameters and their levels of the proposed AIS algorithm. Section 6 produces the computational results compared with solutions of some problems taken from a well-known benchmark. Finally, the conclusion and future points of research is addressed in section 6.

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2. The Assembly Line Balancing Problem

According to the concept of any SALB formulation, an assembly line consists of $j = \{1, \dots, m\}$ stations arranged along a mechanical material handling device. The tasks consecutively moved on from station to another until they reach the end of the line. In general, the line balancing problem consists of optimally assigning the assembly tasks among all stations with respect to some objective. For this purpose, the total amount of work necessary to assemble a product is divided into a set $i = \{1, \dots, n\}$ of tasks. Tasks are indivisible units of work and thus each task i is associated with a processing time t_i. The tasks are performed according to precedence relationship.

The SALBP-1, which is addressed in this work, can be formulated as follows:

Notations:

n	Number of tasks ($i = 1, \dots, n$)
m	Number of stations ($j = 1, \dots, m$)
t_i	Processing time of task i
ct	Cycle time
$P(i)$	The set of immediate predecessors of task i

Decision variables:

$x_{ij} \in \{0,1\}$ 1 if task i is assigned to station j
0 otherwise

$$Minimize\ Z = m.ct - \sum_{j=1}^{m} \sum_{i=1}^{n} t_i . x_{ij} \tag{1}$$

Subject to:

$$\sum_{j=1}^{m} x_{ij} = 1, \forall i = 1, \dots, n \tag{2}$$

$$\sum_{i=1}^{n} t_i \, . \, x_{ij} \le ct, \forall j = 1, \dots, m \tag{3}$$

$$\sum_{j=1}^{m} j \, . x_{ij} \le \sum_{j=1}^{m} j \, . \, x_{kj} \, , \forall k \in P(i) \tag{4}$$

$$x_{ij} \in \{0,1\}$$

The objective function (1) is to minimize the idle time, which is equivalent to minimize the number of stations. Constraints (2) simply say that all tasks have to be assigned in only one station. Constraints

(3) require that for any station, the total processing time of the assigned tasks must not exceed the cycle time, while constraints (4) ensure that the precedence relationships between tasks must be maintained.

3. The Artificial Immune System

Artificial immune system is a field of study that focus on the development of computational models which are emerged from the principles of the biological immune system. AIS algorithms cover huge area of complex computational and engineering problems such as pattern recognition, fault and anomaly detection, data mining and classification, scheduling, machine learning, autonomous navigation, robot navigation, clustering/classification, bio-informatics, image processing, numeric function optimization and computer security (Diabat et al., 2013).

There are different algorithms for AIS such as the negative selection algorithm (Forrest et al., 1994), the cloning selection algorithm (Hunt et al., 1996), the immune genetic algorithm (Chun et al., 1997) and the artificial immune network algorithm (Castro and Zuben, 2001).

4. The Proposed Algorithm

The proposed algorithm is inspired from the clonal selection theory, which was proposed by Burnet (1959). In immune system, there are two kinds of antibodies. The first one is generated from the thymus which is the T-cells and the second kind is generated from the bone marrow which is the B-cells. In case of the B-cells, which is considered in the proposed algorithm, each antibody paratope, receptor, will be tested with the antigen epitope. The closer is this match, the stronger is the bind. This property is the so-called affinity maturation. The higher affinity antibodies will be cloned and mutated to promote a genetic diversity. Accordingly, the proposed algorithm begins by generating an initial population of antibodies. The next step is to use a memory that helps to store the selected cloned antibodies to prevent them from being cloned again. The proposed algorithm also contains an elitism to store the best solution found among all iterations. The iterations of the proposed algorithm include the affinity function calculation, the cloning and mutation procedure and the replacement of the worst member of antibodies. Figure 1. shows the flowchart of the main steps of the proposed algorithm. Additionally, the pseudo code of the proposed algorithm is illustrated as follows:

Start

Generate (P) *initial population with* P_{size} *antibodies.*
Create the memory set $(M) = \Phi$
Create the elitism set $(E) = \Phi$
Elitism value $(EV) = 0$
Counter = 1
Maximum number of iterations $= I_{max}$
Lower bound $(LB) = \left\lceil \frac{Total\ processing\ time}{Cycle\ time(Ct)} \right\rceil$, *where* $\lceil . \rceil$ *is the ceiling function*
Number of stations $(NS) = \infty$
While Counter $\leq I_{max}$ *and* $NS > LB$ *do:*
Calculate the affinity for each antibody in P

$P_a =$ *the set of the highest affinity antibodies in P*
$P_s = P_a - M$
if $P_s \neq \emptyset$ *then:*
$M = M \cup P_s$
Clone each antibody in P_s
Create P_c *which contains all cloned antibodies*
Mutate each antibody in P_c
Evaluate each antibody in P_c
Store the best antibody of P_c *in* A_{best}
Create the set C_{best} *which contains the best antibodies in* P_c
$P = P \cup C_{best}$
Remove the worst antibodies in P
If the evaluation of $A_{best}(EV_{A_{best}}) < EV$ *then:*
$E = A_{best}$
$EV = EV_{best}$
Generate the remaining antibodies of P to reach P_{size}
Counter = Counter + 1
Return the elitism
Stop

For further illustration of the previous algorithm, the algorithm can be subdivided into five parts which are the initial population, the affinity function and the memory of the search, the cloning process and the mutation function, the elitism and the replacement of the worst member of solutions. The next subsections discuss each part of the proposed algorithm.

4.1. The Initial Population

The initial population should contain diversified solutions to allow exploring more search space. For that reason, a random feasible solution algorithm is used to generate each solution in the initial population. The steps of that algorithm begin by generating a random sequence of the problem tasks. According to the sort of such sequence, the priority of the assignable tasks will be determined. The algorithm will continuously assign tasks until the set of unassigned tasks be empty.

The pseudo code of generating the random feasible solution is illustrated as follows:

Start

Generate random tasks sequence (RT)
Unassigned tasks (U_t) *= RT*
Assigned tasks $(A_t) = \{\emptyset\}$
Station number(SN) *= 1*
Station time (ST_{SN}) *= 0*

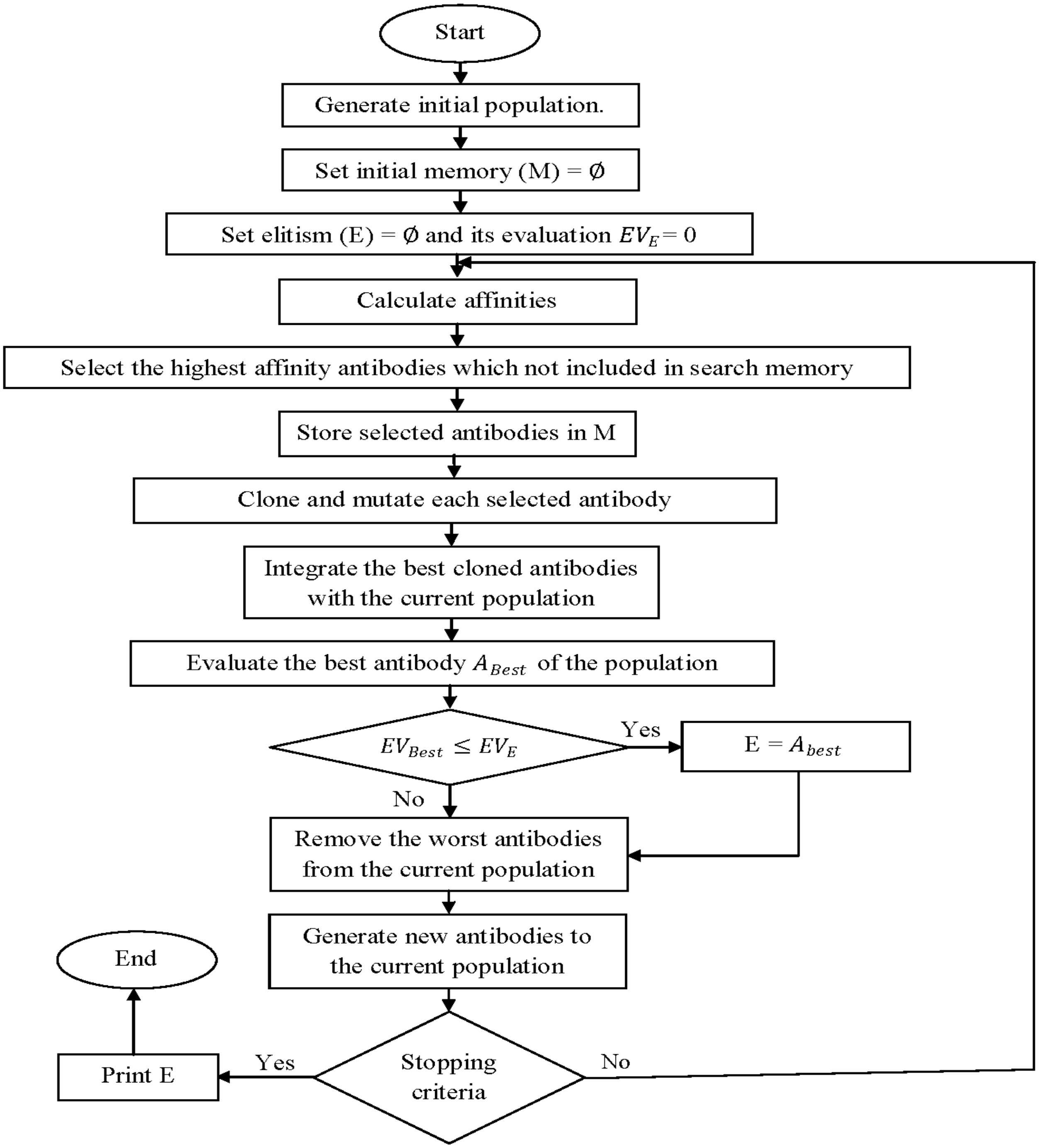

Figure 1. The flowchart of the proposed algorithm

Station's tasks $(S_T) = \{\emptyset\}$
Solution= $\{\emptyset\}$
While $U_t \neq \emptyset$ *do:*
  Counter = 1
  Permissible task (PS) *= 0*

Immediate predecessors test $(IP_{test}) = 0$
Cycle time test $(Ct_{test}) = 0$
While PS = 0 do:
Task = $U_t(Counter)$
Construct the immediate predecessors set (IP) of Task
If $IP = \emptyset$ *or* $IP \cap A_t$ *then* $IP_{test} = 1$
If $IP_{test} = 1$ *then* $PS = 1$
If Task time $(T_{time}) + S_T \leq$ *Cycle time then:*
$ST_{SN} = ST_{SN} + T_{time}$
$A_t = A_t \cup PS$
$S_T = S_T \cup PS$
$U_t = U_t - PS$
$Solution = Solution \cup \{(PS, SN)\}$
If Task time $(T_{time}) + S_T \geq$ *Cycle time then:*
SN = SN + 1
$ST_{SN} = T_{time}$
$S_T = \{PS\}$
$A_t = A_t \cup PS$
$U_t = U_t - PS$
$Solution = Solution \cup \{(PS, SN)\}$
Return solution
Stop

4.2. The Affinity Function and The Memory of The Search

The calculations of the affinity function (*AF*) between each two solutions are applied to extract the most diversified solutions from the population to be prepared for the cloning and mutation process. Two criteria are used to determine the similarity or the affinity between solutions. The first criterion is the smoothness index (*SI*) and the second one is based on the solution structure. The smoothness index shows how the solution is balanced with respect to the idle time. We choose smoothness index criterion because most of the solutions of the population have the same number of stations, but the smoothness index mostly differs. The smoothness index formula found in (Mozdgir et al., 2013; Esmaeilbeigi et al., 2015) as the following equation:

$$SI = \sqrt{\sum_{i}^{m} (ct - st_i)^2}\text{, } where\ st_i\ is\ the\ total\ processing\ time\ of\ station\ i \qquad (5)$$

The solution structure can be represented as the set of all ordered pairs of tasks and their stations. So, the number of unmatched ordered pairs (UO_{ij}) between solution i and solution j will be calculated for each solution i and the maximum number of unmatched ordered pairs will represent the first diversification criterion. The second diversification criterion is the highest Euclidean distance of the smoothness index values between the solution i and the other solutions in the population. The proposed affinity function for solution i can be formulated as follows:

$$AF_i = \max_{j \neq i}\{UO_{ij}\} + \max_{j \neq i}\left\{\sqrt{\left(SI_i - SI_j\right)^2}\right\} \tag{6}$$

After calculating the affinity value for each solution in the population, the higher affinity solution will be selected for the cloning process according to the memory of the search. The memory of the search helps to discard the solutions which are previously selected for the cloning process from being selected again.

4.3. The Cloning Process and The Mutation Functions

In the cloning process, the selected highest affinity solutions will be copied and mutated to produce a number of other solutions by using any local search techniques. After finishing the process the best evaluated cloned solutions will be appended to the current population. The mutation function, which will be selected to mutate the cloned solutions, is basically based on shifting tasks from a selected station. The assignable stations of a selected task for shifting can be determined by calculating the earliest station and the latest station of such task, which represents the station boundaries of such task. The assignable stations are each station of the station boundaries that has enough space to assign the selected task for shifting. The following equations help to calculate the station boundaries of task i:

$$\text{Earliest station of task } i = \max_{k \in IP_i}\left\{\sum_{j=1}^{m} j.x_{kj}\right\}, \tag{7}$$

where IP_i is the set of immediate predecessors of task i

$$\text{Latest station of task } i = \min_{k \in IS_i}\left\{\sum_{j=1}^{m} j.x_{kj}\right\}, \tag{8}$$

where IS_i is the set of immediate successors of task i

4.3.1. The Forward Shifting Function

The forward shifting function (FS) takes one of the assignable stations as an input and then the function tries to shift all tasks of such station to the maximum shifted station between the selected station and the latest station for the currently selected task for shifting. Thus, the function only works as a forward manner and it will ignore the tasks which aren't have available positions in their succeeded stations. The pseudo code the forward shifting function is as follows:

Start

For each task i in station j

Shifting station $(ss) = 0$

Determine the latest station (LS_i)

For each station k in the range $j: LS_i$ do:
if $S_k + t_i \leq ct$ and $k > ss$ then:
$ss = k$
if $ss > 0$ then:
$x_{iss} = 1$ *and* $x_{ij} = 0$
Stop

4.3.2. The Backward Shifting Function

The backward shifting function (BS) works the same like the forward shifting function but it works as a backward manner. The function ignores the tasks which aren't having available positions in their preceded stations. The pseudo code the backward shifting function is as follows:

Start
For each task i in station j
$ss = \infty$
Determine ES_i
For each station k in the range $j: ES_i$ do:
if $S_k + t_i \leq ct$ and $k < ss$ then:
$ss = k$
if $ss < \infty$ then:
$x_{iss} = 1$ *and* $x_{ij} = 0$
Stop

4.3.3. The Forward-Backward Shifting Function

To allow both of the previous mutation functions, the forward-backward shifting function is presented in this section. In the forward-backward shifting function (F-BS), half of the cloned solutions will be mutated by using the forward shifting function and the remaining cloned solutions will be mutated by using the backward shifting function.

4.4. The Elitism

The initial elitism will be constructed as an empty set. Then, after the cloning process per iteration has been done, each solution in the population will be evaluated. If the population includes a solution that has better evaluation than the elitism, then such solution will be the new one. After all iterations have been done, the elitism will be the best solution among all iterations, which is considered to be the output of the algorithm.

4.5. The Replacement of The Worst Member of Solutions

After the cloning and mutation process has been completed, the bad solutions, which have the highest number of stations and the highest smoothness index, should be replaced by new random solutions. Thus, the remaining solutions of the population after removing the worst ones will be generated as the same manner of generating solutions of the initial population.

5. Using Taguchi's Method to Optimize The Parameter of The Proposed Algorithm

The proposed AIS approach works through parameters which have to be tuned to produce better solutions. These parameters are the population size (P_{Size}), the number of iterations (I_{max}), the percentage of the highest affinity solutions (P_a), which are selected for the cloning procedure, the number of the cloned solutions (N_c), the mutation function (*MF*), the percentage of the best cloned solutions (C_{best}) and the percentage of the bad solutions that have to be removed from the population (N_r). These parameters, along with their levels, are summarized in the following table:

Table 1. Parameters and levels of the proposed algorithm

Parameter	Levels
P_{Size}	5, 10 and 15
I_{max}	5, 10 and 15
P_a	0.3, 0.5 and 0.7
N_c	3, 5 and 7
MF	FS, BS, F-BS
C_{best}	0.3, 0.5 and 0.7
N_r	0.3, 0.5 and 0.7

In Table 1 , the values of the parameters are determined empirically to fit the running time consumption. The total number of experiments that should be done to solve one problem according to the aforementioned parameters and levels are equal to $3^7 = 2187$ experiments. Due to the cost and the time of this amount of experiments, Taguchi's method will be used to determine the best combinations of those parameters and levels. The Taguchi's method begins by selecting the predesigned orthogonal array by calculating the total degree of freedom. In this experimentation, one degree of freedom for the total mean and two degrees of freedom for the each parameter. Thus, the total degree of freedom is equal to $1 + (7 \times 2) = 15$. Thus, the appropriate predesigned orthogonal array at least has 15 rows. Table 2 shows the available predesigned orthogonal array $L_{27}(3^{13})$, where parameters are assigned to columns and levels are assigned to rows:

Table 2. Orthogonal array $L_{27}(3^{13})$

No.	P_{Size}	I_{max}	P_a	N_c	*MF*	C_{best}	N_r
1	5	5	0.3	3	FS	0.3	0.3
2	5	5	0.3	3	BS	0.5	0.5
3	5	5	0.3	3	F-BS	0.7	0.7
4	5	10	0.5	5	FS	0.3	0.3
5	5	10	0.5	5	BS	0.5	0.5
6	5	10	0.5	5	F-BS	0.7	0.7
7	5	15	0.7	7	FS	0.3	0.3
8	5	15	0.7	7	BS	0.5	0.5
9	5	15	0.7	7	F-BS	0.7	0.7
10	10	5	0.5	7	FS	0.5	0.7
11	10	5	0.5	7	BS	0.7	0.3
12	10	5	0.5	7	F-BS	0.3	0.5
13	10	10	0.7	3	FS	0.5	0.7
14	10	10	0.7	3	BS	0.7	0.3
15	10	10	0.7	3	F-BS	0.3	0.5
16	10	15	0.3	5	FS	0.5	0.7
17	10	15	0.3	5	BS	0.7	0.3
18	10	15	0.3	5	F-BS	0.3	0.5
19	15	5	0.7	5	FS	0.7	0.5
20	15	5	0.7	5	BS	0.3	0.7
21	15	5	0.7	5	F-BS	0.5	0.3

No.	P_{Size}	I_{max}	P_a	N_c	MF	C_{best}	N_r
22	15	10	0.3	7	FS	0.7	0.5
23	15	10	0.3	7	BS	0.3	0.7
24	15	10	0.3	7	F-BS	0.5	0.3
25	15	15	0.5	3	FS	0.7	0.5
26	15	15	0.5	3	BS	0.3	0.7
27	15	15	0.5	3	F-BS	0.5	0.3

The selected problems are taken from a large set can be found in the open literature http://www.assembly-line-balancing.de/. In case of measuring the problem size according to the number of tasks, the benchmark problems can be divided into three categories. The first category is the small sized problems, which contain number of tasks less than 54 task. The second one is the medium sized problems, which contain tasks varying from 54 to 111. The last category is the large sized problems which contain tasks varying from 112 to 297. For each category, two problems have been selected and solved by the proposed AIS algorithm according to each row of the orthogonal array. The selected benchmark instances are summarized as follows:

Table 3. The selected benchmark problems

Problem Name	Number of Tasks	Cycle time	Problem Size
Buxey	29	27	Small
Hahn	53	2004	
Arcus1	83	3786	Medium
Lutz3	89	75	
Mukherje	94	176	Large
Arcus2	111	5755	

In order to determine the relative importance of each parameter, the signal-to-noise ratio (S/N) has to be calculated. The word signal refers to the desirable value, the response value and the word noise denotes the undesirable value, the standard deviation. The higher S/N ratio indicates the better signal. Therefore, the objective is to maximize the S/N ratio. In the case of minimization problems, the formula of S/N ratio is as follows (Pandey and Panda, 2015):

$$S/N\ ratio = -10Log_{10}(Objective\ function)^2 \quad (9)$$

According to the orthogonal array, 27 trails have been done. For each problem size, the values of the objective function for each trail are reported in Table 4 and the S/N ratios for each trail are shown in Table 5.

Table 4. The values of the objective function for each trail

Trail	Small	Medium	Large
1	581.543	1366.714	1705.215
2	555.9672	1172.105	1850.98
3	547.6529	1082.844	1911.343
4	480.8539	1339.629	1056.938
5	595.3569	1229.882	1494.586

Trail	Small	Medium	Large
6	639.1204	1264.216	2251.514
7	499.3935	1168.43	1871.03
8	625.5423	651.394	906.8336
9	613.0709	1142.808	1573.151
10	573.0924	645.6496	1002.551
11	598.6713	568.9382	959.4434
12	569.337	1290.115	1886.077
13	691.5433	1372.688	1808.844
14	701.2992	1308.347	901.169
15	607.3523	584.0359	1853.198
16	602.9534	1182.143	1001.512
17	612.0562	1278.184	1058.56
18	598.4999	1198.224	1838.034
19	597.2069	1330.859	1013.43
20	584.1771	627.3634	905.8497
21	583.4656	1252.671	1056.684
22	661.6845	1329.775	928.5986
23	605.2124	646.7796	991.1896
24	558.0637	1346.427	1846.721
25	583.9848	568.5583	1001.615
26	678.4185	1358.391	926.4563
27	564.8591	1301.46	1966.983

Table 5. The S/N ratios values for each trail

Trail	Small	Medium	Large
1	-55.2916	-65.0278	-67.2229
2	-54.901	-63.8915	-67.9477
3	-54.7701	-62.9439	-68.2784
4	-53.6403	-64.9919	-62.8188
5	-55.4955	-64.1591	-66.0399
6	-56.1117	-64.535	-69.744
7	-53.9689	-63.7549	-67.9855
8	-55.9251	-58.6054	-61.3304
9	-55.7502	-63.5382	-66.4967
10	-55.1645	-57.9213	-62.2704
11	-55.5438	-56.6527	-61.8357
12	-55.1074	-64.6698	-68.0724
13	-56.7964	-65.2138	-67.747
14	-56.9181	-64.7036	-61.2248
15	-55.6688	-57.0719	-67.9826
16	-55.6057	-63.9322	-62.3309
17	-55.7358	-64.5787	-62.804
18	-55.5413	-63.9118	-67.8386
19	-55.5225	-64.8908	-62.3216
20	-55.3309	-58.2288	-61.3463
21	-55.3203	-64.4411	-62.7807
22	-56.413	-64.9058	-61.5602
23	-55.6382	-57.9522	-62.1419
24	-54.9337	-65.0619	-67.9457
25	-55.328	-57.3759	-62.3556
26	-56.63	-65.0841	-61.5793
27	-55.0388	-64.7622	-68.4967

For the small sized problems, the main effect plot for the means of the objective function value are reported in Figure 2 and the main effect for S/N ratios plot at each level are shown in Figure 3.

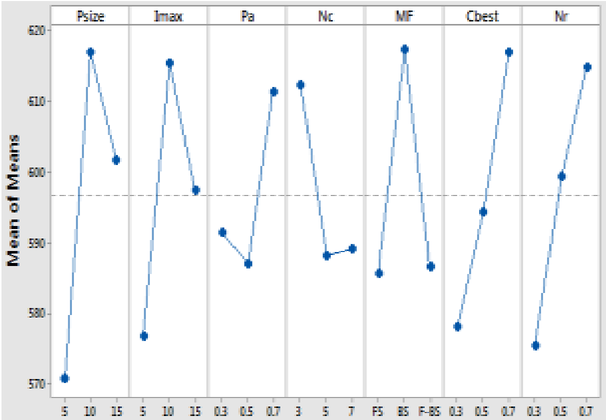

Figure 2. The main effect plot for means for the small sized problem.

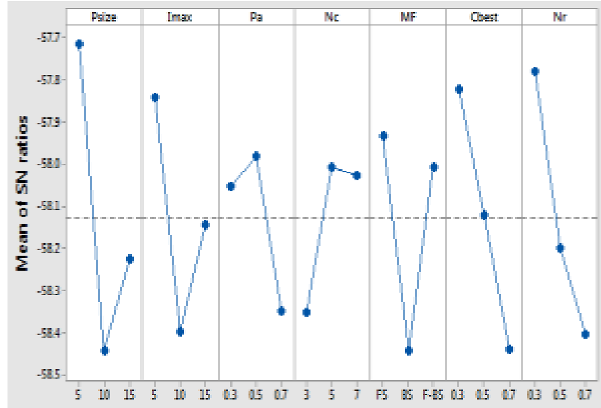

Figure 3. The main effect for S/N ratios plot at each level for the small sized problem.

For the medium sized problems, the main effect plot for the means of the objective function value are reported in Figure 4 and the main effect for S/N ratios plot at each level are shown in Figure 5.

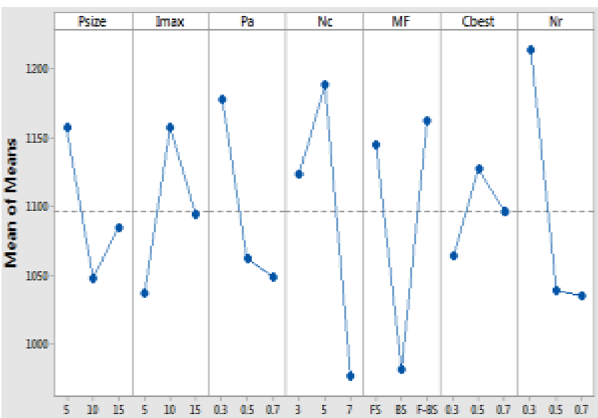

Figure 4. The main effect plot for means for the medium sized problem.

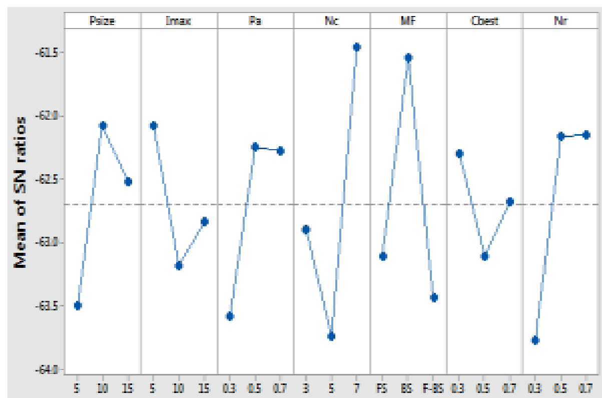

Figure 5. The main effect for S/N ratios plot at each level for the medium sized problem.

For the medium sized problems, the main effect plot for the means of the objective function value are reported in Figure 6 and the main effect for S/N ratios plot at each level are shown in Figure 7.

Figure 6. The main effect plot for means for the large sized problem.

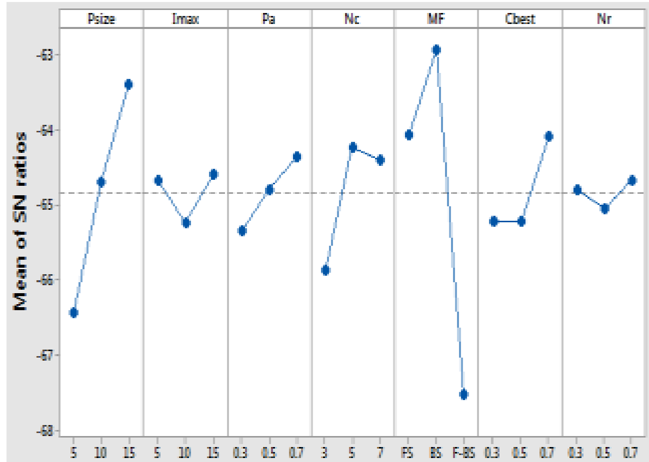

Figure 7. The main effect for S/N ratios plot at each level for the large sized problem.

The aforementioned graphs show the best levels for each parameter according to the problems size. In case of the small sized problem, the best values of the levels according to the sort of the parameters in Table 1 are 5, 5, 0.5, 5, FS, 0.3 and 0.3, respectively. The best values of the levels according to the same sort of the parameters in case of the medium sized problem are 10, 5, 0.7, 7, BS, 0.3 and 0.7, respectively. Eventually, in the same sort of parameters the best value of the levels in the large sized problems are 15, 15, 0.7, 5, BS, 0.7 and 0.7, respectively. The result of the optimization process can be summarized in Table 6 as follows:

Table 6. The optimized parameter levels for each problem size

Parameter	Small	Medium	Large
P_{Size}	5	10	15
I_{max}	5	5	15
P_a	0.5	0.7	0.7
N_c	5	7	5
MF	FS	BS	BS
C_{best}	0.3	0.3	0.7
N_r	0.3	0.7	0.7

6. Notes

After developing and optimizing the parameter levels of the proposed AIS algorithm, which was proposed in Mohammad et al. (2014), the problems that had bad results are produced to the new developed AIS algorithm and the parameter levels of the new algorithm are selected according to each problem size. Table 7 depicts the new results as follows:

Scholl and Klein (1997) bi-directional branch and bound approach S&K

The old proposed artificial immune system approach	AISO
The new proposed artificial immune system approach	AISN
Problem size	n

Table 7: Computational results

Sample problem	n	ct	S&K	AISO	AISN
Barthold2	148	85	51	52	52
		89	49	50	50
Gunther et al.	35	41	14	15	14
Hahn	53	2004	8	9	8
		2338	7	8	7
Kilbridge and Wester	45	92	6	7	7
Mukherjee and Basu	94	222	20	21	20

The highlighted values in Table 7 are the only remaining bad results in case of using the new developed AIS algorithm.

7. Conclusion

This paper is considered to be an extension of the previous work in Mohammad et al. (2014). The parameter levels of the previous work were tuned empirically and, the initial population of it was generated by a real good solution gotten by the bi-directional heuristic based on the positional weight priority rule. Herein, the solutions of the initial population are generated by using a random feasible solution algorithm. The mutation function is further clarified by introducing three mutation functions, which are the forward shifting function, the backward shifting function and the forward-backward shifting function. The parameter levels of the new developed AIS have been optimized by Taguchi's method and the previous bad solutions of the old algorithm have been tested by the new one. The new AIS algorithm along with its optimized parameter levels proofs that it produces better solutions than the last one according to the results which are listed in Table 7.

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Optimizing type II error under Crisp and Fuzzy Environment

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Abstract

In this paper, an approach is developed to solve the problem of non-decision and statistical hypothesis tests when test statistics (Decision making Under Uncertainty) occur as a result of the test cloudy region in One-sample test in case of two tailed in the cases of Crisp and Fuzzy through optimizing type II error. Bootstrap Approach is used in the crisp case because it is a simple method for calculating some of the famous statistics such as Confidence Interval (CI),Testing of Hypothesis, calculating approximated biases and so forth. The developed approach employs some of the statistics that have been calculated from through Bootstrap in determining acceptance and rejection regions and explore the magnitude of the problem that in the case of certainty or in the uncertainty. In the fuzzy case, the membership function is constructed and the critical areas under H_0 and H_1.

Keyword: Bootstrapping, Fuzzy hypothesis; Type II error; P-Value, Membership function; Cloudy area.

1 Introduction

In our daily lives, we find that the events, to which we are exposed, whether at work or in our social environment, are intersecting or overlapping and can be sequential . In practice, we face events, which can be independent or disjoint, both of which are present in very limited manner. This applies for all phenomena under consideration. The variables are also in a relationship with each other and not independents as assumed by the theory as required. These relationships among the variables may be in formed mathematically known forms. The are relationship may be simple or complex, and in some cases, it may be unknown. In some cases the researcher assumes the nature of the relationship between the variables linear, to facilitate the mathematical formulation of the relationship between the variables. Also, in some cases, we can't find a specific form which mathematically meets the relationship. Also in some cases, it difficult to reach a point, where the technique used conforms with the phenomena under study .

The complexity of the mathematical form that meets the phenomenon, and the many factors that affect them, leads to the negligence of some of the phenomena under study, but this can be caused by the negligence of some variables which are difficult to measure. Removal of certain variables in turn affects the accuracy of the results. In addition, the problem is aggravated by the scarcity and high expenses of materials and sampling units in the study, which in turn is reflected in the number of cases used in the research.

Estimation and statistical tests, in traditional form, exposed to a many types of errors, (in crisp case), which have to be studied carefully. Perhaps the most important errors to be studied are errors of measurement sampling error, as well as two famous of errors (Type I

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error and type II error). Most references have stated the significance of the type II error. However, the available literature provides scarce detail, regarding the study of this type of error. Accordingly, this study will be concerned mainly with the detailed study of type II error. This is done in order to optimize the decision. This will also help to clarify the decision and with greater accuracy place the decision in either acceptance region or the rejection region. This would also be done using a suitable sample, which will obviate the need for large statistical sample.

There are a few of Studies concerned with this topics such as: Arnold (1994) determined the parameters of the statistical tests, which should be used for maximizing an aggregated customer's degree of satisfaction. In (1996) Arnold showed, how statistical tests originally constructed to examine crisp hypotheses, can also be applied to fuzzily formulated hypotheses. In particular, criterions α and β are proposed generalizing the probabilities of type I and type II, respectively . Ekel et al. in (2008) in this paper studying the approach to decision making in a fuzzy environment, this is utilized for analyzing multi-criteria optimization models. This can be employed to analyze maximin problems. Akbari (2010) constructed a new method for bootstrap testing hypothesis in fuzzy environment, by defining fuzzy sets of real numbers of the one-sided and the two-sided ordinary (crisp) hypotheses. Arefi et al. (2011) dealt with the problem of testing of hypothesis when both the hypotheses and the available data are fuzzy. This was done by using the construction of fuzzy point estimation, and concept of fuzzy test statistic is based on the α-cuts. Khozaimy et al. (2011) in real life decision problem are often solved by using very limited sets of data points . This paper particularly used normal distribution instead of (Stochastic process cloud - SPC) which is complicated computation . Also if the parameters of a normally distributed variable is not clear, then we have a lack of information, or due to the multiplicity of regression analysis Akbari (2012) showed that, in statistical analysis sometimes the variable measures are not precise, and we want to test the hypotheses of mean and variance. We propose a method to evaluate the fuzzy hypotheses (for one-sample and two-sample) of interest. Nguyen, et al (2015) Introduced a new method, known as fuzzy information, which contributed to the rigorous formulation of statistics with fuzzy data using continuous lattice structure of upper semicontinuous membership function. Hesamian, et al **(2015)** defined fuzzy random variables, the concept of the power of test and p-value is extended to the fuzzy power and fuzzy p-value. This also developed the concepts of type I, fuzzy type II errors.

This shows that statistical science alone, in its traditional form, is limited in most instance. Additional applied science and quantitative methods for example: operations research, particularly simulation systems and fuzzy are needed to facilitate the process of decision making in presence of uncertainty.

Therefore quantitative methods, which take into account the previously mentioned problems must be available.

2 Preliminaries

In this section, representing the definitions the variables in research type II error, p-value, and introduced to research tool as a follow :

2.1 Types of error in testing hypothesis

The following representation for types of these errors, which are decision makers encountering .

- Type I error : The probability of type I error is the likelihood of rejecting (H_0)

when the null hypothesis is true. This probability is usually denoted by (**α**) **Sandra et al. (2009)** .

- Type II error : The probability of a type II error is the likelihood of accepting (H_0)

When the alternative hypothesis is true, this probability is usually denoted by (**β**) **Sandra et al. (2009)**.

Type I error (α) : we can calculate type I error **(α)** as a follows :

$$\alpha = P(H_0 \text{ is rejected} \mid H_0 \text{ is true})$$

Where the value of **(α)** represents the significance level of the test.

The value of (**α)** depends on the sampling distribution for (**μ)** under ($\mathbf{H_0}$) . Either to determine the value (β) which depends on the sampling distribution for (**μ)** under ($\mathbf{H_1}$). This distribution can not determined unless determining the value of the mean ($\boldsymbol{\mu_i}$**)**, of course ($\boldsymbol{\mu_i} \neq \boldsymbol{\mu}$) . Then , it is possible to calculate too many values for type II when we change the mean value ($\boldsymbol{\mu_i}$) under alternative hypothesis . Thus, we conclude that the value of **(α)** is a uniqe while (**β**) take many values , where it is depends on the different values of mean under the alternative hypothesis **Ashor et al. (2000)**.

The relationship between type I error and type II error , we find that the relationship is negative . Whenever there is an increase for type I error that affect type II error decreased and vice versa . This fact the whether the test one taill or two tailed test **Ashor et al. (2000)**.

In general aim of hypothesis testing is to use statistical tests that make (**α** and **β**) as small as possible, which are the chance of error. This goal requires compromise, because making **α** small involves rejecting the null hypothesis less often, whereas making **β** small involves accepting the null hypothesis less often . These action are contradictory, that is, as **(α)** decreases, (**β**) increases, and as (**α**) increases, (**β**) decreases . The usual recommendation is to fix (**α**) at some specific level (for example 0.05) and use that a suitable test , then perform the optimization of the decision **Sandra et al. (2009)** .

2.2 Definition of P-value

The p-value is not probability that the null hypothesis is true. The p-value is a random variable that varies from sample to sample .

It is not appropriate for us to compare the p-value from two distinct experiments,

Or from two distinct experiments, or from tests on two variables measured in the same experiment, and declare that one is more significant than the other.

P-values need not reflect the strength of a relationship.

The vast majority of p-values produced by parametric tests based on the normal distribution are approximations. If the data are " almost " normal , the associated p-values will be almost correct . There exists a class of tests, the permutation tests for which the significance levels are exact if the observations are independent and identically distributed under the null hypothesis or their labels are otherwise exchangeable.

The overall conclusion is that P values can be highly misleading measures of the evidence provided by data against the null hypothesis **Philip et al. (2012)** .

2.3 Tool of the research

The researcher used the IQ test for kids and adults, which was prepared by Prof. Dr. Samia Lotfy Ansari as this test is an accurate means to measure intelligence, as intelligent human formation hypothetical can not be observed directly, but we infer from behavior, phenomenon is the only that can be observed is the behavior or performance whether in the form of work or do or say or think.

This test includes 60 questions, and is required to answer these questions through the person in the laboratory for a period of not more than 45 minutes without exceeding the specified time, at the rate of 45 seconds per question. in average . As such, this test measures the percentage of intelligence accurately .

This test a number of mental processes which is to supplement the numerical strings, the distinction between forms, grasp the meanings of words, and grasp the relationships between letters Away series includes numerical reasoning.

This test was established according to test of the d. Alfred Monzert Alfred W. Munzert to measure the proportion of IQ for individuals 12 years of age to adulthood, but it has been modified in accordance with the Arab environment.

2.4 Population and sample

Researcher targeted young people in a period of adolescence (13-16 years), where the young people are the future leaders but have become in recent influential category in the decision, so it is attracting this category of political currents through the different colors. Since the main purpose of the interview these young people is to these young people in response to the search tool (IQ scale) and where there is agreement on the statement "a healthy mind in a healthy body" Gan researcher targeted this age group through practicing various sports activities at the level of all the games clubs Sports and through the largest two castles of Sport at the level of the Republic in Greater Cairo, two

- **Al Ahly club representatives to the Cairo Governorate.**
- **Zamalek Club representative of the Giza Governorate.**

2.4.1 Determination of the sample

Because the research population is composed by two distinct main stratum - the players during in Al-Ahli – club and players during the Zamalek club , so the researcher use stratified random sample .The researcher selecting sample size to estimate the population mean (μ) approach . **This procedure is called sample size to estimate the population mean (μ) with a bound on the error of estimation (B) Scheaffer et al.** (**1990)**. Because, the aim of the study is to estimate the mean value of the IQ .

Then the sample size = 189.88 $\cong$ 200 .

Due to the stability of the cost among a different games in both clubs so the researcher use Proportional Allocation method in determining the contribution of each of the two classes and age groups in each of them, and the following table presents the results of the research sample allocation and distributed as follows:

2.4.2 The characteristics of the sample

Table 1. : Frequencies & Percentage distributions to represent the sample characteristics .

Dist. Ages / Club-name	13		14		15		16		Total	
	Female	**Male**	**Female**	**Male**	**Female**	**Male**	**Female**	**Male**	**Female**	**Male**
Ahly-club	**10**	**10**	**20**	**17**	**9**	**9**	**21**	**10**	**60**	**46**
% (1)	**50.0**	**50.0**	**54.1**	**45.9**	**50.0**	**50.0**	**63.6**	**36.4**	**56.6**	**43.4**
% (2)	**16.7**	**21.7**	**33.3**	**37.0**	**15.0**	**19.6**	**35.0**	**21.7**	**100**	**100**
% (3)	**47.6**	**38.5**	**52.6**	**60.7**	**60.0**	**47.4**	**60.0**	**55.6**	**55.1**	**50.6**
Zamalek-club	**11**	**16**	**18**	**11**	**6**	**10**	**14**	**8**	**49**	**45**
% (1)	**40.7**	**59.3**	**62.1**	**37.9**	**37.5**	**62.5**	**63.6**	**36.4**	**52.1**	**47.9**
% (2)	**22.4**	**35.6**	**36.7**	**24.4**	**12.2**	**22.2**	**28.6**	**17.8**	**100**	**100**
% (3)	**52.4**	**61.5**	**47.4**	**39.3**	**40.0**	**52.6**	**40.0**	**44.4**	**44.9**	**49.4**
Total	**21**	**26**	**38**	**28**	**15**	**19**	**35**	**18**	**109**	**91**
% (2)	**19.3**	**28.6**	**34.8**	**30.8**	**13.8**	**20.9**	**32.1**	**19.8**	**100**	**100**

1. % was calculated for each dichotomous No. of gender (Male – Female) in each age category in related to total age groups.

2. % was calculated for each dichotomous No. of gender (Male – Female) in each age category in related to total row .

3. % was calculated for each dichotomous No. of gender (Male – Female) in each age category in related to total column (clubs) .

2.5 Studying the quality of the data for the obtained empirical distribution of the data collected

2.5.1 Studying the extremes observations of the sample

Statistical descriptive for IQ-Scores of the sample observations

The following table represents some of statistical measures to describing the IQ-Scores of the sample studying.

Table 2. : Descriptive statistics for IQ-Scores by different ages

Ages	Measures of tendency			Measures of dispersions		Measures of distributions	
	Mean	Median	Mode	Std. Dev.	C.V %	Skewness	Kurtosis
13	103.23	102.00	102	6.25	6.05	0.15	1.80
14	102.45	102.00	102	3.61	3.52	-0.25	4.45
15	100.88	102.00	102	5.75	5.70	-0.40	0.35
16	100.75	100.00	102	3.75	3.72	-0.38	1.68

Table 3. : Descriptive statistics for IQ-Total Scores

Measures of tendency				Measures of dispersions		Measures of distributions	
Mean		Median	Mode	Standard deviations	C.V %	Skewness	Kurtosis
Mean	Tr(mean)(1)						
101.92	101.96	102.00	102	4.84	4.75	0.00	2.53

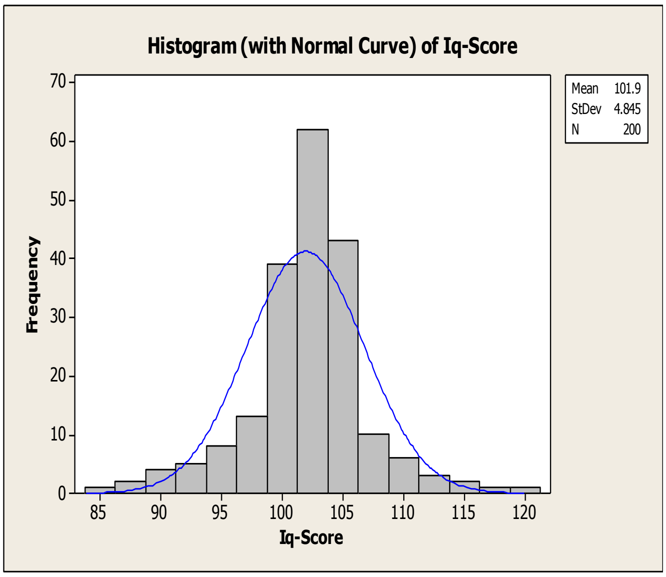
1. Denote to 5 % trimmed mean .

From the previous results of statistics for intelligence of research sample we can found that there is a very limited of odd observations . only representing . However, these views are not effective in the presence of a degree of bias in the statistical indicators values. It has seemed so clearly in all of the value of the coefficient of variation, which came very limited, as well as the value of trimmed mean, which did not differ from the final value of the computational center.

2.5.2 Studying the empirical distribution of the data of the sample study with Normal Curve

2.5.2a The Histogram of the empirical distribution of IQ – Scores

The following the **Histogram** of the empirical distribution of IQ – Scores of the sample study with **Normal Curve**



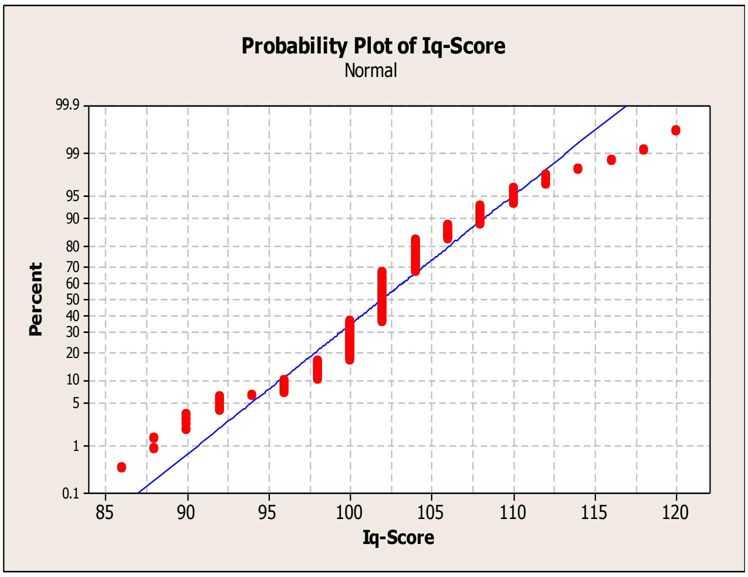


Fig. 1. : Representing the empirical distribution of IQ-Scores of the sample observation with Normal Curve .

Fig. 2. : Representing the empirical distribution of IQ-Scores of the sample observation with Normal Curve .

Previous results confirmed that the distribution of the IQ-Scores of the_sample observations is approximately to the normal distribution as emphasized by the following:

- The results of measure of tendency, that satisfy (mean = median = Mode).
- Results - measures of distributions where the results were to emphasize the (Skewness = 0) & (Kurtosis ≅ 3) .
- The histogram is nearly approximating with the normal curve , which seemed so clearly (Fig. 1), in addition to the distribution of views about the natural curve line, which applied to part of the normal distribution line Views remainder came proportionately.

2.6 The results of Bootstrap with crisp data.

We generate a crisp bootstrap random sample drown randomly with replacement . We need a distribution that estimate the population parameter times under H_0 . We show the following the distribution of the statistic mean computed using 10000 bootstrap samples .

Table 4. : Descriptive statistics for IQ-Total Scores for Bootstrap Results with crisp data

Measures of tendency			Standard deviations	CI with 95 %		Measures of distributions	
Mean	Median	Mode		LL(1)	UL(2)	Skewness	Kurtosis
101.92	101.91	101.94	0.345	101.24	102.61	0.05	0.07

1. **The 5% point is the 500th largest value of the bootstrap distribution .**
2. **The 95% point is the 9500th largest value of the bootstrap distribution.**

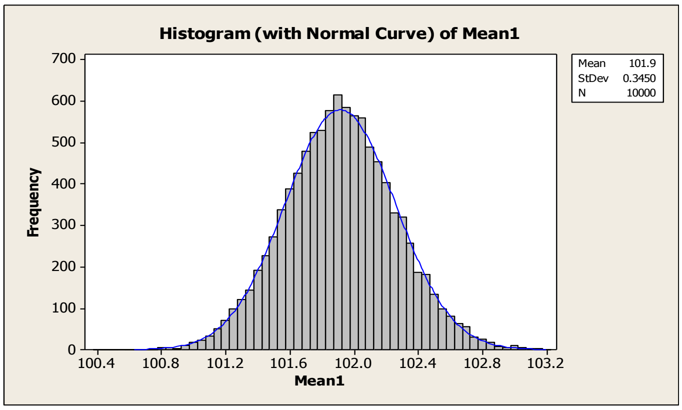


Fig. 3. : Representing the distribution of statistic mean's of IQ-Scores of the bootstrapping samples with Normal Curve .

To make sure if there is the presence of the state of cloudiness, that known as a non-decision .The researcher conducting test about population parameter (mean) for all bootstrap samples (10000 samples) through the null hypothesis and alternative hypothesis , which was determined as follows:

$\mathbf{H_0 : \mu = \mu_0}$ **is to be tested against** $\mathbf{H_1 : \mu \neq \mu_0}$ **.**

$\mathbf{H_0 : \mu = 101.92}$ **V.S** $\mathbf{H_1 : \mu \neq 101.92}$ **.**

Testing H_0 against H_1 by a two-sided , One-sample test about mean , we take each sample as an independent random sample with the same size contains (200 observations)

The following table illustrate the results of the previous hypotheses for all bootstrapping samples.

Table 5. : Frequency and Percentage distribution for study cases in three different situations of the bootstrap Results with crisp data

index	Study Cases	Frequency	%	Cum. %
1-	Sig. cases (False decision)	488	4.88	4.88
2-	Fuzzy cases	117	1.17	6.05
3-	Not Sig. cases (True decision)	9395	93.95	100
	Total	10000	100	

From the previous table shows there is a 117 Cloudy cases (non-decision) by 1.17%, and this a percentage was not bad, because the following reasons.

1- Bootstrap process has completed with a limited and known population parameter , and that the test which was conducted was on these parameters and on the samples that have been drawn from the same population, so the appearance of the no decisions cases within the results with this ratio that means the no decision cases must be valid .

2- Enlarge the sample size that affected of this ratio in the limited form.

3- The spread of a disease in a country for this ratio, is considered disaster.

In addition there exist 488 cases confirms the existence of significant differences between the parameter of the population with the results of bootstrap samples, and this cases represents for 4.88%, in other words it can be said that the test has accuracy about 94% of the results of the bootstrap samples.

It will be a more detailed Studying for " **No decision – cloudy"** in order to convert the "No decision state" to the case of a clear decision whether this decision is the decision by the cases of

- **Sig. cases (Wrong decision) .**
- **Not sig. cases (Right decision).**

Now we can propose the name of "Clear decision" to includes the two groups refer to them. The study will be two groups in some detail later.

So the researcher proposed the following approaches in order to reach a clear solution.

1- The first approach is dominated by the statistical form.

2- The second approach uses a Fuzzy which depends on the results of the bootstrap crispness.

The following representation for the results of the approaches so as follows.

3 Results for the First Approach (Crisp case)

3.1 Studying the Effect of Type II Error on the P-value

It will be the study of the effect of Type II error as (independent variable – explanatory variable) - E on the P-value - C .

Where C is the evaluation procedure i.e the test statistic and the critical value(s) . Also we can say we have ordered paired (E,C) in a universal set (Ω) , each of $E \in [0, 1]$ and $C \in [0, 1]$.

Let P denote to the evaluation procedure for the test statistics, and the critical value(s). where p is the real number $p \in [0 , 1]$ and b denote to the value of type II error (the probability of non-rejecting H_0) the ordered pair (p , b). The crisp set (Ω) , the so-called universe , is assumed to consists of all the sampling procedures , and statistical tests , for instance , of one- or two sided , *T*- or *F*- tests depending on the concrete underlying problem [Arnold 94].

For each group , as a follow :

3.1.1 study the effect of Type II error on the P-value in the Sig. cases

From the results of the figure (4) we get that emphasized the optimal mathematical form that meets the effect of Type II error on the P-value is **Exponentional Form**, also from the figure form we get we have a decreasing the function and not increased.

The following table shows estimate the model parameters of the simple regression analysis using the ordinarily least squares method .

Table 6. : The results of simple regression of effect of type II error on P-value in significant cases

Ln(P-Val)=303.805 – 15.220b_tot **(5.085**)(1) (-49.635**)(1)**	
F-ratio = 2463.609	**, d.f=(1 ; 478)**
Sig. F=0.0000	**, (Sig at 0.01)**
R-Square=83.8%	**S.E = 0.356**

1. Denote to the values of T-test .

** **Denote to there exist a sig. at 0.01 level of sig. for T-test and F-ratio .**

The results of the previous table emphasized that the regression model is highly significance (F-calculated= 2463.609) which confirms the statistical significance at 0.01 level of significance with degree of Freedom (1 , 478). Also that the results confirmed that the changes in the values of P-value due to the changes in the Type II error about 84%.

3.1.2 study the effect of Type II error on the P-value in the Not Sig cases

From the results of the figure (5) we get that emphasized the optimal mathematical form that meets the effect of Type II error on the P-value is **Exponentional Form**, also from the figure form we get increasing function and not decreased.

The following table shows estimate the model parameters of the simple regression analysis using the ordinarily least squares method .

Table 7. : The results of simple regression of effect of type II error on P-value in not significant cases

Ln(P-Val)=0.002 + 7.037b_tot **(40.563**)(1) (230.282**)(1)**	
F-ratio = 53029.72	**, d.f=(1 ; 9393)**
Sig. F=0.0000	**, (Sig at 0.01)**
R-Square=85.0%	**S.E = 0.269**

1. Denote to the values of T-test .

** **Denote to there exist a sig. at 0.01 level of sig. for T-test and F-ratio .**

The results of the previous table emphasized that the regression model is highly significance (F-calculated= 53029.72) which confirms the statistical significance at 0.01 level of significance with degree of Freedom (1 , 9393). Also that the results confirmed that the changes in the values of P-value due to the changes in the Type II error about 85%.

From the above mentioned results we obtain whether the groups are different . And the following are the differences between the two groups :

- **The direction of the relationship (increasing - decreasing).**
- **The values for the standard error for each of the two relationships.**

They also agree in terms of

- **The mathematical form of the relationship.**
- **The degree of explanatory.**
- **The level of significant for the estimated model.**

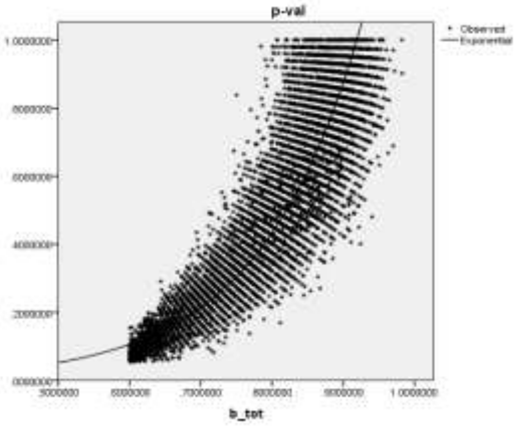


Fig 4. : Representing the relationship between type II error with P-value for the not Sig. cases .

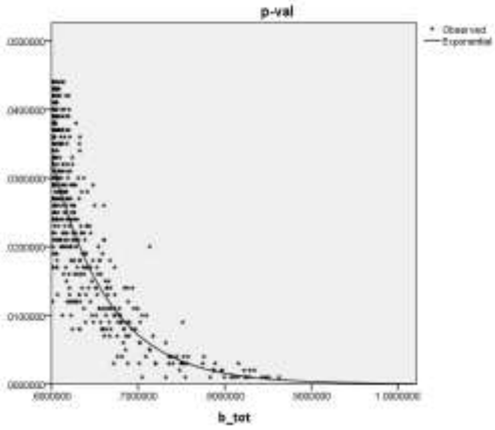


Fig 5. : Representing the relationship between type II error with P-value for the sig. cases .

3.1.3 study the effect of Type II error on the P-value in the cloudy cases

From the results of the figure (6) we get the figure has no clear direction meets the existence for the mathematical form (Mathematical Function) the existence of the effect of the P-value on the type II error. But it is possible to say that it can not measure the relationship between the two variables Using a famous mathematical functions.

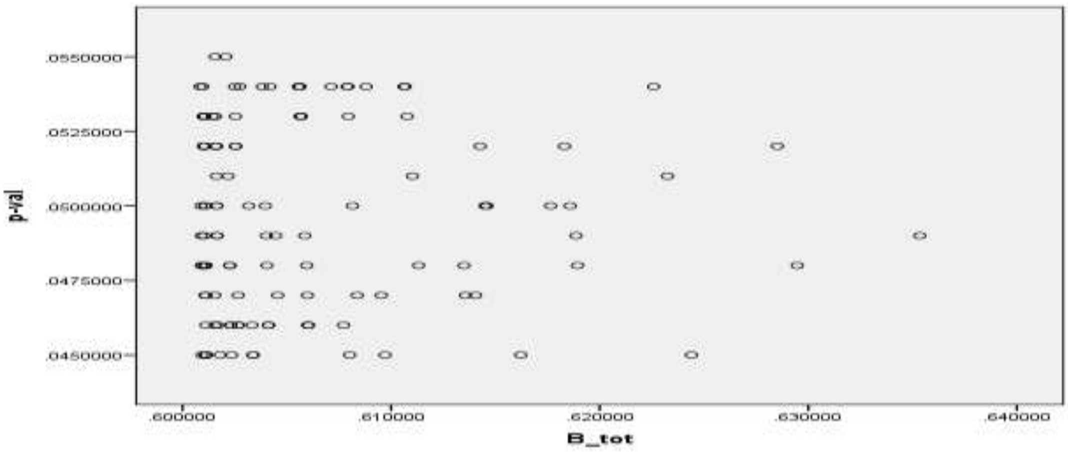


Fig 6. : Representing the relationship between type II error with P-value within the fuzzy cases .

3.2 Study the Discrimination Between Cloudy Group with Each of the Other two Groups

Will be presented the results of the degree of discrimination between the Cloudy group with the following :

- Sig. cases.
- Not Sig. cases.

The following table is encodes the groups as a follows:

Table 8. : Listing for the three different groups and its Codes

Index	Groups	Codes
1-	**Sig. cases (False decision)**	G_0
2-	**Cloudy cases**	G_1
3-	**Not Sig. cases (True decision)**	G_2

Therefore are presented for each of the models are as follows:

Table 9. : The results of discrimination models for Fuzzy Group Cases versus True and false decision groups

Index	Groups in a model	Estimated model parameters and its tests	Total correctly Classified (%)
1-	**(G_0 Vs G_1)**	**Z = 5.717 + 12.545 ln(b_tot)**	**56.4%**
		χ^2 = 44.395 , df =1 , Sig. = 0.000 (sig. at 0.01) **Canonical Corr.= 0.267 , Wilks Lambda=0.929**	
2-	**(G_1 Vs G_2)**	**Z = 1.959 + 8.469 ln(b_tot)**	**94.6%**
		χ^2 = 597.330 , df =1 , Sig. = 0.000 (sig. at 0.01) **Canonical Corr.= 0.247 , Wilks Lambda=0.939**	

<u>The results of the above table confirmed that there is a clear distinction between cloudy group with the third group (code 2) (Group with the right decision), and on the contrary, the results between cloudy group versus the first group (wrong decision)</u>

3.3 Using The Binomial Distribution

For more detail the researcher used the binomial distribution, which one of the famous discrete distributions.

Where that is required what about the probability of belonging the cloudy group to set the right decision and then, this is consider the possibility of success and on the contrary, represents the probability of failure. This procedure will be as follows:

3.3.1 Select a certain sample from a cloudy group.

The selection of the sample results from a cloudy group , which will be a Medium degree in terms of the value of P-value so sample was selected by the median value as the number of cases individual Fuzzy, which represent (N1 = 117), it represents the sample (N1 <59>) that is, they represent a sample No. 59 after an order (ascending or descending).

3.3.2 The results of Bootstrap for fuzzy selected sample .

We generate a crisp bootstrap random sample drown randomly with replacement . We need a distribution that estimate the population parameter times under H_0 .

We show the following the distribution of the statistic mean computed using 10000 bootstrap samples .

Table 10. : Descriptive statistics for IQ-Total Scores for Bootstrap for Fuzzy sample

Measures of tendency			Standard deviations	CI with 95 %		Measures of distributions	
Mean	Median	Mode		LL(1)	UL(2)	Skewness	Kurtosis
101.24	101.24	101	0.342	100.67	101.80	-0.013	-0.094

1. The 5% point is the 500th largest value of the bootstrap distribution .

2. The 95% point is the 9500th largest value of the bootstrap distribution.

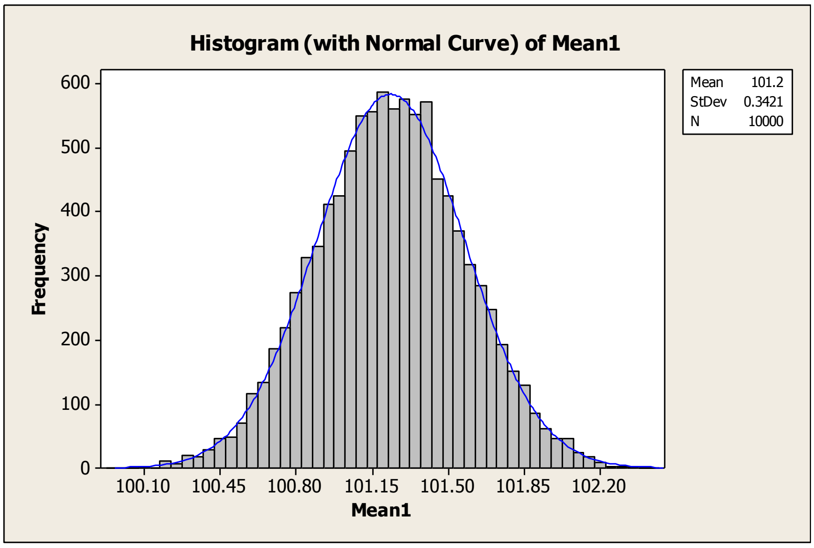


Fig 7. : Representing the distribution of statistic mean's of IQ-Scores of the bootstrapping samples for the Fuzzy sample .

Table 11. : Frequency and Percentage distribution for study cases in three different situations of the bootstrap Results with crisp data

Index	Study Cases	Frequency	%	Cum. %
1-	Sig. cases (False decision)	4931	49.31	49.31
2-	Cloudy cases	356	3.56	52.87
3-	Not Sig. cases (True decision)	4713	47.13	100
	Total	10000	100	

From the previous table shows the presence of 356 cloudy cases (no-decision) with 3.56%, and this represents a significant proportion compared to the previous results (1.17%), but we are not interested in this category at this time. And therefore it will be treated as Sig. cases (False decision) or Not Sig. cases (True decision), and therefore will be reset calculate percentage distribution, which represents 51.1% for the first category (False decision), and 48.9% for the group III (True decision).

We conclude that the opportunity to make a decision between the choosing first decision , and third decision almost equal, but they tend toward the first direction but very slightly.

3.3.3 Using Binomial Distribution .

For estimating the parameter ("$\hat{P}$") of Binomial distribution , depending upon the previous results .

"$\hat{P}$" $\longrightarrow$ **Not sig. case (Right decision).**

Then ("$\hat{P}$" =0.489) , and by apply the Normalization rule .

Where N=117 , P=0.489 , X = 59,60,……..117 . Then

The mean of distribution NP=N $\hat{P}$ =**0.489*117**

The Standard deviation of distribution NPQ= $\sqrt{0.489 * 0.511 * 117}$

Now we want to determine [Pr(X ≥ 59)]

Where 59 represents a the marginal value, which represents the availability of the phenomenon (Right decision) constitutes a large The results were as follows:

Table 12. : The results of Binomial Normalization for resulting of fuzzy sample .

Cumulative Distribution Function [(1)]
Normal with mean = 0 and standard deviation = 1
x P(X <= x)
0.333 0.630433

1. Using Minitab Ver.(16.0)

Then Pr(X ≥ 59) = 1 – Pr(X < 0.333)= 1 – 0.630= 0.37

Which reflects the results are moving more in the opposite direction .

- Sig. cases (False decision) .

4 The Second Approach Uses a Fuzzy Rules Which Depends on the Results of the Crisp Bootstrap .

4.1 General membership function :

Let the null hypothesis (H_0) and the alternative (H_1) be fuzzy sets on the universe Θ , where ($\Theta \subset R$) with membership functions . We have the score of IQ's of the random sample of the players . Also We generate 10,000 random samples (table 14 , fig. 5) , Then we summarize this results by using some of statistics such as :

- **Min , Max.**
- **Quartiles**
- **Some of other Percentiles .**
- **CI with 95% .**

Then by using the above statistics we construct the Critical Regions (C_1 , C_2), $\tilde{\theta}$ = {(100.38,0) , (100.65, 0.025) , (101.68, 0.25), (101.91, .5), (102.14, .75), (01.13 , 0.975) (103.20,100)}

4.2 Fuzzy Hypothesis :

We can define the following fuzzy hypothesis to formulate a fuzzy hypothesis which to facilitate the solve a problem :
H0 : (H : is approximately θ_0) Vs H1 : (H : is not approximately θ_0)
Where θ_0 be a real number and known such as mean value also this hypothesis is a fuzzy simple hypotheses with two-sided hypothesis .
Also in terms of we can write other formulation as a follow:

H_0 : θ is $\tilde{H}_0$ **H_1 : θ is $\tilde{H}_1$ [Akbari 2010 et al] .**

4.3 Membership function To be Clear a decision.

Statistical test that means null hypothesis to be tested against the crisp alternative [Arnold 94] . representing for different cases:

- **We assume that the expert is able**
- **If $mH_0(\theta) = 0$ means that θ does not belong to H_0 at all .**
- **If $mH_0(\theta) = 1$ means that θ completely lies in H_0 .**
- **$mH_1 = 1 - mH_0$.**
- **At least one $\theta \in \Theta$ with $mH_0(\theta) > mH_1(\theta)$, and at least one $\theta \in \Theta$ with $mH_0(\theta) < mH_1(\theta)$.**
- **The membership functions mH_0 and mH_1 there has to be a close cooperation between the statistician and expert who wants to test (H_0) .**
- **Let us assume that $mH_0(\theta)$ and $mH_1(\theta)$ are known and we have to decide whether H_0 is rejected or not. Supposing for the moment that *θ* is known then**
    1. **H_0 is rejected if $mH_0(\theta) < mH_1(\theta)$.**
    2. **H_0 is not rejected if $mH_0(\theta) > mH_1(\theta)$.**
    3. **H_0 may be rejected or not if $mH_0(\theta) = mH_1(\theta)$.**
- **Another problem is how the two fuzzy sets (type II error , P- value are to be aggregated . Although there is a great variety of intersection and others operators . The minimum operator proposed in most commonly used in optimization problems.**

4.4 Construction of membership function

4.4.1 membership function for bootstrapping results for means .

Our Phenomenon Behave Triangular function because we target studying a unique parameter , then consequently , We consider piecewise linear membership functions mH_0 and mH_1 , first we concern for constructing the membership function for of H_o which naming mh_o as a follow :

$$\mu_A(x) = \begin{cases} 0, & x \le a \\ \dfrac{x-a}{m-a}, & a<x\le m \\ \dfrac{b-x}{b-m}, & m<x<b \\ 0, & x\ge b \end{cases}$$

Also , we define Triangular function by "a" is a lower limit , "b" is an upper limit and m value is a parameter which we want to estimate it .

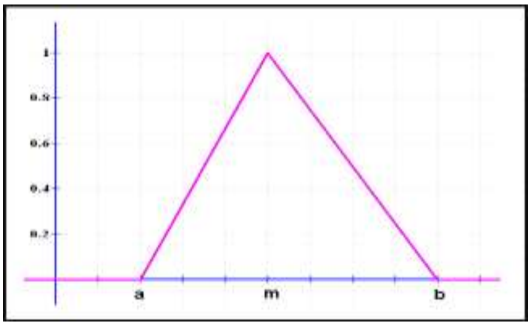


Fig 8. : Representing a Triangular function to construct the membership function for the results of the bootstrapping for a unique parameter .

The following table represents some of crisp statistics for Bootstrapping Distribution will be used to constructing the membership function as a follow :

Table 13. : Some of descriptive statistics for bootstrapping results for distributions for means

Index	Statistics	Values
1-	**Mean , Sd.**	101.9133 , 0.29398
2-	**CI at α :** 0.05 0.01	 101.4200 , 102.4000 101.3000 , 102.5300
3-	**Quartiles:** First , Second , Third	 101.7000 , 101.9100 ,102.1300
4-	**Min ,Max**	100.38 , 103.200

4.4.2 The characteristics of the membership function for bootstrapping results for means .

Table 14. : Frequency and percentage distribution for the result of membership function

Variable characteristics	Frequency	%
$X \le a$	1	0.0
$a < X < m$	5005	50.1
M	134	1.3
$M < X < b$	4859	48.6
$X \ge b$	1	0.0
Total	10000	100

4.3.3 membership function for bootstrapping to facilitate decision making .

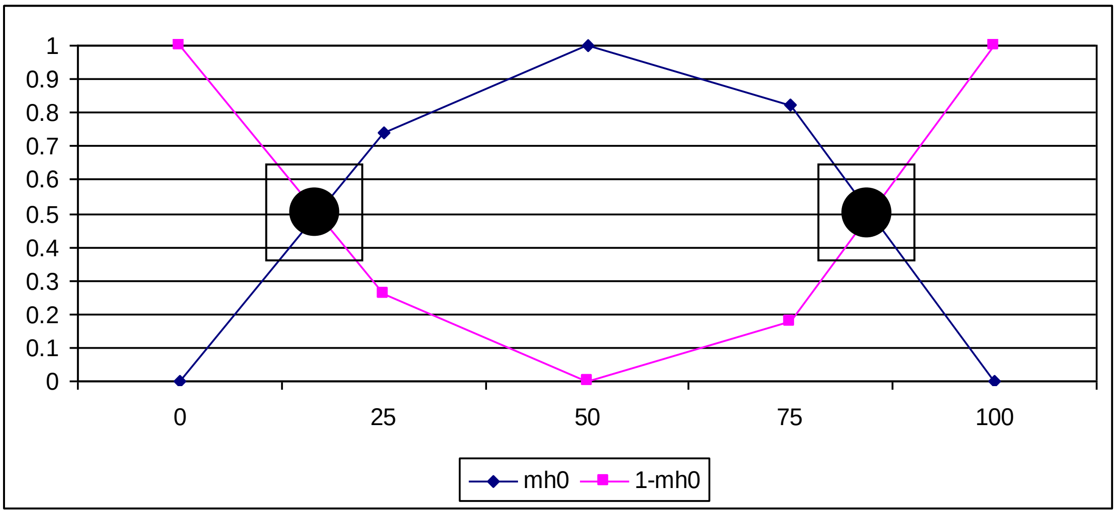


Fig 9. : Representing the membership function for the results of the bootstrapping to take a decision reject or non-reject H_0 .

From the previous figure we able to classify the decision areas as a follow :

Table 15. : The summarize of the decision areas of the previous membership function.

Area	The results for membership function	The decision
1-	**$mH_0(\theta) < mH_1(\theta)$.**	**H_0 is rejected**
2-	**$mH_0(\theta) > mH_1(\theta)$**	**H_0 is not rejected**
3-	**$mH_0(\theta) = mH_1(\theta)$**	**H_0 may be rejected or not**

The following are the frequency distribution and relative .

Table 16. : Frequency distribution of the membership function .

The results for membership function	Frequency	%
$mH_0(\theta) < mH_1(\theta)$.	439	4.4
$mH_0(\theta) > mH_1(\theta)$	9544	95.4
$mH_0(\theta) = mH_1(\theta)$	17	0.2
Total	10000	100

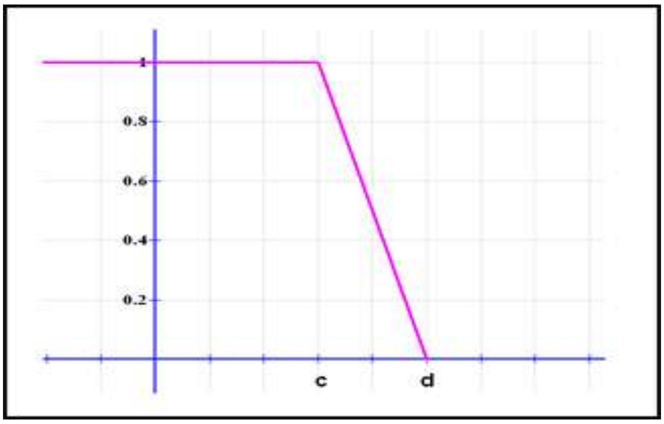
Results from the previous table shows that it can be considered that both the first and second cases represent "The clear decision ", and the third case was a "No decision case " and therefore represents the uncertainty region (cloudy case) , which will be studied in detail. Through studying the membership function on this pivotal point of intersection (± 0.1), through the results during the period, [0.4 , 0.6]. This is after a minimization of type II error, that through the maximizing of the membership function , that through bootstrapping for a sample of 1000 cases . Then determine the value of a membership function using intersecting and maximizing rule .

5 Solve the problem of the cloudy cases

5.1 membership function for type II error .

Our Phenomenon Behave trapezoidal function with special case for right function and we have non-increasing function , as a follow :

$$\mu_A(x) = \begin{cases} 0, & x > d \\ \dfrac{d-x}{d-c}, & c \le x \le d \\ 1, & x < c \end{cases}$$



Where **C** is lower limit (Minimum – Min) and **d** is an upper limit (Maximum - Max)

Fig 10. : Representing a Trapezoidal with right function to construct the membership function for Type II error.

Table 17. : Some of descriptive statistics for bootstrapping results distributions for Type II error

Index	Statistics	Values
1-	**Mean , Sd.**	0.792 , 0.098
2-	**CI at α :** 0.05 0.01	 0.789 , 0.793 0.789 , 0.794
3-	**Quartiles:** First , Second , Third	 0.716 , 0.817 , 0.872

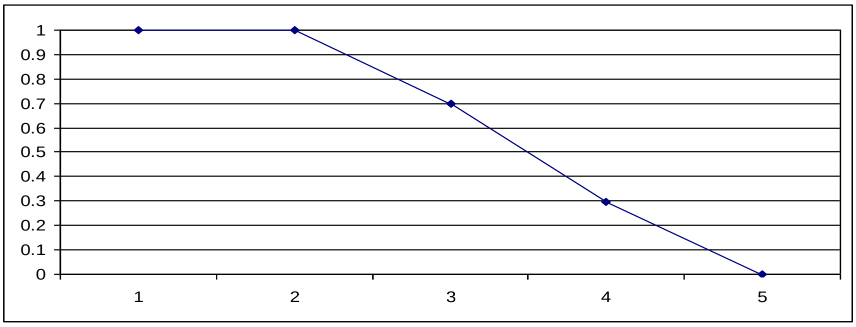


Fig 11. : Representing the membership function for the results of the bootstrapping of type II error .

5.2 The aggregations between the above two membership functions:

To intersect between the two fuzzy sets (type II error-E , bootstrapping results for means-M are to be aggregated) . There are variety of intersection and others operators The minimum operator proposed in most commonly used in optimization problems. Then we have to use MAX to MAX procedure as a follow:

m(E,c) :=min{m1(E),m2(c) }

m(E,c) :=MAX{ $M_{1(A)}$(E), $M_{2(A)}$(c) }

Then, we have to apply the above technique using the following Algorithm :

1) determine the area round the intersected point between mh_0 and mh_1 (± 0.1) and studying this area in detail . Then we concern the values of $M_{2(A)}$(c) within the two real numbers in the interval [0.4 , 0.6] , Then we have 880 cases

2) Use a bootstrap procedure to determine type II error for $M_{1(A)}$(E) as Maximizing.

3) The intersections between the maximizing of $M_{1(A)}$(E) with $M_{2(A)}$(M) .

4) repeat the steps (2,3) as 10000 one's .

5) Evaluate the results and take a suitable decisions procedure .

Then we concern the values of $M_{2(A)}$(c) within the two real numbers in the interval [0.4 , 0.6] , Then we have 880 cases covers this interval with 8.8% over all 10000 cases . The percent 8.8% is very important because this percent used to determine the sample size which will be selected from 880 different samples which equivalent 77 cases randomly selected from 880 cases in each .

The following table represent frequency and percentage distributions for the resulting over all the trials :

Table 18. : Frequencies & Percentage distributions for the resulting of bootstrapping of results of membership functions

The results	Frequencies	Percentage (%)
< 0.50 (Accept H_1)	2418	24.2
> 0.50 (Accept H_0)	7582	75.8
Total	10000	100

From the above results of the table we obtain the results which in cloudy (the intersection between the results of membership function of H_0 and H_1) the results tends to Accept H_0 . Consequently, This results more precise .

6 Conclusions

The problem of the occurrence of the value of test statistics in the cloudy area regardless of the phenomenon under study has a presence. This problem specially increased for appearance in the case of a small sample. Also this problem is very limited and more easier to solving when we use a Fuzzy technique comparing with a crisp case. Fuzzy approach can be used for a small size of random sample while with the crisp approach with accuracy of the results Which it is reflected positively in economic terms on the research budget and the number of papers published in turn affects the speed prepared and completed as well as quality. Finally, the results which obtained using fuzzy approaches more precise than crisp in when we have no decision.

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Tuning the Parameters of Modified Artificial Bee Colony Algorithm for Solving the Container Loading Problem

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ABSTRACT

This paper presents a tuning to the parameters of modified artificial bee colony algorithm for solving the container loading problem to increase the space utilization ratio. This problem is solved in two phases, the parameters of modified artificial bee colony after tuning and a wall building heuristic. In the first phase, modified artificial bee colony with its probabilistic decision rule is used to construct a sequence of boxes. The volumes of boxes are calculated with the wall building heuristic in the second phase. To tune the parameters, Taguchi's methods are used .it helps us to determine the best parameters value. Computational experiments were conducted on benchmark data set and the results obtained from the proposed approach are shown to be comparable with other methods in the literatures.

Key words: Bee Colony, Container Loading Problem, Taguchi's Methods

1. INTRODUCTION

Container Loading Problem (CLP) is an ongoing field and has many uses in the world, such as a transport companies and container distribution [1]. In the area of production and distribution of products the efficient use of transportation mediums such as containers and palettes is of high economic importance. A high use of the applied transportation capacities causes considerable savings. Further effects are the reduction of the good traffic and the protection of natural resources. Computer supported packing methods can considerably contribute to the achievement of these goals. The modeling of practical problems concerning the optimal utilization of transportation devices leads to different kinds of packing problems [2]. Most organization tries to minimize operational cost. CLP has been studied extensively by a lot of researchers with different objective functions and constraints. Majority of the studies focuses on providing solutions from heuristics and meta-heuristic approaches. Methods to obtain a feasible solution include the use of different data structures such as trees and graphs, heuristics algorithms and Meta-heuristic algorithms such as Genetic Algorithms (GA), Simulated Annealing (SA) and Tabu Search (TS). In this paper, tuning the parameters of modified artificial bee colony algorithm is used to solve the CLP. The paper is organized as follows: section two is assigned to the problem definition. Section three is assigned to the Literature review. Wall building heuristic is described in section four. Section five is assigned to Bee algorithm. Taguchi's methods are described in section six. Section seven is assigned to the solution structure.

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2. PROBLEM DEFINTION

CLP is an interesting and NP-hard problem to solve [3,4]. Our problem described as follows: Given a set of n boxes with width (wi), depth (di) and height (hi) and a single container with dimensions (W, D, H) where wi ≤ W, hi ≤ H and di ≤ D, the problem is to pack boxes into the container to maximize the utilization rate of the container [5,6]. The single container loading problem (fig.1) is a three dimensional packing problem in which a large rectangular box (the container) has to be filled with smaller rectangular boxes of different sizes.

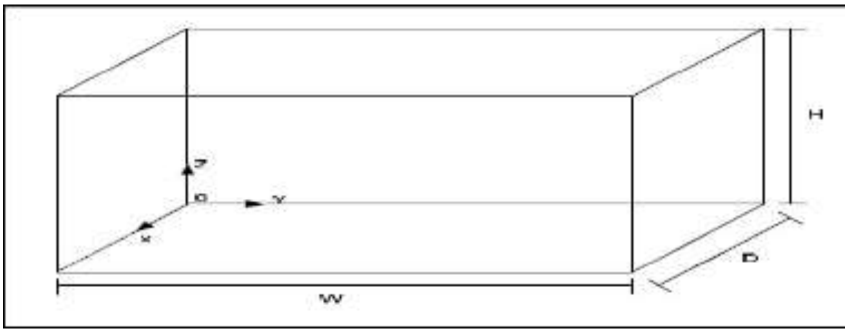

Fig.1 Container dimensional [1]

Container loading problem was formulated by Seow et al. [6]. Given a set of boxes B= $\{b_1,\ldots,b_n\}$ and a container C. Let x_i, y_i and z_i represent the three dimensions of the box b_i, and X, Y and Z represent the three dimensions of the container. The objective of the problem is to select a subset $S \subseteq B$ and assign a position to each box, $b_j \in S$, in the 3D space of container C such that $\sum_{bj \in S} x_j \times y_j \times z_j$ is maximized. The main aim of the CLP is to find the best Space Utilization Ratio (SUR) as formulated as eq.1:

$$\text{Max SUR}= \frac{\sum_{a=1}^{b} Va}{C} \quad \textbf{(1)}$$

Where: b = Quantity of packed box, Va = Volume of the ath packed box, C = Volume of the container. Subjected to the constraints that all boxes must be totally contained in C, no two boxes intersect in 3D space and every edge of the packed box must be parallel or orthogonal to the container walls [7,8]. The problem is solved under the following assumptions:

- The boxes have known dimensions (w_i, d_i, h_i).
- The boxes can be rotated.
- Boxes can be stacked on top of other.

3. LITERATURE REVIEW

Heuristics have been an applicable alternative to find optimal or near optimal packing. Several heuristic procedures have been proposed for solving the CLP. Bortfeldt and Gehring [4] presented a hybrid genetic algorithm (GA) for the container loading problem with boxes of die rent sizes and a single container for loading. Eley [9] showed a new algorithm based on the generation of homogeneous block arrangements was developed in order to solve single container problems. Morabito and Nales [10] presented a new approach to solve unconstrained two dimensional non-guillotine cutting problems, where a rectangular shaped plate must be cut into rectangular shaped pieces and all cuts are orthogonal to one side of the plate. Bortfeldt and Gehring [2] presented a parallel tabu search algorithm for solving the container loading problem with the parallelization approach. Fanslau and Bortfeldt [11] classified approaches for the CLP according to packing heuristic and method type. Bischoff and Ratcliff [12] presented highlighted some important short comings in the existing theoretical literature on container loading problem. Pisinger [13] presented a heuristic algorithm for the Knapsack Container Loading Problem.

4. WALL BUILDING HEURISTIC:

The wall building heuristic introduced by Pham et al [14], is most commonly used and modified by later researchers for its high efficiency and high quality [15,16]. It should be noted that, for each meta-heuristic approach when solving CLP, a wall building heuristic is needed to solve the problem. Without this heuristic, we cannot compute the objective function value of a solution which is volume utilization in the classical CLP.

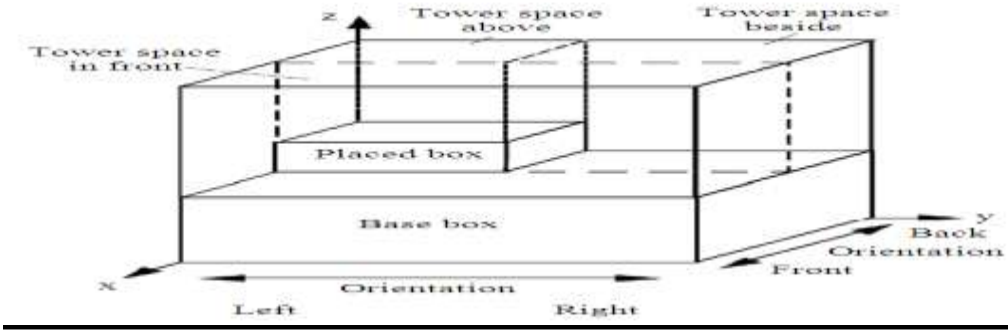

Fig.2. Empty space in front and above of the box [6].

4.1 Layer dimensions

Before starting the filling process, it is important to determine the dimensions of a layer according to priority of arrange box, since the width of a layer must be carefully selected to obtain a good performance [13,17].The width of each layer w_L is set equal to the width of the Box [18,19]. In order to determine the box, first the boxes among the set of available boxes are sorted by width dimension in non-increasing order. Thus, the box with the greatest width dimension is given high priority.

4.2 Fill layer

After the determination of the layer dimensions, it is possible to fill the layers. When the first box is allocated into the layer, three empty new spaces namely beside, in front and above of the packed box are produced. In the given situation (Figs.2), only two of these spaces in front and above occurs as a result of the allocation of the first box into the layer as showing above in fig.2 [1,5].

Once box packed into the container the layer is determine. The layer as shown in Fig.2 and it is desired to pack the next highest priority box in the list of available boxes into the container. If it is possible to allocate this box into this space, then it is packed there and removed from the list of available boxes. The process is repeated more and more for each box in the list of available boxes and for all empty spaces available in current layer until it is not possible to fill empty spaces with a box in the list of available boxes.

5. BEE's COLONY:

Artificial Bee Colony Algorithm (ABC), first introduced by Karaboga in 2005 for real parameter optimization, is a recently introduced optimization algorithm which simulates the gathering behavior of a bee colony [20,21]. ABC classifies into three groups, namely, employed bees, onlooker bees and scout bees. Employed bees search the food around the food source in their memory, mean while they pass their food information to onlooker bees. Onlooker bees tend to select good food sources from those founded by the employed bees, and then further search the foods around the selected food source. Scout bees are translated from a few employed bees for search about food source randomly.

When all employed bees complete their searches, they start sharing their information related to the nectar amounts and the positions of their sources with the onlooker bees on the dance area. An onlooker bee evaluates the nectar information taken from all employed bees and chooses a food source site with a probability related to its nectar amount. This probabilistic selection depends on the fitness values of the solutions in the population. A fitness based selection scheme might be a roulette wheel, ranking based, stochastic universal sampling, tournament selection or another selection scheme [22]. In basic ABC, roulette wheel selection scheme in which each slice is proportional in size to the fitness value is employed as follows:

$$p_i = \frac{\text{fitness}_i}{\sum_{i=1}^{SN} \text{fitness}_i} \quad (2)$$

Where f_{itness} is the fitness value of solution i. obviously, the higher the f_{itness} is, the more probability that the i_{th} food source is selected.

Main steps of the artificial bee colony algorithm are summarized in the following flow chart

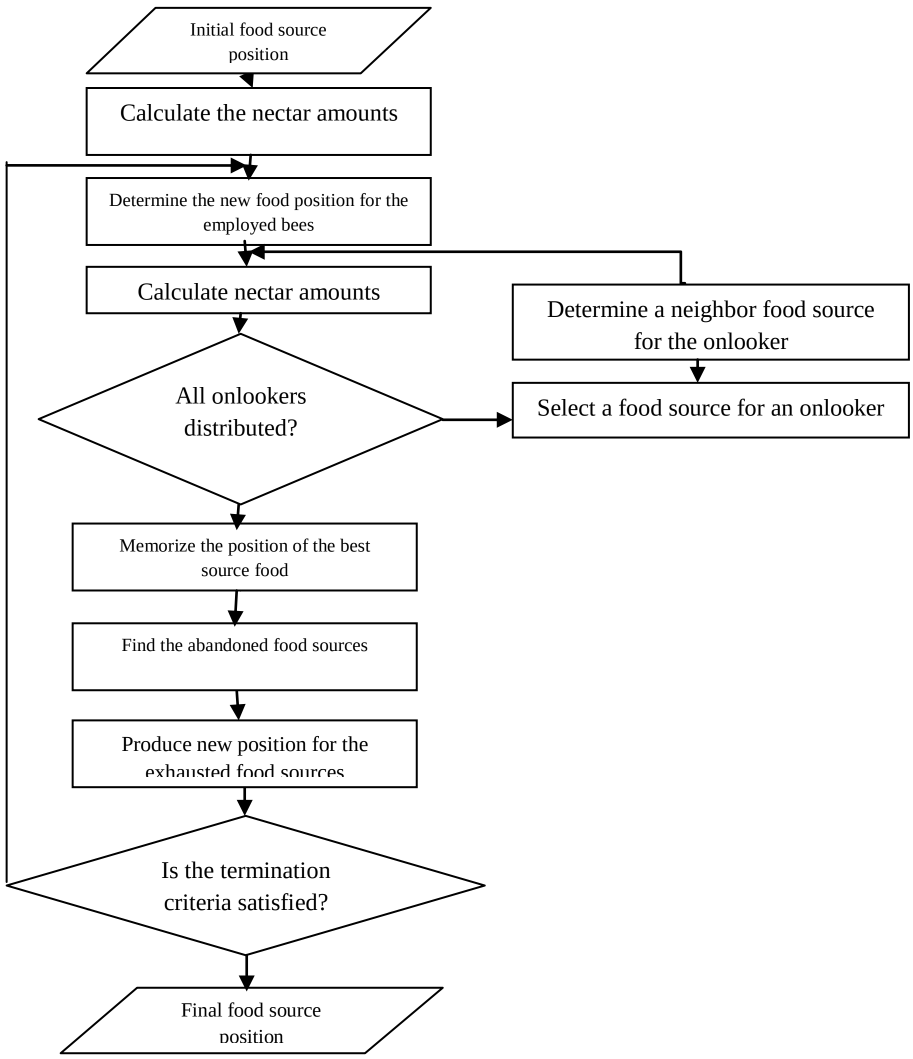

Fig.3. Flowchart of ABC algorithm [28].

6. TAGUCHI'S METHODS:

When the Second World War finished, the army forces found the quality of the telephone system was extremely poor and totally unsuitable for long term communication purposes. To improve the system the army command recommended establishing research facilities in order to develop the Japanese communication system. Genichi Taguchi tries to improve the R&D enhancing and productivity product quality. He found money and time is expended on engineering experimentation and testing [23]. Taguchi developed a new method to optimize the process of engineering experimentation. He thought that the best way to increase quality was to design and build. He developed the techniques which now called *Taguchi Methods*. The main contribution is not in the mathematical formulation of the design of experiments, but the accompanying philosophy. Taguchi Methods produce a unique and powerful quality improvement technique that differs from traditional practices.

6.1 Taguchi's methods to Parameter Design

Taguchi's methods to parameter try to find present the design engineer experimentation d testing with an efficient and systematic method for try to find near optimum design parameters for cost and performance. The objective is selecting the best combination control parameters for this reason the process or product is most robust with respect to noise factors.

Taguchi's method use orthogonal arrays from design of experiments theory to study a large number of variables with a small number of experiments.

Using orthogonal arrays play the main role to reduce the number of experimental configurations to be studied. [24].

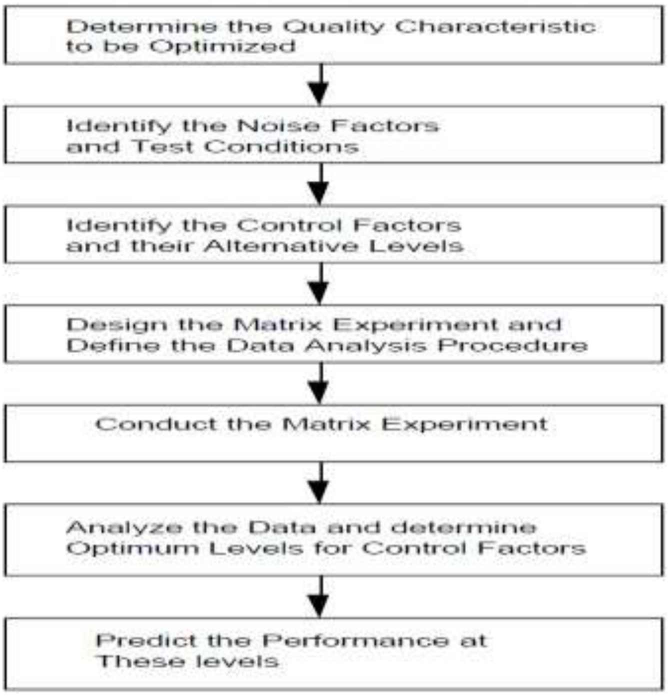

Fig.4 flowchart of Taguchi methods [29]

Taguchi method is applied in four steps [23]:

- Brainstorm the quality characteristics and design parameters important to the product/process.
- Design and conduct the experiments.
- Analyze the results to determine the optimum conditions.
- Run a confirmatory test using the optimum conditions.

6.2 Tuning the parameters of ABC algorithm using Taguchi methods:

The parameters and level are listed below:

Number of bees (population) N

N1=10, N2= 20, N3= 30

Number of selected sites M

M1= 2, M2= 4, M3= 6

Number of elite sites E

E1=1, E2= 2, E3= 3

Iterations I

I1= 100, I2= 300, I3=500

In our problem the total number of experiments that should be done to solve one problem according to the factors and levels are equal to $3^4 = 81$ experiments. According to the cost and the time of this amount of experiments, taguchi's method going to determine the best combinations of those factors and levels. The taguchi's method begins by selecting the orthogonal array by calculating the total degree of freedom. The total degree of freedom equal to $1 + (4 \times 2) = 9$. So, the orthogonal array at least has 9 rows as shown below.

Worksheet 3 ***

↓	C1	C2	C3	C4	C5
	Number of bees	Number of selected sites	Number of elite sites	Iterations	
1	10	2	1	100	
2	10	4	2	300	
3	10	6	3	500	
4	20	2	2	500	
5	20	4	3	100	
6	20	6	1	300	
7	30	2	3	300	
8	30	4	1	500	
9	30	6	2	100	
10					

Fig.5 Orthogonal array of Taguchi methods.

As shown in fig.6 and fig.7 the main effects plot for mean and the SN ratios of Taguchi's methods using Minitab.

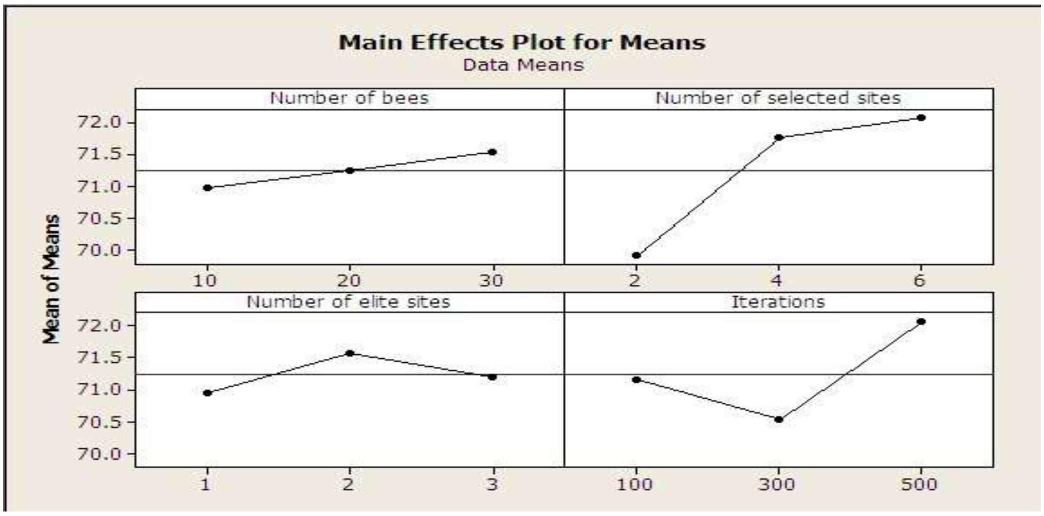

Fig.6 Means effect of Taguchi methods.

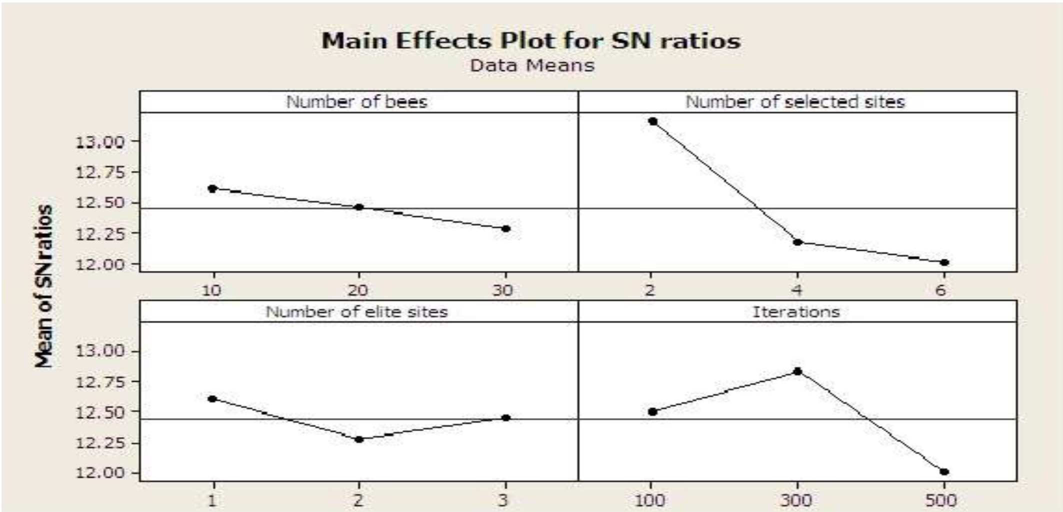

Fig.7 Signal Noise (SN) ratios of Taguchi methods.

7. USING PARAMETERS AFTER TUNING TO SOLVE THE CLP:

In our problem the priorities of the boxes are changed by enabling or disabling the rotation of the boxes. Each bee in the population represents a bit string of length equal to the number of the box types of the given CL problem. Each bit in this string shows an alternative orientation of a box type. The MABC approach makes an analogy to a honey bee colony that tries to find promising food sources in hive. The natural food for aging process of a honey bee colony starts with a number of scout bees from the colony searching for food sources. When scout bees find a rich food source, they begin the so called "waggle dance" in the hive [25].This dance, which is a form of communication between the bees in the colony, includes information about the distance, direction and quality of the food source. Equipped with this important knowledge, the colony sends follower bees more follower bees are sent to more promising food sources to these food sources to collect food. While collecting it, bees evaluate the food level of the source and collect the needed information for the next waggle dance. The employed MABC mimics the food for bagging process of the honey bee colony [26]. There are several parameters should be determined. These key parameters are the number of bee n, the number of selected sites m, the number of elite sites e chosen from m sites, the number of iteration. The step of ABC algorithm after tuning with CLP is summarized as follows:

Randomly generate a set of solution as initial food sources V_i, $i = 1,2,3,\dots,n$

V_1	1	3	2	4	…..	# of boxes

V_2	3	4	5	1	……	# of boxes

.

V_{20}	3	4	5	1	……	# of boxes

Evaluate the fitness f (Vi) of each of the food source xi

F (V1)

F (V2)

.

F (V20)

We chose number of elite sites (for example V1&V2)

For V1 the neighborhood ṽ1 is

$\tilde{v}_1$	3	1	4	2	# of boxes

Evaluate the fitness f(ṽ1) F(ṽ1)

If F(ṽ1) < F(V1), Then we replace V1 by ṽ1

F(ṽ2) < F(V2), Then we replace V2 by ṽ2

For new food: each onlooker bee

F(ṽ1), F(ṽ2)

$P_1 = \tilde{v}1 / (\tilde{v}1 + \tilde{v}2)$

$P_2 = \tilde{v}2 / (\tilde{v}1 + \tilde{v}2)$

If $p_2 > p_1$ the onlooker bees select more nectar food which is ṽ2 repeat that until stop role occurs

COMPUTATIONAL RESULTS:

The computational experiments are conducted on benchmark data set from the literature. The data set consists of 15 problem instances from LN named LN1 to LN15 proposed by Loh and Nee [27], each with a different number of box type and container's dimensions. The characteristic of the problems change from weakly to strongly heterogeneous. The proposed was coded using Microsoft Visual C#. The parameters of the MABCO for test case according to Taguchi's method are set to the following; number of bees (population) n = 30; number of selected sites m= 6; number of elite sites e = 2; iterations= 500 (fig.6).

Table 1: LN test problem results with different methods

Prob no	Loh and Nee[29]	Ngoi et al [30]	Bischof et al [31]	Bischof and Ratcliff[4]	Liang et al [32]	TPMABC proposed
LN1	78.1	62.5	62.5	62.5	62.5	62.5
LN2	76.8	80.7	89.7	90	89.7	91.3
LN3	69.5	53.4	53.4	53.4	53.4	53.4
LN4	59.2	55	55	55	55	55
LN5	85.8	77.2	77.2	77.2	77.2	77.2
LN6	88.6	88.7	89.5	83.1	91.4	90.7
LN7	78.2	81.8	83.9	78.7	84.6	84.7
LN8	67.6	59.4	59.4	59.4	59.4	59.4
LN9	84.2	61.9	61.9	61.9	61.9	61.9
LN10	70.1	67.3	67.3	67.3	67.3	67.3
LN11	63.8	62.2	62.2	62.2	62.2	62.2
LN12	76.3	78.5	76.5	78.5	78.5	78.5
LN13	77	84.1	82.3	78.1	85.6	83.8
LN14	69.1	62.8	62.8	62.8	62.8	62.8
LN15	65.6	59.6	59.5	59.5	59.5	59.5
AVG	74.2	69	69.5	68.6	70.06	70.01

Table 1 showed LN test problem results with different methods. The result after tuning parameters proposed from problem 1 (LN1) to problem15 (LN15). The solution improve in case 2 (LN2) and decrease from 94.4 to 91.3 and problem 13 (LN13) the solution improve from 85.6 to 83.8

CONCLUSION:

In this paper, we solved the container loading problem (CLP) by applying modified artificial bee colony (MABC) after tuning parameters with its probabilistic rule and promising food sources feedback to neighbor with the wall-building heuristic. From the computational experiments, the proposed MABCO generally yield comparable results. The method with tuning parameters has the capability to solve the CLP. Future work will focus on the exploration of the probability function and improve local neighborhood search which might further increase the space utilization ratio.

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Environmental Economics and Feasibility Studies using MCDM for Petroleum and Gas Projects

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Abstract

Petroleum and gas projects are ruled by risks and opportunities within their complex decision making processes. This paper provides a methodology to aid decision making to choose among seven upgrading oil fuel schemes, identified as high diesel production schemes, for a proposed plant capacity of 3.50 million t/y of the atmospheric residue produced in the Egyptian refineries. The problem is to choose the best alternative.

The present paper aims to determine the various objectives involved in the petroleum and gas projects, taking into account environmental, social and economic considerations to maximize the production of LPG and middle distillates (diesel euro 10 ppm). The environmental aspect at the preliminary design phase of petroleum projects is quantified by using a set of metrics or indicators of the sustainability concepts.

A SWOT analysis is a structured planning method used to evaluate the strengths, weaknesses, opportunities and threats involved in a project. The used methodology combines SWOT analysis and Multicriteria Decision Making (MCDM) methods to evaluate and determine the criteria for considering the sustainability aspects.

Keywords: Environmental Economics, Sustainability, MCDM, AHP, TOPSIS, SWOT Analysis, Petroleum and Gas Projects.

1. Introduction

Many countries around the world participate in the petroleum industry, and other stakeholders including political and environmental organizations and multinational companies give the petroleum industry great economic power. Recently, the petroleum industry suffers from risk variability in petroleum project investment decisions due to improved technology, the demand of efficiency and changes in its organizational structure. High financial returns may be realized from oil and gas projects as a result of the high degree of risk due to high investment levels over a long period of time and high-tech resources [1]. Thus, a multiple criteria decision making process is required in order to maximize or minimize the outcomes of each objective [2]. The capital allocation process and related decisions have a critical impact in final performance [3].

One of the important criteria in the petroleum industry is sustainability: the industry must make choices which balance economic concerns and social issues, and act

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so that future generations are not saddled with unnecessary environmental damages [4]. Therefore, decisions must be based on a high level of understanding of the interactions between industrial ecology and sustainability [5].

In order to probably assess the environmental aspects of the preliminary design phase of required chemical processes the industry uses a set of metrics or indicators developed for sustainability guidelines.

One such set was developed by Ruiz-Mercado (2015) whose set of taxonomic classifications and definition of sustainability indicators was based on the GREENSCOPE methodology. The GREENSCOPE methodology proposes an evaluation and design system of sustainability assessment using environmental, economic, energy and efficiency bases.

Ruiz-Mercado (2015) also developed, using high and low level boundaries, a scale of the proper bounds of sustainability [6]. Osmani et al. (2014) proposed three levels of sustainability acceptability: economic, societal and environmental, thus the industry will assess sustainability indicators as one of these three aspects [7].

Currently the capacity needed to fulfill high consumption and demand rates and the demand for lower emission rates soon may exceed natural capacities. Society must develop sustainable systems in order to avoid situations in which ecological services are not available ("a tripping point.") The response of the chemical industry has been to use sustainability methods in the early phases of the design process. This benefits the petroleum industry by increasing the economic and social benefits twofold: one is to lower the amount of needed goods and services; and the second is to minimize emission releases [6].

The rest of the paper is organized as follows: In Section 2, we present the feasibility studies. In Section 3, we describe the SWOT analysis. In Section 4, we present the review of MCDM. In Section 5 we present the proposed algorithm. In Section 6 we present the application for the proposed methodology. Finally, Section 7 presents the conclusion of the work.

2. Feasibility Studies

A feasibility study is a tool for investigating the viability of the prospective project. It is conducted on the very first stages of project development, to reveal whether or not the project is viable from all aspects [8]. A feasibility analysis is an effective analytical tool that can be used to evaluate investments from various perspectives, such as technical, social, economic, legal, financial, market, and organizational [9].

Financial analysis of industrial investment projects is not an isolated activity performed only towards the end of the project design in order to complete a primarily technical study or project proposal and to show the financial implications of a project for promoters and potential investors [10]. A financial decision is dependent on two specific factors, expected return and expected risk, and a financial feasibility analysis is a means for examining those two factors [11].

Financial feasibility is often a predominant factor in feasibility analysis, as most investments are not realized if they do not generate profit for the project owners [9]. Financial feasibility analysis requires detailed information regarding the project operations and financial requirements [12]. In order to evaluate the financial feasibility of an investment project, relevant measurements or criteria need to be specified. Remer and Nieto (1995) categorized the evaluation methods into five basic types: Net present value

(NPV) methods; Rate of return methods; Ratio methods; Payback methods and Accounting methods [13].

The cash flow method considers the time value of money; cash flows are always calculated in several different ways, i.e. using different depreciation methods or inventory listings, which give different profit results. The cash flow method is considered more appropriate for evaluating the financial feasibility of investment projects [9]. There are several different cash flow based methods that can be used to measure the financial feasibility of investment projects, such as the NPV, Internal Rate of Return (IRR), Annual Equivalent Worth, Benefit-Cost Ratio and the Modified Internal Rate of Return. Investors use these quantitative measures to help them decide whether to undertake an investment or not, based on their return requirements [13]. The payback period (PBP) is another method that is sometimes used in financial feasibility analysis. The method determines when the project will break even, i.e. how long it takes for revenues to pay investment outlays [14].

The relation between total initial investment and the number of workers and staff employed is used when comparing alternative technologies. However, when the problem is to choose between alternatives with different labor intensities, it may be advisable to compute the ratio between investment and total costs of personnel. Similarly, the efficiency of personnel employed may be computed by determining the value of output produced by one unit of personnel costs. These ratios, including the capital-output ratio, complement the cash flow and financial analysis in so far as additional information may be obtained with regard to possible risks, suitable investment strategies and positioning of a project in a competitive environment [10].

In evaluating the expenses of a project, Capital Expenditure (CAPEX) used by a company to acquire or upgrade physical assets such as equipment, property, or industrial buildings. The cost of operations or maintenances of a plant known as Operational Expenditures (OPEX) consisting of insurance expenses, plant repair, maintenance expenses, and other ongoing costs. The operating profit margin (operating margin) compares a company’s operating income (earnings before interest and taxes, or EBIT) to sales. It indicates how successful management has been in generating income from operating the business [15].

3. SWOT Analysis

SWOT analysis was first designed and introduced in 1960 by Stanford Research Institute and by early 1975 was generally used as an analytical framework for devising a company's strategies [16]. SWOT analysis is a commonly used tool for analyzing external and internal environments at the same time in the acquisition of a systematic approach and support for a decision situation [17]. The SWOT analysis is a holistic concept which is an analysis of a topic's strengths, weaknesses, opportunities and threats.

Strengths and weaknesses are understood as internal, supporting or hindering factors, which can be directly influenced by the company itself. Opportunities and threats are external, supporting or hindering factors or trends which the company is not able to influence directly [16].

An internal factor is a factor which can bring influence wholly derived from the study object. On the other hand, external factors are factors that originate from the outside environment and can bring positive (opportunities) and negative (threats) influences [17].

A SWOT analysis (alternatively called a SWOT matrix) is a structured planning method used to evaluate the strengths, weaknesses, opportunities, and threats involved in a project or in a business venture [18]. A SWOT analysis can be carried out for a product, place, industry or person. It involves specifying the objectives of the business venture or project and identifying the internal and external factors that are favorable and unfavorable to achieve that objective [16].

Ghazinoory et al. (2011) presented a literature review of SWOT analyses, up to the end of 2009. The origination and historical development and a survey on trends and classifications in SWOT papers including journals, countries, years, people and contents, and a categorical analysis was conducted regarding the scope and area of SWOT application. Additionally, a methodological development of SWOT was discussed [19]. SWOT analysis does not provide a means of systematically determining the relative importance of the criteria or assessing decision alternatives according to the criteria. In order to handle this insufficiency, the SWOT framework is converted into a hierarchic structure [17].

SWOT analysis used in different applications, Farhangi et al. (2012) introduced a strategic look for enhancing the accuracy in collecting strategic data in media organizations instead of using a 4-zonal classical SWOT matrix. The external environment of a media organization is divided into 3 parts: industry or activity environments, closed or public environments; and national or international environments. This new division leads to 12 districts in a SWOT matrix that increases accuracy in recognizing the environment challenges and in choosing more suitable strategies for media organizations [16].

Pesonen et al. (2014) proposed approach to climate-related structural change of the business environment by addressing questions of how usable the climate SWOT is in building climate strategies; whether or not it raises awareness of the life-cycle perspective and climate change strategies and if it is able to produce strategic or operational changes in the pilot organizations [20].

Beloborodko et al. (2015) provided a comparative analysis of the organizational structure and a SWOT analysis of five existing and one potential waste-to-energy clusters and developed suggestions for improved organization of the developing Latvian cluster [21]. Neagu et al. (2015) achieved a dynamic portrait of the Romanian extractive industry after 1989 (through a well-balanced SWOT analysis), and emphasized the role and importance of this industry in the development of Romanian society [22]. Grošelj el al. (2015) proposed a three-phase decision making approach based on the analytic network process and SWOT analysis. The suggested approach was applied to a management problem of Pohorje, a mountainous area in Slovenia. [23].

Baycheva-Merger et al. (2016) analyzed the SWOT factors of the Pan-European C&I in order to determine and prioritize the major constraints and future possibilities of their use and implementation [24]. Shi (2016) argued that despite the looming pollution problems due to the expected surge of coal use, the ASEAN region has many advantages in providing cleaner energy for its green vision, and has employed a SWOT analysis to highlight the potentials ASEAN would be able to reconcile the outlooks for its energy mix [25]. Padash et al. (2016) implemented a comprehensive strategic environmental management plan in the Mond protected area in southern Iran. The protected area was zoned using a multi criteria decision method, a SWOT analysis, an analytical hierarchy process and expert choice software which were applied to weight the factors [26]. Comino et al. (2016) proposed and tested the development of an indicators-based spatial SWOT analysis for a complex territorial system with exceptional multiple values [27].

4. Multi-Criteria Decision Making

In the real world multiple, conflicting and incommensurate criteria often arise thus MCDM methods are used to resolve the problems, in order to assess the decision maker in making a rational, efficient and emplaced decisions which take the important and subjective criteria of the problem into account. [28]

MCDM problems can be categorized as continuous or discrete depending on the domain of the alternatives. When multi criteria conditions are involved, the decision makers can choose from the following techniques: Simple Additive Weighting (SAW), Analytic Hierarchy Process (AHP), ELimination and Choice Expressing Reality (ELECTRE) and the technique for order preference by similarity to ideal solution (TOPSIS) have been developed to help selection in the condition of multi-criteria.

Theoretically a single decision maker uses MADM methods although a growing number of problems in the real world are solved by using group decision making (GDM) by multiple decisions making.

One multiple decision makers are involved in the process, their different opinions in meshed by using GDM which find the most reasonable solution [29].

Recently, group multi-attribute decision making (GMADM) has received considerable attention in the MCDM literature. The TOPSIS is a widely used MADM method initially developed by Hwang and Yoon [30]. It has been applied to a large number of application cases in advanced manufacturing, purchasing and outsourcing, and financial performance measurement [31].

4.1. Analytic Hierarchy Process (AHP)

AHP is a multi-criteria decision making technique that can help express the general decision operation by decomposing a complicated problem into a multilevel hierarchical structure of objective, criteria and alternatives [32]. AHP performs pairwise comparisons to derive relative importance of the variable in each level of the hierarchy and / or appraises the alternatives in the lowest level of the hierarchy in order to make the best decision among alternatives.

AHP is an effective decision making method especially when subjectivity exists and it is very suitable to solve problems where the decision criteria can be organized in a hierarchical way into sub-criteria [33]. AHP is used to determine relative priorities on absolute scales from both discrete and continuous paired comparisons in multilevel hierarchic structures[32].

The prioritization mechanism is accomplished by assigning a number from a comparison scale (Table 2) developed by [34] to represent the relative importance of the criteria. Pairwise comparisons matrices of these factors provide the means for calculation of importance [32].

Table 2: Pairwise Comparison Scale

Intensity of importance	Explanation
1	Two criterion contribute equally to the objective
3	Experience and judgement slightly favor one over another
5	Experience and judgment strongly favor one over another
7	Criterion is strongly favored and its dominance is demonstrated in practice
9	Importance of one over another affirmed on the highest possible order
2, 4, 6, 8	Used to represent compromise between the priorities listed above

The AHP method is based on three principles: first, structure of the model; second, comparative judgment of the criteria and/or alternatives; third, synthesis of the priorities.

In the first step, a decision problem is structured as a hierarchy [34]. AHP initially breaks down a complex multicriteria decision making problem into a hierarchy of interrelated decision elements (criteria, decision alternatives). With the AHP, the objectives, decision criteria and alternatives are arranged in a hierarchical structure similar to a family tree [35].

The second step is the comparison of the criteria and/or the alternatives. Once the problem has been decomposed and the hierarchy is constructed, prioritization procedure starts in order to determine the relative importance of the criteria. In each level, the criteria are compared pairwise according to their levels of influence and based on the specified criteria in the higher level. In AHP, multiple pairwise comparisons are based on a standardized comparison scale of nine levels [33].

Let $C = \{C_j | \; j = 1, 2, \ldots, n\}$ be the set of criteria. The result of the pairwise comparison on n criteria can be summarized in an $(n \; x \; n)$ evaluation matrix A in which every element a_{ij} $(i, j = 1, 2, \ldots, n)$ is the quotient of weights of the criteria. This pairwise comparison can be shown by a square and reciprocal matrix, (Eq. (1)).

$$\boldsymbol{A} = \left(\boldsymbol{a_{ij}}\right)_{\boldsymbol{n\times n}} = \begin{bmatrix} \boldsymbol{a_{11}} & \boldsymbol{a_{12}} \cdot & \cdot \;\cdot \;\cdot \;\cdot \;\cdot & \boldsymbol{a_{1n}} \\ \boldsymbol{a_{21}} & \boldsymbol{a_{22}} & & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \boldsymbol{a_{n1}} & \boldsymbol{a_{n2}} & \cdot \;\cdot \;\cdot \;\cdot \;\cdot \;\cdot & \boldsymbol{a_{nn}} \end{bmatrix} \qquad (\boldsymbol{1})$$

At the last step, each matrix is normalized and be found the relative weights. The relative weights are given by the right eigenvector (w) corresponding to the largest eigenvalue ($\lambda_{\,max}$), as:

$$A_w = \lambda_{\,max} . w \qquad (\boldsymbol{2})$$

If the pairwise comparisons are completely consistent, the matrix A has a rank 1 and $\lambda_{\,max} = n$. In this case, weights can be obtained by normalizing any of the rows or columns of A. It should be noted that the quality of the output of the AHP is related to the consistency of the pairwise comparison judgments.

The consistency is defined by the relation between the entries of A: $aij \; x \; ajk = aik$. The Consistency Index (CI) can be calculated, using the following formula (Saaty, 1980):

$$CI = \frac{\boldsymbol{\lambda_{\,max} - n}}{\boldsymbol{n-1}} \qquad (\boldsymbol{3})$$

Using the final consistency ratio (CR) can conclude whether the evaluations are sufficiently consistent. The CR is calculated as the ratio of the CI and the random index (RI), as indicated in Eq. (4). The number 0.1 is the accepted upper limit for CR. If the final consistency ratio exceeds this value, the evaluation procedure has to be repeated to improve consistency [33]

$$CR = \frac{CI}{RI} \tag{4}$$

Table 3: Random Index [36]

n	1	2	3	4	5	6	7	8	9	10
RI	0,00	0,00	0,58	0,90	1,12	1,24	1,32	1,41	1,45	1,49

4.2. TOPSIS

Hwang and Yoon (1981) developed the TOPSIS method based on the concept that the chosen alternative should have the shortest distance from the ideal solution and the farthest distance from the nadir solution [37]. The method is briefly described as follows:

Step-1: Construct the Normalized Decision Matrix

To transform the various attribute dimensions into non dimensional attributes, which allows comparison across the attributes.

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}} \tag{5}$$

Step-2: Construct the Weighted Normalized Decision Matrix

$$V = [v_{ij}] = [w_j r_{ij}] \tag{6}$$

$$\boldsymbol{V} = \begin{bmatrix} v_{11} & v_{12} \cdots & v_{1j} \cdots & v_{1n} \\ \vdots & \vdots & \vdots & \vdots \\ v_{i1} & v_{i2} & v_{ij} & v_{in} \\ \vdots & \vdots & \vdots & \vdots \\ v_{m1} & v_{m2} \cdots & v_{mj} \cdots & v_{mn} \end{bmatrix} = \begin{bmatrix} w_1 r_{11} & w_2 r_{12} \cdots & w_j r_{ij} \cdots & w_n r_{1n} \\ \vdots & \vdots & \vdots & \vdots \\ w_1 r_{i1} & w_2 r_{11} & w_j r_{ij} & w_n r_{in} \\ \vdots & \vdots & \vdots & \vdots \\ w_1 r_{m1} & w_2 r_{m2} \cdots & w_j r_{mj} \cdots & w_n r_{mn} \end{bmatrix} \tag{7}$$

Step-3: Determine the positive and negative ideal solutions

$$A^+ = \{(\max_i v_{ij} | j\epsilon J), (\min_i v_{ij} | j\epsilon J) | i = 1,2,\dots,m\} = \{v_1^+, v_2^+, \dots, v_j^+, \dots, v_n^+\} \tag{8}$$

$$A^- = \{(\min_i v_{ij} | j\epsilon J), \ (\min_i v_{ij} | j\epsilon J) | i = 1,2,\dots,m\} = \{v_1^-, v_2^-, \dots, v_j^-, \dots, v_n^-\} \tag{9}$$

$where$

J= $\{j = 1,2,\dots,n | j \ \ associated\ with\ benefit\ criteria\}$

$J = \{j = 1,2, \dots, n | j \ \ associated \ with \ cost \ criteria\}$

Step-4: Calculate the separation measure:

a) The separation from the positive ideal sol.

$$S_i^* = \left[\sum_{j=1}^{n}\left(v_{ij} - v_j^*\right)^2\right]^{1/2}, i = 1,2, \dots, m \tag{10}$$

b) The separation from the negative ideal sol.

$$S_i^- = \left[\sum_{j=1}^{n}\left(v_{ij} - v_j^-\right)^2\right]^{1/2}, i = 1,2, \dots, m \tag{11}$$

Step-5: Calculate the relative closeness to the ideal solution:

$$C_i^* = \frac{s_i^-}{(s_i^* + s_i^-)}, 0 < C_i^* < 1, \ i = 1,2, \dots, m \tag{12}$$

$$C_i^* = 1 \ \ if \ \ A_i = A^+$$

$$C_i^* = 0 \ \ if \ \ A_i = A^-$$

Step-6: Rank the preference order:
A set of alternatives can now be preference ranked according to the descending order of C_i^*.

5. Proposed Methodology

The proposed algorithm includes seven steps. The steps of the algorithm are illustrated as follows:

Step 1: Identify the triple bottom line sustainability criteria.
Step 2: Initiate the SWOT matrix for sustainability criteria.
Step 3: Confirm the triple bottom line sustainability criteria.
Step 4: Modify the SWOT matrix for sustainability criteria by any need changes after reviewing the decision makers.
Step 5: Identify the weights of each criterion via the AHP method.
Step 6: Final SWOT matrix for sustainability criteria is obtained.
Step 7: Use the identified weights of each criterion from triple bottom line sustainability criteria to apply the TOPSIS method to get the ranking by Excel.

6. Application

In order to illustrate the methodology, seven upgrading schemes, identified as high diesel production schemes have been evaluated for a proposed plant capacity of 3.5 million t/y of the atmospheric residue produced in the Egyptian refineries. All the details for the seven upgrading schemes are found in [38].

The initial decision-making matrix has been formed according to the TBL sustainability criteria. Applying the AHP method using Transparent choice AHP online software for multi-criteria decision-making. AHP is applied to SWOT matrix. Firstly, pairwise comparisons of the SWOT groups, using the 1-9 Saaty's (1980) comparison scale, are made. Secondly, considering every SWOT group. All pairwise comparisons are performed by the team of experts. Expert team was constituted of EPRI authors of this paper. Thirdly, the overall priority scores of the SWOT factors are calculated. Overall priorities are shown in Table 4 for the outranking Schem-No.1.

Figure 1 shows the outranking alternatives and Figure 2 shows the production group in SWOT matrix by using AHP.

Table 4: The overall priority scores of SWOT groups for the outranking Schem-No.1.

Economic	**Group priority**	**Factor Priority within the group**	**Overall priority of factor**
NPV	0.248	0.345	0.086
IRR		0.151	0.037
PBP		0.153	0.038
Profitability index		0.070	0.017
Cash flow		0.094	0.023
Accounting rate of return		0.016	0.004
CAPEX Ratio		0.048	0.012
OPEX Ratio		0.039	0.010
Operating profit		0.066	0.016
Breakeven point		0.018	0.006
Environmental	**Weight**		
Environmental standards	0.549	0.833	0.457
Global warming potential		0.167	0.091
Social	**Weight**		
Rates of work-related occupational illnesses	0.133	0.667	0.089
Health index		0.333	0.044
Production	**Weight**		
LPG	0.070	0.857	0.060
Middle distillates (diesel euro 10 ppm)		0.143	0.010

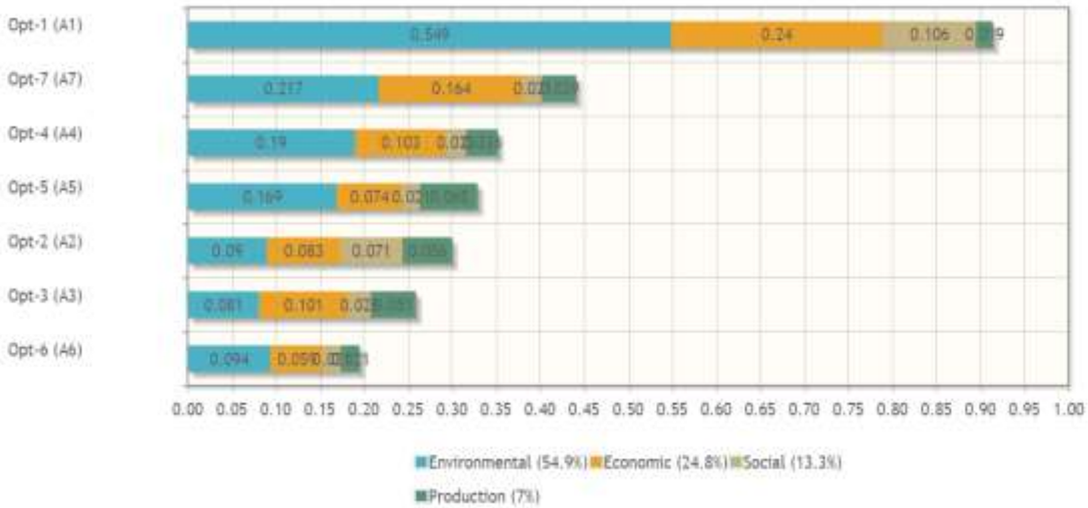

Fig. 1: Ranking alternatives by using AHP

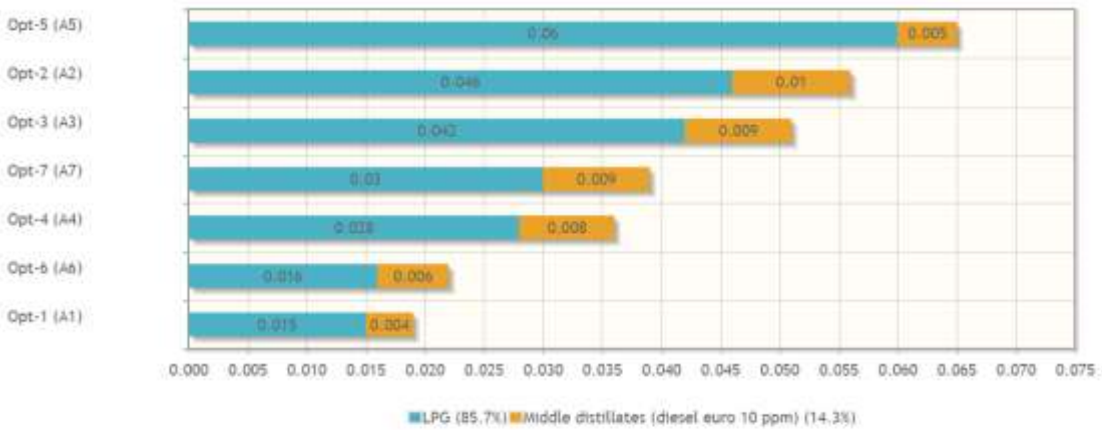

Fig. 2: The production group in SWOT matrix

Use the weights results from AHP method to apply TOPSIS method. Table 5 shows the separation measures, the relative closeness coefficient and the ranking of different options (alternatives). Option 4 had the nearest C* to 1 and selected as the best alternative.

Table 5: The separation measures, the relative closeness coefficient and the ranking of different options (alternatives)

Options	s*	s-	C*	Rank
A1	0.0847	0.0415	0.3291	6
A2	0.0443	0.0811	0.6466	4
A3	0.1198	0.0267	0.1823	7
A4	0.0165	0.1200	0.8792	1
A5	0.0422	0.0825	0.6614	3
A6	0.0246	0.1193	0.8292	2
A7	0.0807	0.0481	0.3734	5

−−→

Options	Rank
A4	1
A6	2
A5	3
A2	4
A7	5
A1	6
A3	7

7. Conclusion

In this paper, we have determined outranking for the alternatives of seven upgrading schemes, identified as high diesel production schemes for a proposed plant capacity of 3.5 million t/y of the atmospheric residue produced in the Egyptian refineries.

The results show that the scheme no.1 is the first alternative and each SWOT group priority for alternative 1: economic (group weight 24.8%), environmental (54.9%), social (13.3%) and production (group weight 7 %), and used the weights results from AHP method to apply TOPSIS method, option 4 had the nearest C* to 1 and selected as the best alternative.

The methodology of SWOT sustainability for petroleum and gas projects has been developed by applying SWOT and MCDM methods: AHP and TOPSIS. The algorithm describes a long-term goal which forms a solid framework for strategic planning in petroleum and gas sustainable projects.

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