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On Solving N -Vehicle Cost Varying Transportation Problem

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Abstract

Solving N –Vehicle Transportation Problem with varying cost due to varying capacity is considered in this paper. The Vogel Approximation Method is applied to determine the cost of each cell instead of the vehicle cost and The unit cost is used by taking the penalty in row and column with the same capacity reaching faster to solution than northwest corner method. The N-vehicle transportation problem cost is also solved with fuzzy numbers using Robust's ranking indices and Vogel Approximation Method reaching to the optimal solution.

Keywords: Transportation Problem, Solid transportation problem, N–Vehicle Transportation Problem, Vogel Approximation Method (VAM), Robust's ranking indices.

1. Introduction

In real life, we face more complex problems like using different models of conveyances, such as goods trains, trucks, ships, cargo flights, etc, which means the route between every source and destination will has more than one cost, and this number of costs equal the available capacities. Solving this problem, we must found the value far as possible allocate to the cheapest route to every capacity. Like this problem called solid transportation problem [14].

We must know the conveyances in solid transportation problem used one time in one trip. If we will use the vehicles more than one time in several trips then we speak about N-Vehicle transportation problem. That's happen when the company has several vehicles with small capacity "may be only two or three vehicles with different capacities" less than the available units "this vehicles can't carry all the units one time" in this case, it must be used the vehicles in more than one trip till transfer all units.

It may be the transportation center has little N-types of vehicles "These vehicles have different capacities" and according to the cost of each one, it will identify the type and how many vehicles that will be used to transfer the units [9].

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The goal of solving Transportation Problem is to transfer a product from supplies to destinations "using only one conveyance, which means one capacity and one cost in the route between every source and destination" by minimum cost [4]. To solve this problem it must be passed throw two stages:

First stage: identify the Initial Basic Feasible Solution, using one of the following (Northwest Corner method or Minimum cost method or Vogel's approximation method (VAM).

Second stage: Identify the optimal solution by evaluating each unused cell to know if we used one or more from them will reduce the cost or no. When all cells have been evaluated and appropriate shifts made, then the problem is solved. We can make this evaluation by the stepping stone method or modified distribution method (MODI) or (u , v) method.

In some situations the decision maker has data as fuzzy to become more reality such as Pandian and Natarajan [11] which it is proposed a new algorithm called zero point method to solve fuzzy transportation problem, then Samuel [15] improved zero point method to solve the problem, and developing unbalanced transportation problem [16]. Also Narayanamoorthy, et al. [8] used Russel's method to solve fuzzy transportation problem. In this paper, the Rssell's method dependent is used to identify (u_i) and (v_j) where (u_i) equal the largest cost in each row and (v_j) equal the largest cost in each column then identify (d_{ij}) where ($d_{ij} = c_{ij} - u_i - v_j$) then choose the most negative cell and allocate as much as possible to this cell. Then go the next most negative cell and repeat the last step till allocate all the supply and demand.

Venkatasubbaiah, et al. [18] developed other method to solve interactive fuzzy goal transportation problem gives the same number of allocations but with improved value.

The N –Vehicle Transportation Problem with varying cost due to varying capacity is considered in this paper, we use VAM to determine the cost to each cell instead of the vehicle cost. So, we use the unit cost taking the penalty in the row and column, reaching to the solution faster than northwest corner method which used in [9].

This paper is organized as follows: Section 2 considers "problem definition". Section 3 introduces "The proposed method: including the steps of solving N-vehicle transportation problem using Vogel's Approximation and applying the optimality test. Section 4 introduces "numerical example". Section 5 shows "Fuzzy transportation problem". It is used ranking fuzzy number and VAM in N vehicle transportation problem in varying cost presenting another numerical example. Finally, Section 6 includes conclusion.

2. Problem Definition

The mathematical model of transportation problem [14]:

$$\min Z = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l} c_{ijk} x_{ijk}$$

$$\sum_{i=1}^{m}\sum_{k=1}^{l} x_{ijk} = D_j , \qquad \text{for } j = 1,2,......n \qquad (2.1)$$

$$\sum_{j=1}^{n}\sum_{k=1}^{l} x_{ijk} = S_i , \qquad \text{for } i = 1,2,......m \qquad (2.2)$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk} = e_k \qquad \text{for } k = 1,2,......l \qquad (2.3)$$

$$\sum_{i=1}^{m} S_i = \sum_{j=1}^{n} D_j = \sum_{k=1}^{l} e_k \qquad x_{ijk} \geq 0 \ , \ \forall i,j,k \qquad (2.4)$$

Where;

S_i : The units available at source i.

D_j : The units demanded at destination j.

e_k : The capacity(e)of k vehicle.

x_{ijk} : The unknown quantity shipped from source (i)to destination (j) by (k) of conveyance.

c_{ijk} : The cost of shipping units from source (i)to destination (j) by (k) of conveyance.

The solid transportation problem, choose the cheaper vehicle available in the route and collect goods till reach exact it's capacity. It was the conveyances (l) has capacities exact equal to amount of goods, but the new here in "N - Vehicle cost varying transportation problem" the transport center has small numbers of conveyances (differ in capacity and cost) ready to transport goods with several trips till transfer all goods as in Table 1.

Table 1 Tableau for cost varying transportation problem.

	D_1	D_2	..	Stock	
S_1	$e^{1}_{111}, e^{2}_{112}, ..., e^{l}_{11k}$	$e^{1}_{121}, e^{2}_{122}, ...e^{l}_{12k}$	..	$e^{1}_{1n1}, e^{2}_{1n2}, ...e^{l}_{1nk}$	S_1
S_2	$e^{1}_{211}, e^{2}_{212}, ...e^{l}_{21k}$	$e^{1}_{221}, e^{2}_{222}, ...e^{l}_{22k}$	..	$e^{1}_{2n1}, e^{2}_{2n2}, ...e^{l}_{2nk}$	S_2
..	..	..	..	..	..
S_m	$e^{1}_{m11}, e^{2}_{m12}, ...e^{l}_{m1k}$	$e^{1}_{m21}, e^{2}_{m22}, ...e^{l}_{m2k}$	..	$e^{1}_{mn1}, e^{2}_{mn2}, ...e^{l}_{mnk}$	S_m
Demand	d_1	d_2	..	d_3	

There are three main differences between solid and N-Vehicel transportation problem:

1. The vehicles capacity is less than the units available at source i, and destination j
2. The conveyance may carry less than its capacity.
3. The cost of conveyance may be multiply more than one time to calculate the cost of transport the goods.

So the above mathematical model formulation in eq. (2.3) changes to eq. (2.5) as follows:

$$\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk} \le e_k \qquad \text{for } k = 1,2,\ldots\ldots l \qquad (2.5)$$

$$\sum_{i=1}^{m} S_i = \sum_{j=1}^{n} D_j \le \sum_{k=1}^{l} e_k \qquad (2.6)$$

3. The proposed Method

To solve transportation problem using our suggested method as follows:

Step1. We have obtain the new transportation cost ($U_{ijk} = \frac{c_{ijk}}{e_{ijk}}$)

Step2. Identify the Initial Basic Feasible Solution using (VAM)

Step3. Improve the solution by applying the optimality test using (MODI)

- **The calculation for obtaining the real transportation cost as follows:**

U_{ijk} (unit cost) transported by each vehicle where $U_{ijk} = \frac{c_{ijk}}{e_{ijk}}$

3.1. Solving N-vehicle transportation problem using Vogel's Approximation

Applying VAM as follows:

1. Balance the given transportation problem if it isn't balanced.
2. Determine the penalty cost to each row by subtracting the lowest cost from the next lowest cost in the row.
3. Determine the penalty cost to each column (for the same capacity) by subtract the lowest cost from the next lowest cost in the same capacity.
4. Select the row or column with the highest penalty cost.
5. Allocate as much as possible to the cell with lowest cost in the row or column.
6. Repeat steps 3, 4, 5 and 6 until all requirements have been met.
7. Compute the total cost to each cell by identifying which vehicle will transport the units as we will explain in the next section (3.2)
8. Compute the total cost to all transportation.

3.2. The method for identifying the cost of each cell.

To identify the cost for each cell(c_{ijk}), it must be follow some steps to know which vehicle will be used and how many trips need to transport such units:

- If $X \le e_1$ or if e_1 trips cost $rc_1 \le c_2$ (where r = the number of trips) then e_1 (vehicle 1) will transport the units.
- If $X > e_1$ and $rc_1 > c_2$ then e_2 will transport the units, but must look at (x),
- If $x \le e_1$ or $rc_1 \le c_2$ then will add (e_1) or rc_1 to the cost
- If $x > e_1$ and $rc_1 > c_2$ Then must add one from the same vehicle (e_2)

Where:

c_k : The cost of the vehicle (k), $k = 1, 2, \ldots. l$

e_k : The capacity of the vehicle (k), $k = 1, 2, \ldots. l$

Where ($C_1 < C_2 < C_3 \ldots < C_k$) and

($e_1 < e_2 < e_3 \ldots < e_k$)

X_{ijk} : The units must transport.

x_{ijk} : The fractional produced when divided $\frac{X_{ijk}}{e_k}$

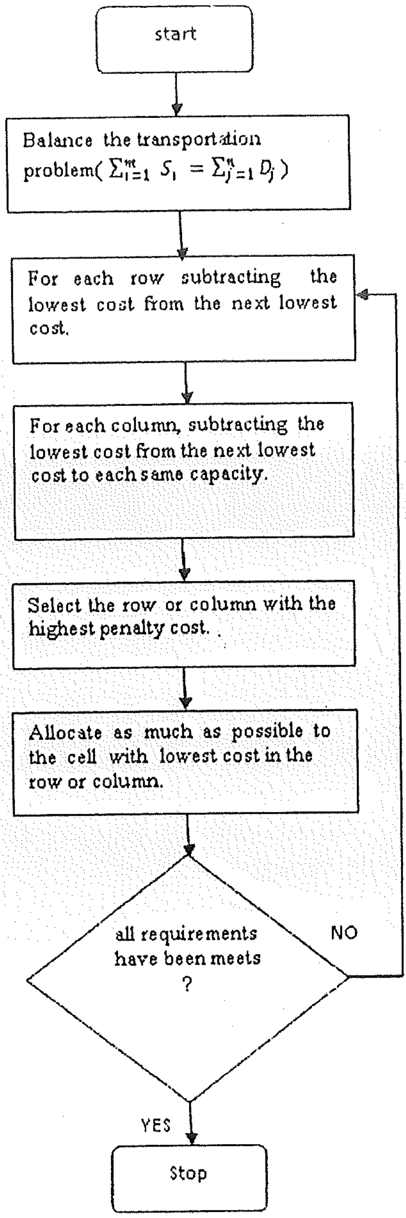
(Divided $\frac{X_{ijk}}{e_k}$ to identify how many numbers of trips will the vehicle takes) the fractional that's less than the capacity of current vehicle so may add other tripe to the current vehicle or use other vehicle types.

3.3 The optimality test

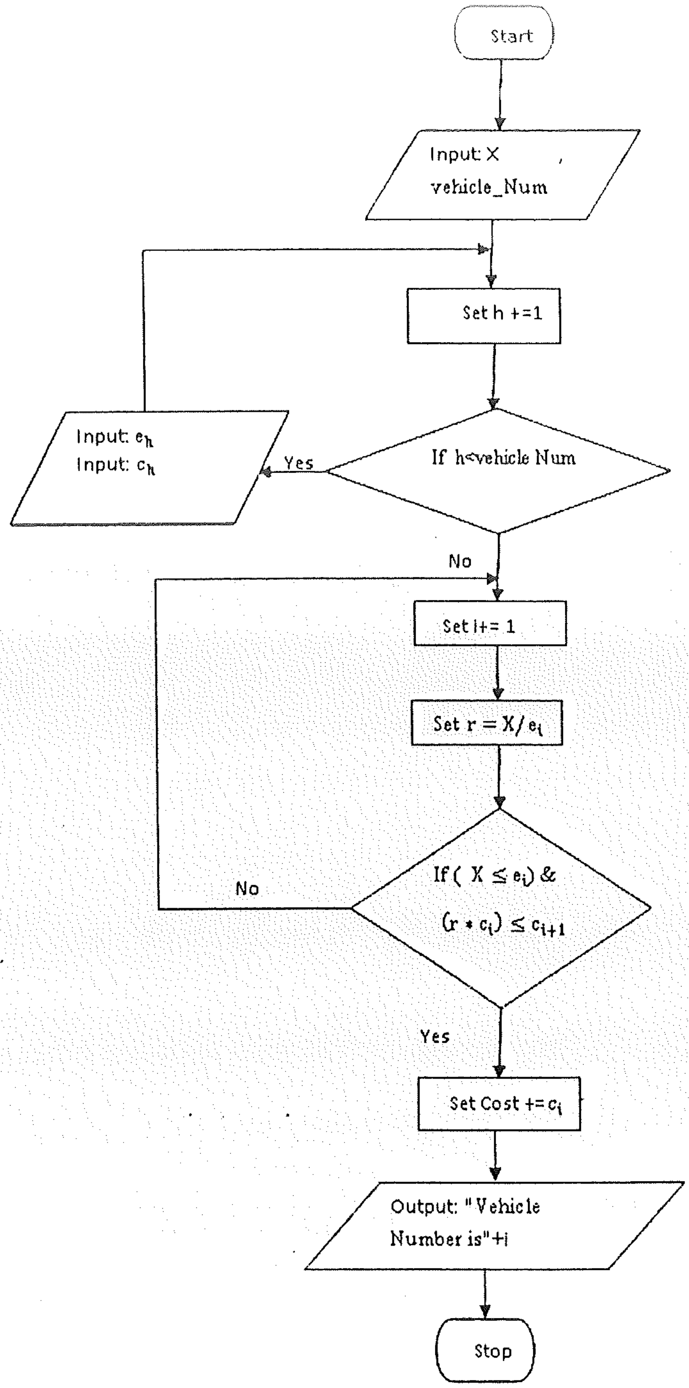
The steps of optimality test:

1. Recalculate the cost to be equal (the cost / units)
2. Determine u_i , $i = 1, 2, \ldots\ldots. m$ and v_j , $j = 1, 2, \ldots\ldots.$ n to all basic cell where $c_{ij} = u_i + v_j$
3. For nun-basic cell determine c_{ij} by introduced the feasible loop to the nun-basic cell to identify the amount of units suitable to the cell then compute the cost to the cell as the basic cell.
4. Determine d_{ij} for each nun-basic cell where $d_{ij} = c_{ij} - u_i - v_j$

- If all $d_{ij} > 0$ then the solution is optimal.



Flowchart 1. Represent the Vogel Approximation Method to solve N -Vehicle Transportation Problem



Flowchart 2. Represent the calculation process to identify which vehicle will transfer the goods.

- If there are $d_{ij} = 0$ then the solution is optimal and alternative solution also exists.
- If there are $d_{ij} < 0$ then the solution is not optimal, to reach the optimal, follow the step 4.
- Go to the most negative cell then produce the loop (use the straight line beginning in the most negative cell and take available path always make right angle in basic cell then end in the same beginning cell).
- For the most negative cell put sign $(+)$ then in the next cells put sequentially sign $(+,-)$.
- Determine the smallest units in the cell have sign$(-)$, then subtract and add this amount of units to the cells according to the sign.
- Repeat the test till reach the optimality solution.

4. Numerical Example:

Example: Consider a 2-vehicles cost varying transportation problem

Table 2 Represent the data of example 1.

	D_1	D_2	D_3	Stock
S_1	4,8	5,10	10,20	48
S_2	2,3	8,16	6,12	52
S_3	7,14	3,6	9,18	25
Demand	75	30	20	

The capacities of vehicles: $e_1 = 6$ and $e_2 = 18$.

Applying the proposed method:

Step (1) for each cell, count U_{ijk} (the unit cost) for each vehicle where $U_{ijk} = \dfrac{c_{ijk}}{e_{ijk}}$ as in Table 3.

Table 3 Iteration 1

	D_1	D_2	D_3	Stock
S_1	4/6, 8/18	5/6, 10/18	10/6, 20/18	**48**
S_2	2/6, 3/18	8/6, 16/18	6/6, 12/18	**52**
S_3	7/6, 14/18	3/6, 6/18	9/6, 18/18	**25**
Demand	**75**	**30**	**20**	

Step (2) Identify the Initial Basic Feasible Solution by (VAM) as in Table 4, 5, 6, 7 & 8.

Table 4 Initial Basic Feasible Solution

	D_1	D_2	D_3	Stock	penalty
S_1	4/6, 8/18	5/6, 10/18	10/6, 20/18	48	1/9
S_2	2/6, 3/18	8/6, 16/18	6/6, 12/18 (20)	32	1/6
S_3	7/6, 14/18	3/6, 6/18	9/6, 18/18	25	1/6
Demand	75	30	0		
	2/6, 5/18	2/6, 4/18	3/6, 6/18		

Table 5 Iteration 3

	D_1	D_2	D_3	Stock	penalty
S_1	4/6, 8/18	5/6, 10/18	10/6, 20/18	48	1/9
S_2	2/6, 3/18 **(32)**	8/6, 16/18	**6/6, 12/18 (20)**	**0**	-
S_3	7/6, 14/18	3/6, 6/18	9/6, 18/18	25	1/6
Demand	**43**	30	**0**		
	2/6, 5/18	2/6, 4/18	- -		

Table 6 Iteration 4

	D_1	D_2	D_3	Stock	penalty
S_1	4/6, 8/18 **(43)**	5/6, 10/18	10/6, 20/18	**5**	1/9
S_2	2/6, 3/18 **(32)**	8/6, 16/18	6/6, 12/18 **(20)**	**0**	-
S_3	7/6, 14/18	3/6, 6/18	9/6, 18/18	25	1/6
Demand	**0**	30	**0**		
	3/6, 6/18	2/6, 4/18	- -		

Table 7 Iteration 5

	D_1	D_2	D_3	Stock	penalty
S_1	4/6, 8/18 **(43)**	5/6, 10/18 **(5)**	10/6, 20/18	**0**	1/9
S_2	2/6, 3/18 **(32)**	8/6, 16/18	6/6, 12/18 (20)	**0**	-
S_3	7/6, 14/18	3/6, 6/18 **(25)**	9/6, 18/18	**0**	1/6
Demand	**0**	**0**	**0**		
	- -	2/6, 4/18	- -		

Table 8 The final tableau of initial basic feasible solution.

	D_1	D_2	D_3	Stock
S_1	4/6, 8/18 (43)	5/6, 10/18 (5)	10/6, 20/18	48
S_2	2/6, 3/18 (32)	8/6, 16/18	6/6, 12/18 (20)	52
S_3	7/6, 14/18	3/6, 6/18 (25)	9/6, 18/18	25
Demand	75	30	20	

Step (3) identify the cost for each cell as in Table 9 then apply the optimality test as in Table 10 and 11

Table 9 The initial basic feasible solution cost.

	D_1	D_2	D_3	Stock
S_1	4,8 (24/43)	5,10 (5/5)	10,20	48
S_2	2,3 (6/32)	8,16	6,12 (18/20)	52
S_3	7,14	3,6 (12/25)	9,18	25
Demand	75	30	20	

Table 10 The optimality test Iteration 1.

	v_1	v_2	v_3	
u_1	4,8 24/43	5,10 5/5	10,20	0
u_2	2,3 6/32	8,16	6,12 18/20	-255/688
u_3	7,14	3,6 12/25	9,18	-13/25
	24/43	1	4371/3440	

Table 11 The optimality test Iteration 2.

	v_1	v_2	v_3	
u_1	4,8 (43)	5,10 (5)	10,20 $d>0$	0
u_2	2,3 (32)	8,16 $d>0$	6,12 (20)	-255/688
u_3	7,14 $d>0$	3,6 (25)	9,18 $d>0$	-13/25
	24/43	1	4371/3440	

There is no negative d_{ij} so, the final results in (Table 12) is the same result as in (Table 8) after Applying VAM so, we reach to the optimal and final solution with a few steps.

Table 12 The optimal solution

	D_1	D_2	D_3	**Stock**
S_1	4,8 (43)	5,10 (5)	10,20	**48**
S_2	2,3 (32)	8,16	6,12 (20)	**52**
S_3	7,14	3,6 (25)	9,18	**25**
Demand	**75**	**30**	**20**	

Table.13 The summary of the results including the total cost

The basic cells	The quantity of units	Cost of vehicle (1)	No. Trips	Cost of vehicle (2)	No. Trips	The total cost
x_{11}	43	(4)	-	(8)	3	24
x_{12}	5	(5)	1	(10)	-	5
x_{21}	32	(2)	-	(3)	2	6
x_{23}	20	(6)	1	(12)	1	18
x_{32}	25	(3)	-	(6)	2	12
Summation	125units					65$

So, the final total cost = 65$ corresponding to the optimal solution.

5 Fuzzy Transportation Problem:

In real life, the problem is more complex than that simple form where we may deal with fuzzy transportation problem. The value of cost, supply and demand may be appeared as fuzzy number. The mathematical model of fuzzy transportation problem is as follows [12]:

$$\min \ \tilde{Z} \approx \sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{c}_{ij}\tilde{x}_{ij} \qquad (5.1)$$

$$s.to. \sum_{j=1}^{n} \tilde{x}_{ij} \approx \tilde{S}_i, \qquad i = 1,\dots,m. \qquad (5.2)$$

$$\sum_{i=1}^{m} \tilde{x}_{ij} \approx \tilde{D}_j, \qquad j = 1,\dots,n. \qquad (5.3)$$

$$\sum_{i=1}^{m} \tilde{S}_i \approx \sum_{j=1}^{n} \tilde{D}_j, \qquad (5.4)$$

$$\tilde{x}_{ij} \geq \tilde{0} \qquad \forall i, j$$

Where;

$\tilde{S}_i$: The uncertain units available at source i.

$\tilde{D}_j$: The uncertain units demanded at destination j.

$\tilde{e}_k$: The uncertain capacity (e) of k vehicle.

$\tilde{x}_{ijk}$: The uncertain unknown quantity shipped from source (i) to destination (j) by (k) of conveyance.

$\tilde{c}_{ijk}$: The uncertain cost of shipping units from source (i) to destination (j) by (k) of conveyance.

Some definitions:

Definition (1): A fuzzy set A^* with membership $\mu_A(x)$ is defined by [13] $A^* = \{(x, \mu_A(x)) : x \in A \text{ and } \mu_A(x) \in [0,1]\}$. The fuzzy number $\tilde{a} = (a_1, a_2, a_3, a_4)$ is a fuzzy subset from the real line (R) has the membership function $\mu_{\tilde{a}}(a)$ with the following conditions:

$\mu_{\tilde{a}}(a)$ is a continuous mapping from R to the closed interval [0, 1],

$\mu_{\tilde{a}}(a) = 0$ for every $a \in (-\infty, a_1]$,

$\mu_{\tilde{a}}(a) = 0$ is strictly increasing and continuous on $[a_1, a_2]$,

$\mu_{\tilde{a}}(a) = 1$ for every $a \in [a_2, a_3]$,

$\mu_{\tilde{a}}(a)$ is strictly decreasing and continuous on $[a_3, a_4]$, and

$\mu_{\tilde{a}}(a) = 0$ for every $a \in [a_4, +\infty]$

Definition (2): A fuzzy number $\tilde{a}$ is a trapezoidal fuzzy number denoted by (a_1, a_2, a_3, a_4) where a_1, a_2, a_3 and a_4 are real numbers and its member ship function $\mu_{\tilde{a}}(x)$ is given below [13]:

$$\mu_{\tilde{a}}(x) = \begin{cases} 0 & for \;\; x \le a_1 \\ (x - a_1)/(a_2 - a_1) & for \;\; a_1 \le x \le a_2 \\ 1 & for \;\; a_2 \le x \le a_3 \\ (a_4 - x)/(a_4 - a_3) & for \;\; a_3 \le x \le a_4 \\ 0 & for \;\; x \ge a_4 \end{cases}$$

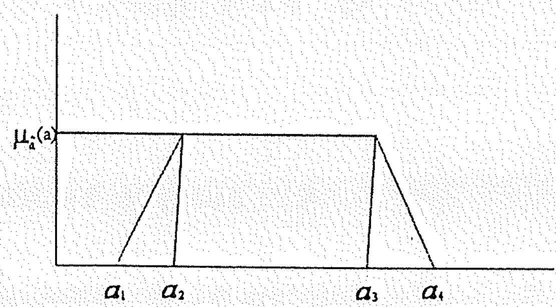


Fig.1: A trapezoidal fuzzy number

5.1 Ranking Fuzzy Number:

The Fuzzy number may be triangular or trapezoidal, and to compare between two or multi fuzzy numbers we go to use one of the ranking methods. Several methods are introduced for ranking of fuzzy numbers. The measure of a fuzzy number is obtained by the average of two side areas, left side area and right side area consider the following fuzzy number [1]:

$\tilde{A}_w = (\underline{A}(r), \overline{A}(r))$, where $0 \le r \le \omega$ (5.5)

When a normal fuzzy number is meant, the fuzzy number is shown as follows: $\tilde{A} = (\underline{A}(r), \overline{A}(r))$, where $0 \le r \le 1$ (5.6)

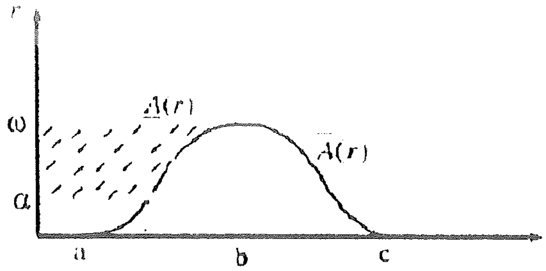


Fig.2 The presented quantity

As shown in Fig 2, which represents quantity as a summation of the dotted area and the cross-hatched area

$$M_{\alpha}(\tilde{A}_w) = \frac{1}{2}\int_{\alpha}^{w} \{\underline{A}(r) + \overline{A}(r)\}dr = \frac{1}{2}\int_{\alpha}^{w} \underline{A}(r)dr + \frac{1}{2}\int_{\alpha}^{w} \overline{A}(r)\,dr \qquad (5.7)$$

With the trapezoidal fuzzy number definition we can express $M(\tilde{B})$ as

$$\tilde{B} = (\underline{B}(r), \overline{B}(r)), \qquad (0 \le r \le 1) \qquad (5.8)$$

So , we can rewrite (5.7) as follows:

$$M_0^{Tra}(\tilde{B}) = \frac{1}{2}\int_0^1 \{\underline{B}(r) + \overline{B}(r)\}dr = \frac{1}{4}[m + n + \delta + \beta]. \qquad (5.9)$$

5.2 Fuzzy N -Vehicle Cost Varying Transportation Problem

The N-vehicle transportation problem cost may be come as fuzzy numbers which are realistic and general in nature; we will solve this problem by using Robust's ranking indices and obtain the optimal solution using last steps.

Example: Consider a 2-vehicles cost varying transportation problem the capacities of vehicles: $e_1 = 6$ and $e_2 = 18$, and the costs as follows:

Table 14 The data

	D_1	D_2	D_3	Stock
S_1	(9,10,12,13), (11,13,14,16)	(11,13,14,15), (14,16,18,20)	(4,5,7,8), (6,8,9,11)	(50,70,90,105)
S_2	(5,8,10,12), (7,9,12,14)	(6,7,9,11), (9,11,13,14)	(3,5,6,8), (5,6,7,8)	(30,60,80,93)
S_3	(10,11,12,13), (12,14,16,18)	(8,10,11,13), (13,14,15,16)	(6,7,9,10), (7,9,10,12)	(20,35,45,85)
Demand	(25,40,55,83)	(28,39,44,76)	(47,86,116,124)	

In this example, each cell has two fuzzy costs because there are two vehicles to transport the units. For example the route between (S_1, D_1) the cost of (e_1) = (9,10,12,13) and the cost of (e_2) = (11,13,14,16). Using ranking method, the cost becomes as in Table 15

So, each cost in Table 14 appears as trapezoidal fuzzy number, we find the summation of cost then divided it by 4 as follows:

For example the route between (S_1,D_1), the cost of (e_1) =(9,10,12,13) the rank of this cost = $\frac{9+10+12+13}{4}=11$, and the rank of the second vehicle cost (e_2) in the same route = $\frac{11+13+14+16}{4}=13.5$

Table 15 Iteration 1 - after applying the ranking method

	D_1	D_2	D_3	Stock
S_1	(11),(13.5)	(13.25),(17)	(6),(8.5)	78.75
S_2	(8.75),(10.5)	(8.25),(11.75)	(5.5),(6.5)	65.75
S_3	(11.5),(15)	(10.5),(14.5)	(8),(9.5)	46.25
Demand	50.75	46.75	93.25	

Step (2) Identify the Initial Basic Feasible Solution applying VAM as in Table 16 and 17

Table 16 it shows the Iteration 2

	D_1	D_2	D_3	Stock	penalty
S_1	(1.83), (0.75)	(2.21), (0.94)	(1), (0.47)	78.75	0.28
S_2	(1.46), (0.58)	(1.38), (0.65)	(0.92), (0.36)	65.75	0.22
S_3	(1.92), (0.83)	(1.75), (0.81)	(1.33), (0.53)	46.25	0.28
Demand	50.75	46.75	93.25		
	0.38, 0.17	0.38, 0.15	0.08, 0.11		

Table 17 The Initial Basic Feasible Solution:

	D_1	D_2	D_3	Stock
S_1	(11), (13.5)	(13.25), (17)	(6), (8.5) 78.75	78.75
S_2	(8.75), (10.5) 50.75	(8.25), (11.75) 0.5	(5.5), (6.5) 14.5	65.75
S_3	(11.5), (15)	(10.5), (14.5) 46.25	(8), (9.5)	46.25
Demand	50.75	46.75	93.25	

Step (3) applying the optimality test as in Table 18 and 19

Table 18 The optimality test Iteration 1.

	v_1	v_2	v_3	
u_1	(11), (13.5)	(13.25), (17)	(6), (8.5) 42.5/78.75	0
u_2	(8.75), (10.5) 31.5/50.75	(8.25), (11.75) 8.25/0.5	(5.5), (6.5) 6.5/14.5	-0.09
u_3	(11.5), (15)	(10.5), (14.5) 43.5/46.25	(8), (9.5)	-15.65
	0.71	16.59	0.54	

Table 19 The optimality test Iteration 2.

	v_1	v_2	v_3	
u_1	(11), (13.5) $d > 0$	(13.25), (17) $d > 0$	(6), (8.5) 78.75	0
u_2	(8.75), (10.5) 50.75	(8.25), (11.75) 0.5	(5.5), (6.5) 14.5	-0.09
u_3	(11.5), (15) $d > 0$	(10.5), (14.5) 46.25	(8), (9.5) $d > 0$	-15.65
	0.71	16.59	0.54	

The final results as in Table 20 with the corresponding total cost = 132.25$.

Table.20 Final results and corresponding total cost

The basic cells	The quantity of units	Cost of vehicle (1)	No. Trips	Cost of vehicle (2)	No. Trips	The total cost
x_{13}	78.75	(6)	-	(8.5)	5	42.5
x_{21}	50.75	(8.75)	-	(10.5)	3	31.5
x_{22}	0.5	(8.25)	1	(11.75)	-	8.25
x_{23}	14.5	(5.5)	-	(6.5)	1	6.5
x_{32}	46.25	(10.5)	-	(14.5)	3	43.5
Summation	190.75units		1		12	132.25$

6- Conclusion

In this paper, the VAM is used to solve N -vehicle transportation problem, where the cost is defined by (vehicle cost/ vehicle capacity) then applying the penalty by taking the difference between the two smallest costs in each row and the same on columns reaching to the initial basic solution. The proposed method is faster than the method proposed applying northwest corner method It is also applied VAM in fuzzy N-vehicle transportation problem using Robust's ranking indices reaching to the optimal solution.

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Solving Rough Interval Cost of Transportation Problem with Source and Destination Parameters

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Abstract:

The rough-interval linear programming method is developed for transportation problem to represent uncertain parameters. In transportation problem, the cost of transportation from source to destination may depend on many factors which may not be deterministic in nature. In this paper, the unit cost of transportation with source and destination are considered to be the rough interval cost. The problem is solved by two methods, the traditional and proposed method. In proposed method the objective and constraints are roughness. The results of many cases are different and the others are the same.

Keywords: Transportation problem, Optimization, Rough Set, Rough interval, rough-interval linear programming (RILP)

1. Introduction

Transportation problem is one of the important linear programming problems. Many efficient methods have been developed to solve a transportation problem where the unit cost of transportation, demand and supply are deterministic .In many practical situations, the cost of transportation may be not deterministic in nature. The cost depends upon many factor such as road condition, traffic load causing delay in delivery; break down of vehicles, labor problem for loading and unloading which may increase the cost. All these factors are uncertain which cannot be predicted before due to insufficient information, unpredictable environment [2, 3].

Transportation problem is studied in various types of uncertain environment such as fuzzy numbers. In this paper, we develop a transportation model with rough interval parameters. In real life, the information is available in imprecise nature. There is an inherent degree of vagueness or uncertainty under consideration problem.

Rough set was pioneered by Pawlak in mid 1980s and since that time, the rough set theory has become at hot topic of great interest to most researchers in several fields which applied on many domains such as expert systems, machine learning and decision theory.

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A rough set concept is different from ordinary set as well as fuzzy set. An object in an ordinary set is completely determined. An object in a fuzzy set is fuzzily determined while in rough set the object is approximately determined based on some knowledge. Thus knowledge in the rough approach is necessarily connected with the variety of classification patterns related to specific parts of the real [10]. This paper is organized as follows; section 2 introduces "rough set theory, kinds of mathematical programming problems and rough interval" including some definitions and examples. Section 3 considers "rough-interval linear programming". Section 4 presents the transportation problem in crisp and rough interval form. Section 5 includes explaining "rough interval using traditional and proposed method". Section 6 contains "two illustration examples". Section 7 presents "conclusion".

2- Preliminaries:

2.1-Rough Set Theory

Rough set theory (RST) has been proven to be an excellent mathematical tool for dealing with uncertain and vague data descriptions of objects [7].

The main idea of rough set is the approximation of subsets or partitions of the universe U. For fixed $x \in U$, the available information can be used to calculate its degree belonging to a subset X of U where X can be regarded as a concept to be approximated. X is said to be a definable set (or exact set) if X is union of some equivalence classes, X is said to be a rough set respect to (U,R).The lower approximation of X by R is defined as $\underline{RX} = \{x \in U \,|[x]_R \subseteq X\}$ and the upper approximation is defined as $\overline{RX} = \{X \in U \,|[X]_R \cap X \neq \varnothing\}$. Clearly, the lower approximation $\underline{RX}$ is the greatest definable set contained in X, and the upper approximation $\overline{RX}$ is the least definable set containing X. The boundary region of rough set X is defined as $BN_R(X) = \overline{RX} - \underline{RX}$. The boundary region does not represent the area which cannot be classified, to X nor to its complement U-X, is the boundary region of the definable set an empty set because the upper approximation of the definable set are equal. The basic concepts of rough set theory are shown in Fig 1 [9, 11]

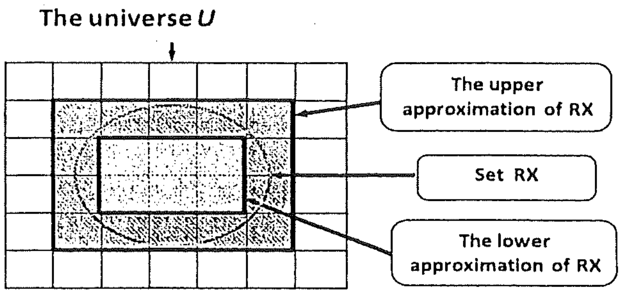

Fig 1 Rough set[11]

2.2 Kinds of rough mathematical programming problems:

The rough mathematical programming is considered when the mathematical programming of any parameters in objective function or constraint is roughness. The rough programming problems can be broadly classified as:

- **1st case**: mathematical programming with rough feasible set and crisp objective function.

- **2nd case**: mathematical programming with crisp feasible set and rough objective function.

- **3rd case**: mathematical programming with rough feasible set and rough objective function

The rough feasibility arise only in1st and 3rd cases, where the feasible set is a rough set and the solutions have different degrees of feasibility as follows:

-A solution of x is surely-feasible if it belongs to the lower approximation of the feasible set

-A solution of x is possibly-feasible if it belongs to the upper approximation of the feasible set

-A solution of x is surely-not feasible if it does not belongs to the upper approximation of the feasible set [7]

Now we consider the description of transportation quantity. If the transportation quantity is less than 130 tons, then the universe can be set as [0, 130]. With his or her knowledge and experience, the decision maker (DM) may divide the universe into five equivalent classes as follows:

- [0, 25]: The DM believes that the transportation quantity belonging to this interval is ''low.''

- (25, 30): The DM believes that the transportation quantity belonging to this interval is in transition from ''low'' to ''middle.''

- [30, 55): The DM believes that the transportation quantity belonging to this interval is ''middle.''

- (55, 60): The DM believes that the transportation quantity belonging to this interval is in transition from ''middle'' to ''high.''

- [60, 130]: The DM believes that the transportation quantity belonging to this interval is ''high.''.

Then the partition {[0, 25], (25, 30), [30, 55], (55, 60), [60, 130]} given by the DM can be regarded as the available information. For another person, he or she may make a vague description of ''low,'' ''middle,'' and ''high.'' Assume that he or she regards that the ''low'' transportation quantity is from 0 to 35, the ''middle'' transportation quantity is from 25 to

60, and the ''high'' transportation quantity is from 50 to 130. According to the partition of the DM, the transportation quantity interval [0, 25] is a surely low transportation quantity interval, the transportation quantity interval [0, 30) is a possibly low transportation quantity interval, and any transportation quantity in (25, 30) has rough membership from 1 to 0. Similar analysis may be carried out for the concepts of ''middle'' and ''high'' (see Fig. 2) [11, 12].

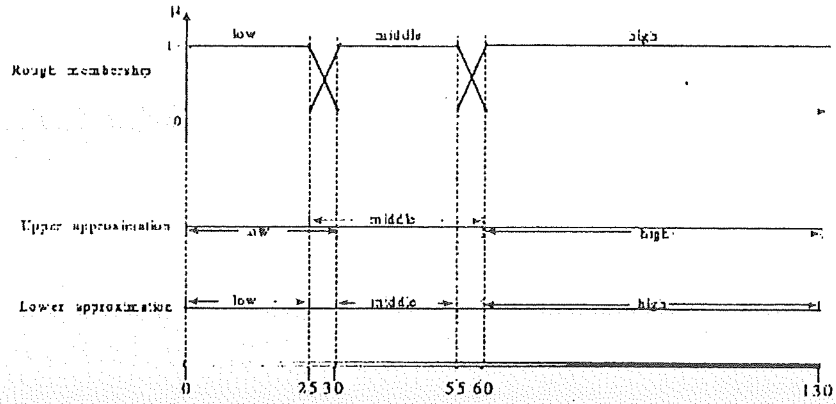

Fig 2 Rough description for transportation quantity [12]

2.3-Rough interval

Rough interval (RI) is a particular case of a rough set. The lower and upper approximation interval concepts satisfy the mathematical definition of the rough set`s upper and lower approximation adapted to intervals. Arithmetic of rough interval can be expressed as follows:

$$RI = ([a,b],[c,d]) \text{ where } (c \le a \le b \le d)$$

RI comprises of two parts: upper approximation interval ([c,d]) and a lower approximation interval ([a,b]) .For conveniences, it is denoted the upper and lower approximation interval of RI as $RI^*$and RI_* respectively. So, in normal cases, the variable takes value in the lower approximation and in special cases that the takes the value in upper approximation, that is to say there is no value taken the variable outside the upper approximation interval[6].

Definition 1: let x denote a closed and bounded set of real numbers. A rough interval X^I is defined as an interval with known lower and upper bounds but unknown distribution information for x:

$X^I = [X^L : X^U]$ where X^L and X^U are lower and upper approximation intervals of X^I, respectively[5].

X^L and X^U are conventional intervals and X^L □ X^U.

Definition 2: let R^R denote a set of rough intervals, A rough interval vector X^I is a tupelo of rough intervals, and a rough interval matrix whose elements are rough intervals:

$$X^{I} = \{X_i^{I} = [X_i^{L} : X_i^{U}] | \forall i\} \quad , X^{I} \in R^{R(1\times n)}$$
$$X_{ij}^{I} = \{X_{ij}^{I} = [X_{ij}^{L} : X_{ij}^{U}] | \forall i, j\} \quad , X_{ij}^{I} \in R^{R(m\times n)}$$

The operation for rough-interval vector and matrices is defined to be analogous to those for real vectors and matrices. . More definitions related to the operation methods for rough intervals and the modeling approach for RILP are all provided.

Definition 3: For rough interval X^{RI}, we have:

$$X^{RI} \geq 0, \text{ iff } X^{L}, X^{U} \geq 0$$
$$X^{RI} \leq 0, \text{ iff } X^{L}, X^{U} \leq 0$$

Definition 4: Let $* \in \{+,-,\times,\div\}$ be a binary operation on interval number X^I and Y^I when X^I, Y^I ≥ 0 we have[5] :

$$X^{I}+Y^{I} = [[X^{a}+Y^{a}]:[X^{b}+Y^{b}]]$$
$$X^{I}-Y^{I} = [[X^{a}-Y^{a}]:[X^{b}-Y^{b}]]$$
$$X^{I}+Y^{I} = [[X^{a}\times Y^{a}]:[X^{b}\times Y^{b}]]$$
$$X^{I}+Y^{I} = [[X^{a}\div Y^{a}]:[X^{b}\div Y^{b}]]$$

As X^L , X^U , Y^L and Y^U are conventional intervals, the above operations can be further transferred to the following functions if letting $X^L =[X^a, X^b]$, $X^U=[X^c, X^d]$, $Y^L =[Y^a,Y^b]$, $Y^U=[Y^c, Y^d]$where X^a, X^b, X^c, X^d Y^a, Y^b, Y^c and Y^d are deterministic numbers denoting the lower and upper bounds of X^L , X^U , Y^L and Y^U

$$X^{RI}+Y^{RI} = ([X^{a}+Y^{a}],[X^{b}+Y^{b}]:[X^{c}+Y^{c}],[X^{d}+Y^{d}])$$
$$X^{RI}-Y^{RI} = ([X^{a}-Y^{b}],[X^{b}-Y^{a}]:[X^{c}-Y^{d}],[X^{d}-Y^{c}])$$
$$X^{RI} * Y^{RI} = ([X^{a}*Y^{a}],[X^{b}*Y^{b}]:[X^{c}*Y^{c}],[X^{d}*Y^{d}])$$
$$X^{RI} \div Y^{RI} = ([X^{a}\div Y^{b}],[X^{b}\div Y^{a}]:[X^{c}\div Y^{d}],[X^{d}\div Y^{c}])$$

These transformation processes are based on the operation method for conventional intervals [12].

Example:

Let X^{RI} =([4, 8],[2, 10]),and Y^{RI} =([2, 5],[1, 7]) be two rough intervals then the rough interval arithmetic defined by[12]

$$X^{RI}+Y^{RI} = ([6,13]:[3,17])$$
$$X^{RI}-Y^{RI} = ([-1,6]:[-5,9])$$
$$X^{RI} * Y^{RI} = ([8,40]:[2,70])$$
$$X^{RI} \div Y^{RI} = ([4/5,4]:[2/7,10])$$

Definition 6: Let A^{RI} and B^{RI} denote two sets of rough intervals .A rough - interval linear programming model can be formulated as follows:

$$\begin{aligned} \max \quad & f^{RI} = C^{RI} \cdot X \\ \textit{subject to} \quad & A^{RI} \cdot X \leq B^{RI} \\ & X \geq 0 \end{aligned} \qquad (1)$$

Where $A^{RI} \in R^{RI\,(m\times n)}$, $B^{RI} \in R^{RI(m\times 1)}$, $C \in \{\Pi\}^{R}$ and Π denotes a set of deterministic parameters , X^{RI} denotes a set decision variables, f^{RI} denotes objective function value.

3. Rough-interval linear programming

Prior to proposing rough-interval linear programming, a set of definitions for rough intervals need to be provided. Generally, a rough interval comprises two parts: an upper approximation interval (UAI) and a lower approximation interval (LAI). The variable is assumed to be inside of its UAI and cannot get the qualitative value outside of this interval. The second element of rough interval LAI, can also be defined on this basis [4].

4. Transportation problem:

4.1. The crisp model of transportation:

Suppose that there are m sources and n destinations. Let ai be the number of supply units available at sources i (i=1,2,3....m) and let bj the number of demands units required at destination j (j=1,2,3....n). Let cij represents the unit transportation cost for transportation the units from source i to destination j. The objective is to determine the number of units to be transported from source i to destination j, so that the total transportation cost is minimum. Let x_{ij} be the decision variable which denotes the number of units shipped from sources i to destination j[8].

$$\begin{aligned} \textit{Minimiza} \quad & f = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} \\ \textit{subject to} \quad & \sum_{j=1}^{n} x_{ij} \leq a_i \quad , \; i = 1,2,........m \\ & \sum_{i=1}^{m} x_{ij} \geq b_j \quad , \; j = 1,2,........n \\ & x_{ij} \geq 0 \end{aligned} \qquad (2)$$

4.2. The rough interval model of transportation:

The cost c_{ij} is a rough interval variable of the form let

$$C_{ij}^{RI} = ([c_{ij}^{a}, c_{ij}^{b}]:[c_{ij}^{c}, c_{ij}^{d}])$$

$$\text{where } c_{ij}^{c} \leq c_{ij}^{a} \leq c_{ij}^{b} \leq c_{ij}^{d}$$

The supply a_i is a rough interval $a_i^{RI} = ([a_i^{a}, a_i^{b}]:[a_i^{c}, a_i^{d}])$

The demand b_j is rough interval $b_j^{RI} = ([b_j^a, b_j^b]:[b_j^c, b_j^d])$ and the objective function becomes a rough variable of the form $f^{RI} = ([f^a, f^b]:[f^c, f^d])$

$$
\begin{aligned}
&Minimiza \quad f^{RI} = \sum_{i=1}^{m}\sum_{j=1}^{n} c^{RI}{}_{ij}\, x_{ij} \quad , \mathrm{RI} = a, \mathrm{b}, \mathrm{c}, \mathrm{d} \\
&subject\ to \quad \sum_{j=1}^{n} x_{ij} = a^{RI}{}_i \quad , \ i = 1, 2, \ldots\ldots.\mathrm{m} \\
&\qquad\qquad \sum_{i=1}^{m} x_{ij} = b^{RI}{}_j \quad , \ j = 1, 2, \ldots\ldots.\mathrm{n} \\
&\qquad\qquad \sum_{i=1}^{m} a_i^{RI} = \sum_{j=1}^{n} b_j^{RI} \\
&\qquad\qquad x_{ij} \geq 0
\end{aligned} \tag{3}
$$

5. Rough interval using traditional and proposed methods

5.1 The traditional method

Divide the model to four sub-model A, B,C and D and solve them separating of each other. The results corresponding to the model is f^a , f^b, f^c, f^d and X^a , X^b, X^c, and X^d, respectively.

5.2 The proposed method

The steps of proposed method as follows:

Step 1: Formulate the problem

Step 2: Solve the second -desired upper-bound sub-model UAI[d] and obtain solutions of X_{opt}^d *and* f_{opt}^d

Step 3: Solve the first lower sub-model UAI[c], and add constraint $X_{ij} \leq X_{opt}^d$ to obtain the solution of X_{opt}^c *and* f_{opt}^c

Step 4: Solve the second upper sub-model LAI [b], and add constraint $X_{ij} \geq X_{opt}^c$ to obtain the solution of X_{opt}^b *and* f_{opt}^b

Step 5: Solve the first lower sub-model LAI[a], and add constraint $X_{ij} \geq X_{opt}^c$ and $X_{ij} \leq X_{opt}^b$ to obtain the solution of X_{opt}^a *and* f_{opt}^a

Step 6: Incorporate the solutions of all sub-models to obtain primal solutions as follows:

$$f^{RI} = [[f_{opt}^a, f_{opt}^b]:[f_{opt}^c, f_{opt}^d]]$$

$$X^{RI} = [[X_{opt}^a, X_{opt}^b]:[X_{opt}^c, X_{opt}^d]]$$

6. Illustration examples

6.1-Example 1:

	B1	B2	B3	B4	S
A1	([6,7]:[5,9])	([6,7]:[5,9])	([5,7]:[4,8])	([6,7]:[5,9])	([7,9]:[6,10])
A2	([9,11]:[8,12])	([15,17]:[14,18])	([16,17]:[14,19])	([18,21]:[17,22])	([17,21]:[16,22])
A3	([1,13]:[10,14])	([3,5]:[2,6])	([9,11]:[8,12])	([3,4]:[1,5])	([16,18]:[14,20])
D	([10,12]:[9,13])	([2,4]:[1,5])	([13,15]:[12,16])	([15,17]:[14,18])	([40,48]:[36,52])

Consider a problem with three sources (i= 1,2,3) and four destination (j=1,2,3,4). The unit of transportation cost are rough interval, the availability at each sources, demand of each destination are rough interval

The problem is solved by two methods, applying Lingo package reaching to the final results.

The traditional method:

A					
	B1	B2	B3	B4	S
A1	6	6	5	6	7
A2	9	15	16	18	17
A3	11	3	9	3	16
D	10	2	13	15	40

$f^a = 284$

$X^a = (0,0,7,0,10,1,6,0,0,1,0,15)$

B					
	B1	B2	B3	B4	S
A1	7	7	7	7	9
A2	11	17	17	21	21
A3	13	5	11	4	18
D	12	4	15	17	48

$f^b = 421$

$X^b = (0,3,6,0,12,0,9,0,0,1,0,17)$

C					
	B1	B2	B3	B4	S
A1	5	5	4	5	6
A2	8	14	14	17	16
A3	10	2	8	1	14
D	9	1	12	14	36

$f^c = 208$

$X^c = (0,0,6,0,9,1,6,0,0,0,0,14)$

d					
	B1	B2	B3	B4	S
A1	9	9	8	9	10
A2	12	18	19	22	22
A3	14	6	12	5	20
D	13	5	16	18	52

$f^d = 506$

$X^d = (0,0,10,0,13,3,6,0,0,2,0,18)$

f^{RI}=[[284,421]:[208,506]],

X_{12}=[[0,0]:[0,3]], X_{13}=[[6,7]:[6,10]] X_{21}=[[10,12]:[9,13]], X_{22} =[[1,1]:[0,3]], X_{23}=[[6,6]:[6,9]] ,X_{32} =[[1,1]:[0,2], X_{34}=[[15,17]:[14,18]]

The proposed method:

Step 1: divide the original model into LAI [a ,b] and UAI [c ,d]

LAI[a,b]					
	B1	B2	B3	B4	S
A1	([6,7])	([6,7])	([5,7])	([6,7])	([7,9])
A2	([9,11])	([15,17])	([16,17])	([18,21])	([17,21])
A3	([11,13])	([3,5])	([9,11])	([3,4])	([16,18])
D	([10,12])	([2,4])	([13,15])	([15,17])	([40,48])

UAI[c,d]					
	B1	B2	B3	B4	S
A1	([5,9])	([5,9])	([4,8])	([5,9])	([6,10])
A2	([8,12])	([14,18])	([14,19])	([17,22])	([16,22])
A3	([10,14])	([2,6])	([8,12])	([1,5])	([14,20])
D	([9,13])	([1,5])	([12,16])	([14,18])	([36,52])

Then, classifying each one into sub-model according to the lower and upper approximation interval, LAI[a] LAI[b],and UAI[c] UAI[d] respectively

LAI

A					
	B1	B2	B3	B4	S
A1	6	6	5	6	7
A2	9	15	16	18	17
A3	11	3	9	3	16
D	10	2	13	15	40

B					
	B1	B2	B3	B4	S
A1	7	7	7	7	9
A2	11	17	17	21	21
A3	13	5	11	4	18
D	12	4	15	17	48

UAI

C					
	B1	B2	B3	B4	S
A1	5	5	4	5	6
A2	8	14	14	17	16
A3	10	2	8	1	14
D	9	1	12	14	36

D					
	B1	B2	B3	B4	S
A1	9	9	8	9	10
A2	12	18	19	22	22
A3	14	6	12	5	20
D	13	5	16	18	52

Step 2 : solve UAI[d] then obtaining the solution of X^d_{opt} *and* f^d_{opt}

X^d = (0,0,10,0,13,3,6,0,0,2,0,18) $f^d = 506$

Step 3 : Solve UAI[c] after adding the following constraints

X_{13} <=10, X_{21} <=13 , X_{22} <=3, X_{23} <=6, X_{32} <=2 , X_{34} <=18

Reaching to the following solution of X^c_{opt} *and* f^c_{opt}

X^c = (0,0,6,0,9,1,6,0,0,0,0,14) $f^c = 208$

Step 4: Solve LAI[b] add the following constraints

X_{13} >=6 , X_{21} >=9 , X_{22} >=1 , X_{23} >=6 , X_{34} >=14

Reaching to the following solution of X^b_{opt} *and* f^b_{opt}

X^b = (0,0,9,0,12,3,6,0,0,1,0,17) f^b = 421

Step 5: Solve LAI[a] and adding the following constraints

X_{13} >=6 , X_{21} >=9 , X_{22} >=1 , X_{23} >=6 , X_{34} >=14 and X_{13}<=9, X_{21} <=12 , X_{22} <=3 , X_{23} <6 , X_{32} <=1, X_{34} <=17

Reaching to the following solution of X^a_{opt} *and* f^a_{opt},

X^a = (0,0,7,0,10,1,6,0,0,1,0,15) f^a = 284

Step 6 : Incorporate the solution of all sub-models to obtain primal solution as follows

f^{RI}=[[284,421]:[208,506]], X_{13}=[[7,9]:[6,10]] X_{21}=[[10,12]:[9,13]], X_{22}=[[1,3]:[1,3]] X_{23}=[[6,6]:[6,6]], X_{32}=[[1,1]:[0,2], X_{34}=[[15,17]:[14,18]]

6.2-Example 2:

	B1	B2	B3	S
A1	([7,9]:[6,10])	([10,11]:[8,12])	([11,13]:[10,12])	([24,27]:[22,28])
A2	([6,8]:[5,9])	([9,10]:[7,11])	([5, 7]:[4,8])	([15,18]:[14,19])
A3	([8,10]:[7,11])	([13,15]:[12,16])	([8,10]:[7,11])	([22,24]:[21,26])
D	([13,16]:[12,17])	([25,28]:[23,30])	([23,25]:[22,26])	([61,69]:[57,73])

Consider a problem with three sources (i= 1,2,3) and three destination (j=1,2,3). The unit of transportation cost are rough interval, the availability at each sources, demand of each destination are rough interval

The traditional method:

A	B1	B2	B3	S
A1	7	10	11	24
A2	6	9	5	15
A3	8	13	8	22
D	13	25	23	61

f^a = 495

X^a = (0, 24, 0, 0, 1, 14, 13, 0, 9)

B	B1	B2	B3	S
A1	9	11	13	27
A2	8	10	7	18
A3	10	15	10	24
D	16	28	25	69

f^b = 666

X^b = (0, 27, 0, 0, 1, 17, 16, 0, 8)

C	B1	B2	B3	S
A1	6	8	10	22
A2	5	7	4	14
	7	12	7	21
D	12	23	22	57

f^c = 382

X^c = (0, 22,0 ,0 ,1 ,13,12 ,0 ,9)

D	B1	B2	B3	S
A1	10	12	12	28
A2	9	11	8	19
A3	11	16	11	26
D	17	30	26	73

f^d = 780

X^d = (0, 28, 0, 0, 2, 17, 17, 0,9)

f^{RI}=([495,666]:[382,780]), X_{12}=([24,27]:[22,28]), X_{22}=([1,1]:[1,2])

X_{23}=([14,17]:[13,17]), X_{31} =([13,16]:[12,17]), X_{33}=([8,9]:[9,9])

Applying the suggested method:

Step 1: divide the original model into LAI [a ,b] and UAI [c ,d]

[a,b]	B1	B2	B3	S
A1	([7,9])	([10,11])	([11,13])	([24,27])
A2	([6,8])	([9,10])	([5,7])	([15,18])
A3	([8,10])	([13,15])	([8,10])	([22,24])
D	([13,16])	([25,28])	([23,25])	([61,69])

[c,d]	B1	B2	B3	S
A1	([6,10])	([8,12])	([10,12])	([22,28])
A2	([5,9])	([7,11])	([4,8])	([14,19])
A3	([7,11])	([12,16])	([7,11])	([21,26])
D	([12,17])	([23,30])	([22,26])	([57,73])

Then, classifying each one into sub-model according to the lower and upper approximation interval, LAI[a] LAI[b],and UAI[c] UAI[d] respectively

LAI

A	B1	B2	B3	S
A1	7	10	11	24
A2	6	9	5	15
A3	8	13	8	22
D	13	25	23	61

B	B1	B2	B3	S
A1	9	11	13	27
A2	8	10	7	18
A3	10	15	10	24
D	16	28	25	69

UAI

C	B1	B2	B3	S
A1	6	8	10	22
A2	5	7	4	14
A3	7	12	7	21
D	12	23	22	57

D	B1	B2	B3	S
A1	10	12	12	28
A2	9	11	8	19
A3	11	16	11	26
D	17	30	26	73

Step 2 : solve UAI[d] then, obtaining the solution of X^{d}_{opt} *and* f^{d}_{opt}

X^d = (0,28,0,0,2,17,17,0,9) f^d = 780

Step 3 : Solve UAI[c] after adding the following constraints

X_{12} <=28, X_{22} <=2 , X_{23} <=17, X_{31} <=17, X_{33} <=9 ,

Reaching to the solution of X^{c}_{opt} *and* f^{c}_{opt} X^c = (0,22,0,1,13,12,0,9) f^c = 380

Step 4: Solve LAI[b] after adding the following constraints

$X_{12}>=22$, $X_{22}>=1$, $X_{23}>=13$, $X_{31}>=12$, $X_{33}>=9$

Reaching to X^{b}_{opt} *and* f^{b}_{opt} $X^b = (0,27,0,1,1,16,15,0,9)$ $f^b = 667$

Step 5: Solve LAI[a] after adding the following constraints

$X_{12}>=22$, $X_{22}>=1$, $X_{23}>=13$, $X_{31}>=12$, $X_{33}>=9$ and $X_{12}<=27$,

$X_2<=1$, $X_{22}<=1$, $X_{23}<=16$, $X_{31}<=15$, $X_{33}<=9$

Reaching to solution of X^{a}_{opt} *and* f^{a}_{opt}, $X^a = (0,24,0,0,1,14,13,0,9)$ $f^a = 495$

Step 6: Incorporate the solutions of all sub-model to obtain primal solution as follows

f^{RI}=([495,666]:[380,780]), X_{12}=([24,27]:[22,28]), X_{21}=([0,0]:[0,1]),

X_{22}=([1,1]:[1,2]) , X_{23}=([13,16]:[13,17]), X_{31}=([13,15]:[12,17]), X_{33}=([9,9]:[9,9])

7-Conclusion

This paper considers a formulation of an interval rough transportation problem which it is solved by two methods. The traditional and proposed method are applied in form of the third case which the objective and constraints are roughness. The solution of suggested method in many cases is different in results but some cases with the same result.

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On multi-objective linear programming problems with inexact rough interval-fuzzy coefficients

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Abstract

This paper deals with a multi-objective linear programming problem with an inexact rough interval fuzzy coefficients IRFMOLP. This problem is considered by incorporating an inexact rough interval fuzzy number in both the objective function and constrains. The concept of "Rough interval" is introduced in the modeling framework to represent dual-uncertain parameters. A suggested solution procedure is given to obtain rough interval solution for IRFLP(w) problem. Finally,two numerical example is given to clarify the obtained results in this paper.

Keywords

Multi-objective linear programming; Rough-interval fuzzy coefficients; Weighting method; Rough interval solution.

1 Introduction

The theory of rough sets proposed by Pawlak (1982) [16], can be regarded as an effective mathematical vehicle for dealing with imprecise and ambiguous data analyses, which can be subsequently applied to pattern recognition, machine learning, and knowledge discovery [1-7]. The equivalence relation is a key notion in Pawlak's rough set model. All equivalence

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classes form a partition of a universe of discourse. Using equivalence classes, an arbitrary subset can be approximated by two subsets called the lower approximation and the upper approximation. However, the equivalence relation is a stringent condition that may limit the applications of rough sets in practical problems. Hence, various extensions of Pawlak's rough set were developed from an equivalence relation into a more general mathematical concept, e.g., binary relations by Slowinski (2000) [18]. The theory of rough set deals with approximation of an arbitrary subset of universe by two definable or observable subsets called lower and upper approximation. Dubois and Prade (1990) [5] pointedly that the rough fuzzy set is a special case of the fuzzy rough set. The notion of interval-valued fuzzy set was suggested for the first time by Turksan (1986) [19] and Gorse (1986) [9]. They are applied to the fields of the approximate inference gave Miyamoto (2004)[14] fuzzy sets and α-level sets .Shaocheng (1994)[17] introduced two kinds of linear programming with fuzzy numbers. They are called interval number and fuzzy number linear programming. Guijun and Xiapong (1998)[10] define interval-valued fuzzy number and interval distribution numbers and gave their extended operations. Zhong et al. (1994)[23] study fuzzy random linear programming having fuzzy random variable coefficients and the decision vector of fuzzy random variable. It is generally accepted that these two theories are related, but a listing and complementary, to each other Miyamoto (2004) [14], XU (2012) [20] and Yao (1996) [21]. establishes a rough multiple objective programming model for a solid transportation problem. Gong et al., (2008) [8]and Li et al., (2007) [12], introduce an interval-valued fuzzy information system by means of iterating the classical Pawlak's rough set theory with the interval-valued fuzzy set theory and oles cusses the basic rough set theory for the interval-valued fuzzy information system. There are at least two approaches for the development of the fuzzy rough sets theory: The constructive and axiomatic approaches (Yao (1998) [22]). In the constructive approach, the relation to the universe is the primitive notion. The lower and upper approximation operator are contracted by means of this notion. On the other hand, the axiomatic approach takes the lower and upper approximation operator as primitive notion. In this approaches, a set of axioms is used to characterize approximation operators. Zhang (2012) [24] presents a general framework for the study of interval type-2 rough fuzzy sets by using both constructive and axiomatic approaches. In this paper, multi-objective linear programming problem with an inexact rough interval fuzzy coefficients IRFMOL is introduced.The problem is transformed into the corresponding IRFLP problem using the weighting method, A solution procedure is given to obtain the rough interval solution for the IRFLP(w) problem.

The paper is organized as in the following sections: In section 2, some preliminaries are introduced. In section 3, problem formulation is introduced as specific definition and properties. In section 4, a solution procedure is given to obtain a rough interval solution for the IRFMOLP problem in section 3. In section 5, numerical example is given to clarify the obtained results. Finally, some concluding remarks are reported in section 6.

2 Preliminaries

In this section, we introduce some notions related to the IRFMOLP problem. The following concepts can be found in [11], [15] and [24]. Number of α-cuts, leading to a series of dual intervals being generated. Among these intervals, the internal has two limits (i.e. $\tilde{a}_N^{-L}(\alpha)$ *and* $\tilde{a}_N^{+L}(\alpha)$) reflect the conservative feature of fuzzy information, while the external ones (i.e. $\tilde{a}_N^{-U}(\alpha)$ *and* $\tilde{a}_N^{+U}(\alpha)$) correspond to the optimistic characteristics. Let $F_N^R(R)$ denote the set of all compact rough fuzzy numbers on real line (R), that is, $\tilde{a}^R \in F_N^R(R)$. It is $\tilde{a}^L$ lower bounded membership grades consist of a membership function located inside the other one formed by the upper bounded grades $\tilde{a}^U$, $\tilde{a}^R$ Satisfies:

1. $\tilde{a}^R$ is normal
(i.e $\exists\, x\ \in \tilde{a}_\alpha^R\ =\ \{x : \tilde{a}^L \geq\ \alpha,\ \tilde{a}^U \geq \alpha,\ \tilde{a}^L \subseteq \tilde{a}^U,\ \alpha\ \in\ (0,1]\}$) such that $a^R(x) = 1$.
2. For any $\alpha\ \in\ (0,1]$, $a_\alpha^R = \left[[a(\alpha)^{-U}, a(\alpha^{+U})] : [a(\alpha)^{-L}, a(\alpha)^{+L}]\right]$ is a rough interval number on R such that $[a_\alpha^{-L}, a_\alpha^{+L}] \subseteq [a_\alpha^{-U}, a_\alpha^{+U}]$.

Definition 1 *Suppose that*

$$a_\alpha^R = \left[[a^{-U}(\alpha), a^{+U}(\alpha)] : [a^{-L}(\alpha), a^{+L}(\alpha)]\right],$$

$$b_\alpha^R = \left[[b^{-U}(\alpha), b^{+U}(\alpha)] : [b^{-L}(\alpha), b^{+L}(\alpha)]\right].$$

we define

1.

$$a_\alpha^R + b_\alpha^R = \left[[a^{-U}(\alpha) + b^{-U}(\alpha),\ a^{+U}(\alpha) + b^{+U}(\alpha)] : [a^{-L}(\alpha) + b^{-L}(\alpha), a^{+L}(\alpha) + b^{+L}(\alpha)]\right],$$

2.

$$a_\alpha^R - b_\alpha^R = \left[[a^{-U}(\alpha) - b^{+U}(\alpha),\ a^{+U}(\alpha) - b^{-U}(\alpha)] : [a^{-L}(\alpha) - b^{+L}(\alpha), a^{+L}(\alpha) - b^{-L}(\alpha)]\right],$$

3.

$$a_\alpha^R \times b_\alpha^R = \left[[a^{-U}(\alpha) \times b^{-U}(\alpha), a^{+U}(\alpha) \times b^{+U}(\alpha)] : [a^{-L}(\alpha) \times b^{-L}(\alpha), a^{+L}(\alpha) \times b^{+L}(\alpha)]\right],$$

4. The order relation " $\leq$ " is defined by

$$a_\alpha^R \leq b_\alpha^R\ \ iff\ \ a^{+L}(\alpha) \leq b^{+L}(\alpha)\ \ and\ \ a^{-U}(\alpha) \leq b^{-U}(\alpha).$$

5. Let $\{\ \left[[a^{-U}(\alpha_i), a^{+U}(\alpha_i)] : [a^{-L}(\alpha_i), a^{+L}(\alpha_i)]\right],\ \ i\ \in\ I\ \}$ *I is the index set and* $\wedge_{i\in\ I}\, a(\alpha_{i\in I})^{-U} > -\infty$, *then*

$$\wedge_{i\in I}\ \left[[a^{-U}(\alpha_i), a^{+U}(\alpha_i)] : [a^{-L}(\alpha_i), a^{+L}(\alpha_i)]\right] =$$
$$\left[[\wedge_{i\in I}\ a^{-U}(\alpha_i), \wedge_{i\in\ I}\ a^{+U}(\alpha_i)] : [\wedge_{i\in I}\ a^{-L}(\alpha_i), \wedge_{i\in I}\ a^{+L}(\alpha_i)]\right],$$

6. *let* $\{ \ \left[[a^{-U}(\alpha_i), a^{+U}(\alpha_i)] : [a^{-L}(\alpha_i), a^{+L}(\alpha_i)]\right], \quad i \in I \ \}$ *I is the index set and* $\vee_{i\in\ I}\, a(\alpha_{i\in I})^{+U} < +\infty$, *then*

$$\vee_{i\in I}\ \left[[a^{-U}(\alpha_i), a^{+U}(\alpha_i)] : [a^{-L}(\alpha_i), a^{+L}(\alpha_i)]\right] =$$

$$\left[[\vee_{i\in I}\ a^{-U}(\alpha_i), \vee_{i\in\ I}\ a^{+U}(\alpha_i)] : [\vee_{i\in I}\ a^{-L}(\alpha_i), \vee_{i\in I}\ a^{+L}(\alpha_i)]\right].$$

Definition 2 *Let* $F_N(R)$ *denote the set of all compact fuzzy numbers, that is, for any* $\tilde{a} \in F_N(R)$, *satisfies:*

1. $\tilde{a}$ *is normal (i.e* $\exists\ x\ \in a_\alpha\ =\ \{x : \tilde{a}(x) \geq\ \alpha,\ \alpha\ \in\ (0,1]\}$*) such that* $\tilde{a}(x) = 1$,
2. *For any* $\alpha\ \in\ (0,1]$, $a_\alpha = \left[a^L(\alpha), a^U(\alpha)\right]$ *is closed interval number on* R, $a^L_\alpha \leq a^U_\alpha$.

Suppose that, $a_\alpha = \left[a^L(\alpha), a^U(\alpha)\right]$, $b_\alpha = \left[[b^L(\alpha), b^U(\alpha)\right]$, *we define*

1.

$$a_\alpha + b_\alpha = [a^L(\alpha), a^U(\alpha)] + [b^L(\alpha), b^U(\alpha)] = [a^L(\alpha) + b^L(\alpha), a^U(\alpha) + b^U(\alpha)], \qquad (1)$$

2.

$$a_\alpha - b_\alpha = [a^L(\alpha), a^U(\alpha)] - [b^L(\alpha), b^U(\alpha)] = [a^L(\alpha) - b^U(\alpha), a^U(\alpha) - b^L(\alpha)], \qquad (2)$$

3.

$$a_\alpha \cdot b_\alpha = [a^L(\alpha), a^U(\alpha)] \cdot [b^L(\alpha), b^U(\alpha)] = [a^L(\alpha)b^L(\alpha) \wedge a^U(\alpha)b^L(\alpha) \wedge a^L(\alpha)b^U(\alpha)$$
$$\wedge a^U(\alpha)b^U(\alpha), a^L(\alpha)b^L(\alpha) \vee a^L(\alpha)b^U(\alpha) \vee a^U(\alpha)b^L(\alpha) \vee a^U(\alpha)b^U(\alpha)], \qquad (3)$$

4. *The order relation "* $\leq$ *" is defined by*

$$[a^L(\alpha), a^U(\alpha)] \leq [b^L(\alpha), b^U(\alpha)]\ iff\ a^L(\alpha) \leq b^L(\alpha), a^U(\alpha) \leq b^U(\alpha),$$

5. *Let* $\{[a^L(\alpha_i), a^U(\alpha_i)],\ i \in I\} \subset R$, *I is the index set and* $\wedge_{i\in\ I}\, a(\alpha_{i\in I})^L > -\infty$, *then*

$$\wedge_{\alpha_i}[a^L(\alpha_i), a^U(\alpha_i)] = [\wedge_{\alpha_i}a^L(\alpha_i), \wedge_{\alpha_i}a^U(\alpha_i)], \qquad (4)$$

6. *Let* $\{[a^L(\alpha_i), a^U(\alpha_i)],\ i \in I\} \subset R$, *Iis the index set and* $\vee_{i\in\ I}\, a(\alpha_{i\in I})^U < \infty$, *then*

$$\vee_{\alpha_i}[a^L(\alpha_i), a^U(\alpha_i)] = [\vee_{\alpha_i}a^L(\alpha_i), \vee_{\alpha_i}a^U(\alpha_i)]. \qquad (5)$$

Lemma 1 *Let* $\tilde{a},\ \tilde{b} \in F_N(R)$, *then for any* $\alpha \in (0,1]$, *we have*

$$(a\ * b)_\alpha =\ a_\alpha\ * b_\alpha \qquad (6)$$

$*$ *is any operation defined in (1)-(6).*

3 Problem formulation

Consider the rough fuzzy multiobjective linear programming problem (IRFMOLP), as in the following form:

- Model 1

$$IRFMOLP \qquad \min_{x^R \in \tilde{M}^R} \left(\tilde{f}^R(\tilde{D}^R, \tilde{X}^R) = \tilde{D}^R \tilde{X}^R \right) \tag{7}$$

Subject to:

$$\tilde{M}^R(\tilde{A}, \tilde{B}) = \{ \tilde{x}^R : \ \tilde{A}^R \ \tilde{X}^R \leq \ \tilde{B}^R, \ \ \tilde{x}^R \geq 0 \}$$

where $\tilde{D}^R = [\ \tilde{d}_{li}^R]_{k \times n}$, $\tilde{A}^R = [\ \tilde{a}_{ji}^R]_{m \times n}$, $\tilde{B}^R = (\ \tilde{b}_1^R, \ \tilde{b}_2^R, ..., \ \tilde{b}_m^R)$, $X^R = (x_1^R, x_2^R, ..., x_n^R)^T$, and $\tilde{D}^R$; $\tilde{A}^R$; $\tilde{B}^R$ are compact rough fuzzy numbers. The above problem can be reformulated by use the weighting method as in the following form:

- Model 2

$$IRFLP(w) \qquad \min_{\tilde{x}^R \in \ \tilde{M}^R} \left(\tilde{f}^R(\tilde{C}^R, \tilde{X}^R) = \tilde{C}^R \ \tilde{X}^R \right) \tag{8}$$

Subject to:

$$\tilde{M}^R = \{ x^R : \ \tilde{A}^R \ X^R \leq \ \tilde{B}^R, \ \ x^R \geq 0 \}$$

where $[\tilde{C}]_n^R = [W]_{1 \times k} [\tilde{D}]_{k \times n}^R$, $w \in \Omega = \{ w_i \geq 0 \ , i = 1, 2, ..., k, \ \ \sum_{i=1}^k w_i = 1 \}$, $\tilde{D}^R = [\ \tilde{d}_{li}^R]_{k \times n}$, $\tilde{A}^R = [\ \tilde{a}_{ji}^R]_{m \times n}$, $\tilde{B}^R = (\ \tilde{b}_1^R, \ \tilde{b}_2^R, ..., \ \tilde{b}_m^R)$, $X^R = (x_1^R, x_2^R, ..., x_n^R)^T$, and $\tilde{C}^R$; $\tilde{A}^R$; $\tilde{B}^R$ are compact rough fuzzy numbers.

Definition 3 *The rough fuzzy vector $x^{*R}(\tilde{A}^{*R}, \tilde{B}^{*R})$ which satisfies the conditions in model (2), is said to be a rough fuzzy optimal solution of mode (2) IRFLP(w) if*

$$\tilde{f}^R(\tilde{C}^R, \tilde{x}^{*R}) \leq \tilde{f}^R(\tilde{C}^{*R}, \tilde{x}^R)$$

for each $\tilde{x}^{*R} \in \tilde{M}^R(\tilde{A}^R, \tilde{B}^R)$, furthermore if x^{*R} is a rough fuzzy vector. then it is said to be a rough fuzzy optimal solution of model 2: for any $\alpha \in (0, 1]$ the $\alpha-$level cuts of c_i^{*R}, a_{ji}^{*R}, b_j^{*R}, x_α^{*R} are rough intervals, furthermore, if x^{*R} is a rough fuzzy vector, then it is said to be a rough fuzzy optimal solution of model 2: for any $\alpha \in (0, 1]$ the $\alpha-$level cuts of c_i^{*R}, a_{ji}^{*R}, b_j^{*R}, x_α^{*R} are rough intervals denote them by:

$$(c_i^{*R})_\alpha = \left[[c_i^{*-U}(\alpha), c_i^{*+U}(\alpha)] : [c_i^{*-L}(\alpha), C_i^{*+L}(\alpha)] \right],$$

$$(a_{ji}^{*R})_\alpha = \left[[a_{ji}^{*-U}(\alpha), a_{ji}^{*+U}(\alpha)] : [a_{ji}^{*-L}(\alpha), a_{ji}^{*+L}(\alpha)] \right],$$

$$(b_j^{*R})_\alpha = \left[[b_j^{*-U}(\alpha), b_j^{*+U}(\alpha)] : [b_j^{*-L}(\alpha), b_j^{*+L}(\alpha)] \right],$$

$$(x_i^{*R})_\alpha = \left[[x_i^{*-U}(\alpha), x_i^{*+U}(\alpha)] : [x_i^{*-L}(\alpha), x_i^{*+L}(\alpha)] \right],$$

$i = 1, 2, ..., n \quad j = 1, 2, ..., m$

Definition 4 *(Rough fuzzy efficient solution) The rough fuzzy vector $x^{*R}(\tilde{A}^{*R}, \tilde{B}^{*R})$ which satisfies the condition in model 1 IRFMOLP, is called a rough fuzzy efficient solution of mode 1 IRFMOLP if and only if there does not exist another $x^R(\tilde{A}^{*R}, \tilde{B}^{*R}) \in \tilde{M}^R$ such that*

$$\tilde{f}_i(\tilde{C}^{*R}, \tilde{x}^R) \leq \tilde{f}_i(\tilde{C}^{*R}, \tilde{x}^{*R})$$

for all i and $f_i(x^R) \neq f_i(x^{*R})$ for at least one $i = 1, 2, ..., k$

Remark 1 *For any $w^* \in \Omega$, any rough fuzzy optimal solution of modle 2 is an rough fuzzy efficient solution of problem modle 1*

Also, for any $\alpha \in (0, 1]$ we transfer model 2 to two boundary models, an upper approximation fuzzy interval programming problem $\tilde{P}_U^U$ and lower approximation fuzzy interval programming problem $\tilde{P}_L^L$ as follows:

$$\tilde{P}_U^U : \qquad \min_{x \in \tilde{M}^U} \left(\tilde{f}^U = \tilde{C}^U \tilde{X}^U\right) \tag{9}$$

Subject to:

$$\tilde{M}^U(\tilde{A}^U, \tilde{B}^U) = \{x \, : \tilde{A}^U X \leq \tilde{B}^U, \quad x \geq 0\}$$

$$\tilde{P}_L^L : \qquad \min_{x \in \tilde{M}^L} \left(\tilde{f}^L = \tilde{C}^L \tilde{X}^L\right) \tag{10}$$

Subject to:

$$\tilde{M}^L(\tilde{A}^L, \tilde{B}^L) = \{x \, : \tilde{A}^L X \leq \tilde{B}^L, \quad x \geq 0\}$$

4 Solution procedure

1. Convert model (2) to two boundary corresponding to submodels P_U^U, P_L^L, respectively, if the objective is to be minimized, then P_U^U is desired.
2. Convert each boundary model to two corresponding to submodels the upper and lower approximation intervals, respectively.
3. Solve the first desired upper bounded submodel and obtain solution of x^{+U} and P_U^{+U}.
4. Add constraint $x^{+L} \leq x^{+U}$ to the second upper bounded submodel, solve it and obtain solution of x^{+L} and P_L^{+L}.
5. Add constraint $x^{-L} \leq x^{+L}$ to the first first lower bounded submodel, solve it and obtain solution of x^{-L} and P_L^{-L}.
6. Add constraint $x^{-U} \leq x^{-L}$ to the first second lower bounded submodel, solve it and obtain solution of x^{-U} and P_U^{-U}.

7. Incorporate the solution of all submodel to the obtain optimal solution as following :

$$x_\alpha^R = \left[[x_\alpha^{-U}, x_\alpha^{+U}] : [x_\alpha^{-L}, x_\alpha^{+L}]\right] \qquad f_\alpha^R = \left[[f_\alpha^{-U}, f_\alpha^{+U}] : [f_\alpha^{-L}, f_\alpha^{+L}]\right]$$

After deduce the following programming problems for a upper approximation interval and lower approximation interval

- Model 3:
  1. $$P_{-U}^{-U} : \qquad \min_{x_\alpha \in M_\alpha^{-U}} C_\alpha^{-U} X_\alpha$$
  2. $$P_{-L}^{-L} : \qquad \min_{x_\alpha \in M_\alpha^{-L}} C_\alpha^{-L} X_\alpha$$
- Model 4:
  1. $$P_{+U}^{+U} : \qquad \min_{x_\alpha \in M_\alpha^{+U}} C_\alpha^{+U} X_\alpha$$
  2. $$P_{+L}^{+L} : \qquad \min_{x_\alpha \in M_\alpha^{+L}} C_\alpha^{+L} X_\alpha$$
- Model 5:
  1. $$P_{+U}^{-U} : \qquad \min_{x_\alpha \in M_\alpha^{+U}} C_\alpha^{-U} X_\alpha$$
  2. $$P_{+L}^{-L} : \qquad \min_{x_\alpha \in M_\alpha^{+L}} C_\alpha^{-L} X_\alpha$$
- Model 6:
  1: $$P_{-U}^{+U} : \qquad \min_{x_\alpha \in M_\alpha^{-U}} C_\alpha^{+U} X_\alpha$$

2.

$$P^{+L}_{-L}: \qquad \min_{x_\alpha \in \ M_\alpha^{-L}} C_\alpha^{+L} X_\alpha$$

- Model7:

1. $P^{+U}_{\mp U}$ and $P^{-U}_{\mp U}$

$$\min_{x_\alpha \in M_\alpha^{(-U,+U)}} C_\alpha^{+U} X_\alpha \quad and \quad \min_{x_\alpha \in M_\alpha^{(-U,+U)}} C_\alpha^{-U} X_\alpha$$

2. $P^{+L}_{\mp L}$ and $P^{-L}_{\mp L}$

$$\min_{x_\alpha \in M_\alpha^{(-L,+L)}} C_\alpha^{+L} X_\alpha \quad and \quad \min_{x_\alpha \in \ M_\alpha^{(-L,+L)}} C_\alpha^{-L} X_\alpha$$

- Model 8:

1.

$$P^{+U}_{\mp U}: \qquad \min_{x_\alpha \in \ M_\alpha^{(-U,+U)}} C_\alpha^{+U} X_\alpha$$

2.

$$P^{+L}_{\mp L}: \qquad \min_{x_\alpha \in \ M_\alpha^{(-L,+L)}} C_\alpha^{+L} X_\alpha$$

It is clear that for any $\alpha \in (0,1]$ Models (3-8) are linear programming problem with rough interval numbers solved them by simplex method .

Lemma 2 *Suppose that* $(\tilde{A})^L \geq 0$ *(i.e* $a_{ji}^{-L} \geq 0$ *and* $a_{ji}^{+L} \geq 0$ *) for any* $\alpha \in (0,1]$ *if* $\tilde{x} \in \tilde{M}$, *then* $x_\alpha^{-L} \in M_\alpha^{-L}$ *and* $x_\alpha^{+L} \in M_\alpha^{+L}$. *Similarly, when* $\tilde{A}^U \geq 0$ *and* $\tilde{x} \in \tilde{M}^U$ *then* $x_\alpha^{-U} \in M_\alpha^{-U}$ *and* $x_\alpha^{+U} \in M_\alpha^{+U}$.

Lemma 3 *Suppose that* $(\tilde{A})^L \leq 0$ *(i.e* $a_{ji}^{-L} \leq 0$ *and* $a_{ji}^{+L} \leq 0$ *) for any* $\alpha \in (0,1]$ *if* $\tilde{x}^L \in \tilde{M}^L$, *then* $x_\alpha^{-L} \in M_\alpha^{+L}$ *and* $x_\alpha^{+L} \in M_\alpha^{-L}$. *Similarly, when* $\tilde{A}^U \leq 0$ *and* $\tilde{x}^U \in \tilde{M}^U$ *then* $x_\alpha^{-U} \in M_\alpha^{+U}$ *and* $x_\alpha^{+U} \in M_\alpha^{-U}$.

Lemma 4 *Suppose that* $A^{-L} \leq 0$ *and* $A^L \geq 0$ *(i.e* $a_{ji}^{-L} \leq 0$ *and* $a_{ji}^{+L} \geq 0$ *) for any* $\alpha in(0,1]$ *if* $\tilde{x} \in \tilde{M}^{(-L,+L)}$, *then* $x_\alpha^{+L} \in M_\alpha^{(-L,+L)}$. *Similarly, when* $A^{-U} \leq 0$ *and* $A^{+U} \leq 0$; $\tilde{x} \in \tilde{M}^{(-U,+U)}$, *then* $x_\alpha^{+U} \in M_\alpha^{(-U,+U)}$

5 Basic results

In the following we shall prove some theorems which point out that a rough fuzzy optimal solution of models $\tilde{P}^U_U$ and $\tilde{P}^L_L$ may by resolved into a class of optimal solution of modules (model 3-8)

Theorem 1 *Suppose that $\tilde{A}^U \geq 0$ and $\tilde{C}^U \geq 0$ if $\tilde{x}^{*U}$ is upper approximation (rough) fuzzy optimal solution of models $\tilde{P}^U_U$, then for any $\alpha \in (0,1]$, we have*

1.

$$\eta^{-U}_\alpha = \min_{x^U_\alpha \in M^{+U}_\alpha} C^{-U}_\alpha X^U_\alpha, \qquad \eta^{+U}_\alpha = \min_{x^U_\alpha \in M^{-U}_\alpha} C^{+U}_\alpha X^U_\alpha$$

2.

$$\tilde{\eta}^U = \min_{\tilde{x}_\alpha \in \tilde{M}^U} \tilde{C}^U \tilde{X}^U, \qquad \tilde{\eta}^U_\alpha = (\eta^{-U}_\alpha, \eta^{+U}_\alpha) \tag{11}$$

3. *$(x^*)^{+U}$, $(x^*)^{-U}$ are upper approximation (rough) optimal solution of models (5) (problems P^{-U}_{+U}, P^{-L}_{+L}) and (6)(problems P^{+U}_{-U}, P^{+L}_{-L}), respectively.*

Proof Suppose that $\tilde{x}^{*U}$ is upper approximation fuzzy solution of model $P^{(U)}$, that is,

$$\tilde{x}^{*U} \in \tilde{M}^U \quad and \quad (\tilde{C}^U)^T\, \tilde{x}^{*U} = Min\ (\tilde{C}^U)^T\, x^U \tag{12}$$

Since $\tilde{A}^{*U} \geq 0$, by lemma(1), we have $(x^*)^{-U}_\alpha \in M^{-U}_\alpha$, $(x^*)^{+U}_\alpha \in M^{+U}_\alpha$,

$$M^{-U}_\alpha = \{x^{-U}_\alpha : \tilde{x} \in \tilde{M}\} \quad and \quad M^{+U}_\alpha = \{x^{+U}_\alpha : \tilde{x} \in \tilde{M}\}$$

from eqs (1) (3), (6) and since $\tilde{C}^{*U} \geq 0$, then the the first side of eqn12

$$\begin{aligned}
[(C^{*-U})_\alpha (x^*)^{*+U}_\alpha, (C^{*+U}_\alpha)(x^*)^{-U}_\alpha] &= [\sum_{j=1}^{n} (c^{*-U}_j)_\alpha (x^*_j)^{+U}_\alpha, \sum_{j=1}^{n} (c^{*+U}_j)_\alpha)(x^*_j)^{*-U}_\alpha] \\
&= \sum_{j=1}^{n} [(c^{*-U}_j)_\alpha (x^*_j)^{+U}_\alpha,\ (c^{*+U}_j)_\alpha (x^*_j)^{-U}_\alpha] \\
&= \sum_{j=1}^{n} [(c^{*-U}_j)_\alpha,\ (c^{*+U}_j)_\alpha)][(x^*_j)^{-U}_\alpha,\ (x^*_j)^{+U}_\alpha] \\
&= \sum_{j=1}^{n} (\tilde{c}^{*U}_j)_\alpha (\tilde{x}^*_j)^U_\alpha \\
&= [\tilde{C}^{*U}\, \tilde{X}^{*U}]_\alpha
\end{aligned} \tag{13}$$

From eqs (1),(3),(4),(5) and (6), then the second side of eqn(12)

$$\begin{aligned}\left(\min_{\tilde{x}^U\in\tilde{M}^U}(\tilde{C}^U)^T\,\tilde{x}^U\right)_\alpha &= \min_{\tilde{x}^U_\alpha\in\tilde{M}^U}\left((\tilde{C}^U)^T\,\tilde{x}^U\right)_\alpha\\ &= \min_{\tilde{x}^U_\alpha\in\tilde{M}^U}\left(\sum_{j=1}^{n}(\tilde{c}^U_j)_\alpha(\tilde{x}_j)^U_\alpha\right)\\ &= \min_{\tilde{x}^U_\alpha\in\tilde{M}^U}\left(\sum_{j=1}^{n}[(c^{-U}_j)_\alpha,\ (c^{+U}_j)_\alpha)][(x_j)^{-U}_\alpha,\ (x_j)^{+U}_\alpha]\right)\\ &= \min_{\tilde{x}^U_\alpha\in\tilde{M}^U}\left(\sum_{j=1}^{n}[(c^{-U}_j)_\alpha(x_j)^{+U}_\alpha,\ (c^{+U}_j)_\alpha)\ (x_j)^{-U}_\alpha]\right)\\ &= \min_{\tilde{x}^U_\alpha\in\tilde{M}^U}\ [(c^{-U})_\alpha(x)^{+U}_\alpha,\ (c^{+U})_\alpha)\ (x)^{-U}_\alpha]\end{aligned}$$

$$= [\min_{\tilde{X}^U_\alpha\in\tilde{M}^U}\ (C^{-U})_\alpha(X)^{-U}_\alpha,\ \min_{\tilde{X}^U_\alpha\in\tilde{M}^U}\ (C^{+U})_\alpha)\ (X)^{+U}_\alpha] \tag{14}$$

From eqs (13) and (14) there followes (11), we also obtain, for any $\alpha\in(0,1]$

$$(C^{+U})_\alpha(X)^{*-U}_\alpha = \min_{X_\alpha\in M^{-U}}\ (C^{+U})_\alpha X_\alpha \tag{15}$$

$$(C^{-U})_\alpha(X)^{*+U}_\alpha = \min_{X_\alpha\in M^{-U}}\ (C^{-U})_\alpha X_\alpha \tag{16}$$

eqs (15) and (16) show that the part of the theorem is correct, too the proof is compleat.

Theorem 2 *Suppose that $\tilde{A}^U\geq 0$ and $\tilde{C}^U\leq 0$ if $\tilde{M}^{-U}_\alpha\subset\{x^{-U}_\alpha:\tilde{x}^U\in\tilde{M}^U\}$ and $\tilde{M}^{+U}_\alpha\subset\{x^{+U}_\alpha:\tilde{x}^U\in\tilde{M}^U\}$ $\tilde{x}^{*U}$ is upper approximation (rough) fuzzy optimal solution of models $\tilde{P}^U_U$, then for any $\alpha\in(0,1]$, we have*

1.

$$\eta^{-U}_\alpha = \min_{x_\alpha\in\ M^{+U}_\alpha}\ C^{-U}_\alpha\ X_\alpha \qquad \eta^{+U}_\alpha = \min_{x_\alpha\in\ M^{-U}_\alpha}\ C^{+U}_\alpha\ X_\alpha$$

2.

$$\tilde{\eta}^U = \min_{\tilde{x}^U_\alpha\in\ \tilde{M}^U}\ \tilde{C}^U\ \tilde{X}^U \qquad \tilde{\eta}^U_\alpha = (\eta^{-U}_\alpha,\eta^{+U}_\alpha) \tag{17}$$

3. x^{*+U}, x^{*-U} are upper approximation (rough) optimal solution of models (5) (problems $P^{-U}_{+U}(\alpha)$ $P^{-L}_{+L}(\alpha)$) and (6) (problems $P^{+U}_{-U}(\alpha)$, $P^{+L}_{-L}(\alpha)$), respectively;

Proof Suppose that $\tilde{x}^{*U}$ is upper approximation fuzzy optimal solution of model $\tilde{P}_U^U$ (problem 9), that is

$$\tilde{x}^{*U} \in \tilde{M}^U \quad and \quad (\tilde{C}^U)^T\, x^{*U} = \min(\tilde{C}^U)^T\, x^U \tag{18}$$

Since $\tilde{A}^U \leq 0$ by lemma 1, we have $(x^*)_\alpha^{-U} \in M_\alpha^{-U}$, $(x^*)_\alpha^{+U} \in M_\alpha^{+U}$, $M_\alpha^{-U} = \{x_\alpha^{-U} : \tilde{x}^U \in \tilde{M}^U\}$ and $M_\alpha^{+U} = \{x_\alpha^{+U} : \tilde{x}^U \in \tilde{M}^U\}$
From eqs (1) (3), (6) and since $\tilde{C}^U \geq 0$, then the the first side of eqn18

$$\begin{aligned}
[(C^{*-U})_\alpha (x^*)_\alpha^{+U}, (C_\alpha^{*+U})(x^*)_\alpha^{-U}] &= [\sum_{j=1}^{n} (c_j^{*-U})_\alpha (x_j^*)_\alpha^{+U}, \sum_{j=1}^{n} (c_j^{*+U})_\alpha)(x_j^*)_\alpha^{-U}] \\
&= \sum_{j=1}^{n} [(c_j^{*-U})_\alpha (x_j^*)_\alpha^{+U},\ (c_j^{*+U})_\alpha)(x_j^*)_\alpha^{-U}] \\
&= \sum_{j=1}^{n} [(c_j^{*-U})_\alpha,\ (c_j^{*+U})_\alpha)][(x_j^*)_\alpha^{-U},\ (x_j^*)_\alpha^{+U}] \\
&= \sum_{j=1}^{n} (\tilde{c}_j^{*U})_\alpha (x_j^*)_\alpha^U
\end{aligned}$$

$$= [\tilde{C}^{*U}\ \tilde{X}^{*U}]_\alpha \tag{19}$$

From eqs (1),(3),(4),(5) and (6), then the second side of eqn(18)

$$\begin{aligned}
\left(\min_{\tilde{x}^U \in \tilde{M}^U} (\tilde{C}^U)^T\, x^U\right)_\alpha &= \min_{\tilde{x}_\alpha^U \in \tilde{M}^U} \left((\tilde{C}^U)^T\, x^U\right)_\alpha \\
&= min_{\tilde{x}_\alpha^U \in \tilde{M}^U} \left(\sum_{j=1}^{n} (\tilde{c}_j^U)_\alpha (x_j^*)_\alpha^U\right) \\
&= \min_{\tilde{x}_\alpha^U \in \tilde{M}^U} \left(\sum_{j=1}^{n} [(c_j^{-U})_\alpha,\ (c_j^{+U})_\alpha)][(x_j)_\alpha^{-U},\ (x_j)_\alpha^{+U}]\right) \\
&= \min_{\tilde{x}_\alpha^U \in \tilde{M}^U} \left(\sum_{j=1}^{n} [(c_j^{-U})_\alpha (x_j)_\alpha^{+U},\ (c_j^{+U})_\alpha)\ (x_j)_\alpha^{-U}]\right) \\
&= \min_{\tilde{x}_\alpha^U \in \tilde{M}^U} \left(\ [(C^{-U})_\alpha (x)_\alpha^{+U},\ (C^{+U})_\alpha)\ (x)_\alpha^{-U}]\right)
\end{aligned}$$

$$= [\min_{\tilde{x}_\alpha^U \in \tilde{M}^U} (C^{-U})_\alpha (x)_\alpha^{+U}, \min_{\tilde{x}_\alpha^U \in \tilde{M}^U} (C^{+U})_\alpha)\ (x)_\alpha^{-U}] \tag{20}$$

From eqs (19) and (20) there followes (17), also obtain, for any $\alpha \in (0, 1]$

$$(C^{+U})_\alpha (X^*)_\alpha^{-U} = Min_{X_\alpha \in M^{-U}}\ (C^{+U})_\alpha X_\alpha \tag{21}$$

$$(C^{-U})_\alpha (X^*)_\alpha^{+U} = \min_{X_\alpha \in M^{+U}} (C^{-U})_\alpha (X)_\alpha^U \tag{22}$$

eqs (21) and (22) show that the part of the theorem is correct, too the proof is compleat.

Corollary 1 *Suppose that* $A_\alpha^{-U} \leq 0$, $A_\alpha^{+U} \geq$, $C_\alpha^{-U} \leq 0$, *and* $C_\alpha^{+U} \geq 0$ *let* $M_\alpha^{(-U,+U)} \subset \{x_\alpha^{+U} : \tilde{x}^U \in \tilde{M}^U\}$, *where* $\alpha \in (0,1]$ *if* $\tilde{x}^{*U}$ *is upper approximation (rough) fuzzy optimal solution of model* $\tilde{P}_U^U$, *then for any* $\alpha \in (0,1]$, x^{*U} *is upper approximation (rough) fuzzy optimal solution of model 8.*

Example 1 : *Consider the following rough fuzzy linear programming problem:*
$(IRFMOLP)_1$

$$\max \begin{pmatrix} \tilde{1}^R & \tilde{2}^R \\ \tilde{3}^R & \tilde{2}^R \end{pmatrix} \begin{pmatrix} \tilde{x}^R \\ \tilde{x}^R \end{pmatrix}$$

Subject to :

$$\begin{pmatrix} \tilde{2}^R & \tilde{6}^R \\ \tilde{8}^R & \tilde{6}^R \\ \tilde{3}^R & \tilde{1}^R \end{pmatrix} \begin{pmatrix} \tilde{x}_1^R \\ \tilde{x}_2^R \end{pmatrix} \leq \begin{pmatrix} \tilde{27}^R \\ \tilde{45}^R \\ \tilde{15}^R \end{pmatrix}, \qquad \begin{pmatrix} \tilde{x}_1^R \\ \tilde{x}_2^R \end{pmatrix} \geq 0$$

Suppose for $\alpha = 0.5$ the $\alpha-$ Cut of the rough fuzzy numbers in the above can be write as:

$$\max \begin{pmatrix} [[0,2]:[0.5,1.5]] & [[1,3]:[1.5,2.5]] \\ [[2,4]:[2.5,3.5]] & [[1,3]:[1.5,2.5]] \end{pmatrix} \begin{pmatrix} x_1^R \\ x_2^R \end{pmatrix}$$

subject to :

$$\begin{pmatrix} [[1,3]:[1.5,2.5]] & [[5,7]:[5.5,6.5]] \\ [[7,9]:[7.5,8.5]] & [[5,7]:[5.5,6.5]] \\ [[2,4]:[2.5,3.5]] & [[0,2]:[0.5,1.5]] \end{pmatrix} \begin{pmatrix} x_1^R \\ x_2^R \end{pmatrix} \leq \begin{pmatrix} [[26,28]:[26.5,27.5]] \\ [[44,46]:[44.5,45.5]] \\ [[14,16]:[14.5,15.5]] \end{pmatrix}$$

where

$$\begin{pmatrix} x_1^R \\ x_2^R \end{pmatrix} = \begin{pmatrix} \left[[x_1^{-L}, x_1^{+L}] : [x_1^{-U}, x_1^{+U}]\right] \\ \\ \left[[x_2^{-L}, x_2^{+L}] : [x_2^{-U}, x_2^{+U}]\right] \end{pmatrix}$$

Convert the $(IRFMOLP)_1$ linear programming problem to single objective by weighting method $f(x) = w_1 f_1 + w_2 f_2$, *at* $w_1 = w_2 = 0.5$, follows:
$IRFLP_1(w)$

$$\max \begin{pmatrix} [[1,3]:[1.5,2.5]] & [[1,3]:[1.5,2.5]] \end{pmatrix} \begin{pmatrix} x_1^R \\ x_2^R \end{pmatrix}$$

Subject to :

$$\begin{pmatrix} [[1,3]:[1.5,2.5]] & [[5,7]:[5.5,6.5]] \\ [[7,9]:[7.5,8.5]] & [[5,7]:[5.5,6.5]] \\ [[2,4]:[2.5,3.5]] & [[0,2]:[0.5,1.5]] \end{pmatrix} \begin{pmatrix} x_1^R \\ x_2^R \end{pmatrix} \leq \begin{pmatrix} [[26,28]:[26.5,27.5]] \\ [[44,46]:[44.5,45.5]] \\ [[14,16]:[14.5,15.5]] \end{pmatrix}$$

Now, we divided a problem above to a problems upper approximation problem $P_U^U(\alpha)$ and lower approximation problem $P_L^L(\alpha)$ follows

$P_L^L(\alpha)$:

$$Max \begin{pmatrix} [1.5, 2.5] & [1.5, 2.5] \end{pmatrix} \begin{pmatrix} \left[x_{1\alpha}^{-L}, x_{1\alpha}^{+L}\right] \\ \\ \left[x_{2\alpha}^{-L}, x_{2\alpha}^{+L}\right] \end{pmatrix}$$

Subject to :

$$\begin{pmatrix} [1.5, 2.5] & [5.5, 6.5] \\ [7.5, 8.5] & [5.5, 6.5] \\ [2.5, 3.5] & [0.5, 1.5] \end{pmatrix} \begin{pmatrix} \left[x_{1\alpha}^{-L}, x_{1\alpha}^{+L}\right] \\ \\ \left[x_{2\alpha}^{-L}, x_{2\alpha}^{+L}\right] \end{pmatrix} \leq \begin{pmatrix} [26.5, 27.5] \\ [44.5, 45.5] \\ [14.5, 15.5] \end{pmatrix}$$

$P_U^U(\alpha)$:

$$\max \begin{pmatrix} [1, 3] & [1, 3] \end{pmatrix} \begin{pmatrix} \left[x_{1\alpha}^{-U}, x_{1\alpha}^{+U}\right] \\ \\ \left[x_{2\alpha}^{-U}, x_{2\alpha}^{+U}\right] \end{pmatrix}$$

Subject to :

$$\begin{pmatrix} [1, 3] & [5, 7] \\ [7, 9] & [5, 7] \\ [2, 4] & [0, 2] \end{pmatrix} \begin{pmatrix} \left[x_{1\alpha}^{-U}, x_{1\alpha}^{+U}\right] \\ \\ \left[x_{2\alpha}^{-U}, x_{2\alpha}^{+U}\right] \end{pmatrix} \leq \begin{pmatrix} [26, 28] \\ [44, 46] \\ [14, 16] \end{pmatrix}$$

Since $C^U \geq 0$, $A^U \geq 0$ and $\alpha = 0.5$ of $P_U^U(\alpha)$, then the $P_U^U(\alpha)$ problem corresponds to the following model in the model 6 P_{-U}^{+U} as:

$$P_{-U}^{+U} \qquad \max \begin{pmatrix} 3 & 3 \end{pmatrix} \begin{pmatrix} x_{1\alpha}^{+U} \\ \\ x_{2\alpha}^{+U} \end{pmatrix}$$

Subject to :

$$\begin{pmatrix} 1 & 5 \\ 7 & 5 \\ 2 & 0 \end{pmatrix} \begin{pmatrix} x_{1\alpha}^{+U} \\ \\ x_{2\alpha}^{+U} \end{pmatrix} \leq \begin{pmatrix} 26 \\ 44 \\ 14 \end{pmatrix}.$$

Follows the upper approximation (rough) solution of problem P_{-U}^{+U}, $x_{1\alpha}^{+U} = 3$, $x_{2\alpha}^{+U} = 4.6$. Since $C^L \geq 0$, $A^L \geq 0$ and $\alpha = 0.5$ of $P_{-L}^{+L}(\alpha)$, then the $P_L^L(\alpha)$ problem with add constraints $x_{1\alpha}^{+L} \leq 3$ and $x_{2\alpha}^{+L} \leq 4.6$ corresponds to the following model in the (model 6) P_{-L}^{+L} as:

$$P_{-L}^{+L} \qquad \max \begin{pmatrix} 2.5 & 2.5 \end{pmatrix} \begin{pmatrix} x_{1\alpha}^{+L} \\ \\ x_{2\alpha}^{+L} \end{pmatrix}$$

Subject to :

$$\begin{pmatrix} 1.5 & 5.5 \\ 7.5 & 5.5 \\ 2.5 & 0.5 \\ 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x_{1\alpha}^{+L} \\ \\ x_{2\alpha}^{+L} \end{pmatrix} \leq \begin{pmatrix} 26.5 \\ 44.5 \\ 14.5 \\ 3 \\ 4.6 \end{pmatrix}$$

follows the lower approximation (rough) solution $x_{1\alpha}^{+L} = 3, \quad x_{2\alpha}^{+L} = 4$.
Now, from the $P_L^L(\alpha)$ problem, we have $C^L \geq 0$ and $A^L \geq 0$, then $P_L^L(\alpha)$ with add $x_{1\alpha}^{-L} \leq 3$ and $x_{2\alpha}^{-L} \leq 4$ corresponds to the following model in the model 5 P_{+L}^{-L} as:

$$P_{-L}^{+L} \qquad \max \begin{pmatrix} 1.5 & 1.5 \end{pmatrix} \begin{pmatrix} x_{1\alpha}^{-L} \\ \\ x_{2\alpha}^{-L} \end{pmatrix}$$

Subject to :

$$\begin{pmatrix} 2.5 & 6.5 \\ 8.5 & 6.5 \\ 3.5 & 1.5 \\ 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x_{1\alpha}^{-L} \\ \\ x_{2\alpha}^{-L} \end{pmatrix} \leq \begin{pmatrix} 27.5 \\ 45.5 \\ 15.5 \\ 3 \\ 4 \end{pmatrix}$$

follows the lower approximation (rough) solution $x_{1\alpha}^{-L} = 3, \quad x_{2\alpha}^{-L} = 3.076$
Now, from the $(P_U^U(\alpha))$ problem, we have $C \geq 0$ and $A \geq 0$, then a problem $P_U^U(\alpha)$ with add $x_{1\alpha}^{-U} \leq 3$ and $x_{2\alpha}^{-U} \leq 3.076$ corresponds to the following in model 5 $P_{+U}^{-U}W$ as:

$$P_{+U}^{-U} \qquad \max \begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} x_{1\alpha}^{-U} \\ \\ x_{2\alpha}^{-U} \end{pmatrix}$$

Subject to :

$$\begin{pmatrix} 3 & 7 \\ 9 & 7 \\ 4 & 2 \\ 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x_{1\alpha}^{-U} \\ \\ x_{2\alpha}^{-U} \end{pmatrix} \leq \begin{pmatrix} 28 \\ 46 \\ 16 \\ 3 \\ 3.076 \end{pmatrix}$$

follows the upper approximation (rough) solution $x_{1\alpha}^{-U} = 2.54, \quad x_{2\alpha}^{-U} = 2.90$. then for $w_1 = w_2 = 0.5$ and for $\alpha = 0.5$ the rough fuzzy optimal solution of $RFLP_1(w)$ is

$$x^R = (x_1^R, x_2^R) = [[2.5, 3] : [3, 3]], [[2.9, 4.6] : [3.076, 4]]$$

with rough fuzzy objective value is

$$f^R = [[5.44, 22.8] : [9, 11, 17.5]]$$

Example 2

$$\min \begin{pmatrix} \tilde{4}^R & \tilde{2}^R \\ \tilde{2}^R & \tilde{3}^R \end{pmatrix} \begin{pmatrix} \tilde{x}^R \\ \\ \tilde{x}^R \end{pmatrix}$$

Subject to :

$$\begin{pmatrix} -\tilde{2}^R & -\tilde{3}^R \\ -\tilde{3}^R & -\tilde{2}^R \end{pmatrix} \begin{pmatrix} \tilde{x}_1^R \\ \\ \tilde{x}_2^R \end{pmatrix} \leq \begin{pmatrix} -\tilde{4}^R \\ -\tilde{3}^R \end{pmatrix}$$

Suppose for $\alpha = 0.5$ the $\alpha-$ Cut of the rough fuzzy linear multiobjective problem are as:
$IRFMOLP_2$

$$\min \begin{pmatrix} [[3,5]:[3.5,4.5]] & [[1,3]:[1.5,2.5]] \\ [[1,3]:[1.5,2.5]] & [[2,4]:[2.5,3.5]] \end{pmatrix} \begin{pmatrix} x_1^R \\ \\ x_2^R \end{pmatrix}$$

subject to :

$$\begin{pmatrix} [-[1,3]:-[1.5,2.5]] & [-[3,4]:-[2.5,3.5]] \\ [-[2,4]:-[2.5,3.5]] & [-[1,3]:-[1.5,2.5]] \end{pmatrix} \begin{pmatrix} x_1^R \\ \\ x_2^R \end{pmatrix} \leq \begin{pmatrix} [-[3,5]:-[3.5,4.5]] \\ [-[2,4]:-[2.5,3.5]] \end{pmatrix}$$

Subject to

$$\begin{pmatrix} x_1^R \\ \\ x_2^R \end{pmatrix} = \begin{pmatrix} \left[[x_1^{-L}, x_1^{+L}] : [x_1^{-U}, x_1^{+U}]\right] \\ \\ \left[[x_2^{-L}, x_2^{+L}] : [x_2^{-U}, x_2^{+U}]\right] \end{pmatrix}.$$

Convert the $IRFMOLP_2$ to single objective by weighting method $f(x) = w_1 f_1 + w_2 f_2$, *at* $w_1 = w_2 = 0.5$, , follows:
$IRFLP_2(w)$

$$\min \begin{pmatrix} [[2,4]:[2.5,3.5]] & [[1.5,3.5]:[2,3]] \end{pmatrix} \begin{pmatrix} x_1^R \\ \\ x_2^R \end{pmatrix}$$

Subject to :

$$\begin{pmatrix} [-[1,3]:-[1.5,2.5]] & [-[3,4]:-[2.5,3.5]] \\ [-[2,4]:-[2.5,3.5]] & [-[1,3]:-[1.5,2.5]] \end{pmatrix} \begin{pmatrix} x_1^R \\ \\ x_2^R \end{pmatrix} \leq \begin{pmatrix} [-[3,5]:-[3.5,4.5]] \\ [-[2,4]:-[2.5,3.5]] \end{pmatrix}$$

Now, we divided a problem above to a problems upper approximation problem $P_U^U(\alpha)$ and lower approximation problem $P_L^L(\alpha)$ follows

$$P_L^L(\alpha) \qquad \min \begin{pmatrix} [2.5,3.5] & [2,3] \end{pmatrix} \begin{pmatrix} \left[x_{1\alpha}^{-L}, x_{1\alpha}^{+L}\right] \\ \\ \left[x_{2\alpha}^{-L}, x_{2\alpha}^{+L}\right] \end{pmatrix}$$

Subject to :

$$\begin{pmatrix} -[1.5,2.5] & -[2.5,3.5] \\ -[2.5,3.5] & -[1.5,2.5] \end{pmatrix} \begin{pmatrix} \left[x_{1\alpha}^{-L}, x_{1\alpha}^{+L}\right] \\ \\ \left[x_{2\alpha}^{-L}, x_{2\alpha}^{+L}\right] \end{pmatrix} \leq \begin{pmatrix} -[3.5,4.5] \\ -[2.5,3.5] \end{pmatrix}$$

$$P_U^U(\alpha) \qquad \min \begin{pmatrix} [2,4] & [1.5,3.5] \end{pmatrix} \begin{pmatrix} \left[x_{1\alpha}^{-U}, x_{1\alpha}^{+U}\right] \\ \\ \left[x_{2\alpha}^{-U}, x_{2\alpha}^{+U}\right] \end{pmatrix}$$

Subject to :

$$\begin{pmatrix} -[1,3] & -[2,4] \\ -[2,4] & -[1,3] \end{pmatrix} \begin{pmatrix} \left[x_{1\alpha}^{-U}, x_{1\alpha}^{+U}\right] \\ \\ \left[x_{2\alpha}^{-U}, x_{2\alpha}^{+U}\right] \end{pmatrix} \leq \begin{pmatrix} -[3,5] \\ -[2,4] \end{pmatrix}$$

Since $C^U \geq 0$, $A^U \leq 0$ and $\alpha = 0.5$ of $P_U^U(\alpha)$, then the $P_U^U(\alpha)$ problem corresponds to the following (in model 6) P_{-U}^{+U} as:

$$P_{-U}^{+U} \qquad \min \begin{pmatrix} 4 & 3.5 \end{pmatrix} \begin{pmatrix} x_{1\alpha}^{+U} \\ \\ x_{2\alpha}^{+U} \end{pmatrix}$$

subject to :

$$\begin{pmatrix} -1 & -2 \\ -2 & -1 \end{pmatrix} \begin{pmatrix} x_{1\alpha}^{+U} \\ \\ x_{2\alpha}^{+U} \end{pmatrix} \leq \begin{pmatrix} -3 \\ -2 \end{pmatrix}.$$

Follows the upper approximation (rough) solution of problem P_{-U}^{+U} $x_{1\alpha}^{+U} = 0.333$, $x_{2\alpha}^{+U} = 1.333$ since, $C^L \geq 0$ and $A^L \leq 0$ of $P_L^L(\alpha)$, then the $P_L^L(\alpha)$ problem with add constraints $x_{1\alpha}^{+L} \leq 0.333$ and $x_{2\alpha}^{+L} \leq 1.333$ corresponds to the following in model 6 P_{-L}^{+L} as:

$$P_{-L}^{+L} \qquad \min \begin{pmatrix} 3.5 & 3 \end{pmatrix} \begin{pmatrix} x_{1\alpha}^{+L} \\ \\ x_{2\alpha}^{+L} \end{pmatrix}$$

subject to :

$$\begin{pmatrix} -1.5 & -2.5 \\ -2.5 & -1.5 \\ 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x_{1\alpha}^{+L} \\ \\ x_{2\alpha}^{+L} \end{pmatrix} \leq \begin{pmatrix} -3.5 \\ -2.5 \\ 0.333 \\ 1.333 \end{pmatrix}$$

have the lower approximation (rough) solution of problem P_{-L}^{+L} $x_{1\alpha}^{+L} = 0.25$, $x_{2\alpha}^{+L} = 1.25$ Now, from the $P_L^L(\alpha)$ problem, we have $C^L \geq 0$ and $A^L \leq 0$, then $P_L^L(\alpha)$ with add

$x_{1\alpha}^{-L} \leq 0.25$ and $x_{2\alpha}^{-L} \leq 1.25$ corresponds to the following (in model 5) P_{+L}^{-L} as:

$$P_{+L}^{-L} \qquad \min \begin{pmatrix} 2.5 & 2 \end{pmatrix} \begin{pmatrix} x_{1\alpha}^{-L} \\ x_{2\alpha}^{-L} \end{pmatrix}$$

subject to :

$$\begin{pmatrix} -2.5 & -3.5 \\ -3.5 & -2.5 \\ 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x_{1\alpha}^{-L} \\ x_{2\alpha}^{-L} \end{pmatrix} \leq \begin{pmatrix} -4.5 \\ -3.5 \\ 0.25 \\ 1.25 \end{pmatrix}$$

follows the lower approximation (rough) solution of problem P_{+L}^{-L} $x_{1\alpha}^{-L} = 0.1667$, $x_{2\alpha}^{-L} = 1.1667$

Now, from the $P_U^U(\alpha)$ problem, we have $C^L \geq 0$ and $A^L \geq 0$, then a problem $P_L^L(\alpha)$ with add $x_{1\alpha}^{-U} \leq 0.1667$ and $x_{2\alpha}^{-U} \leq 1.1667$ corresponds to the following (in model 5) P_{+U}^{-U} as:

$$P_{+U}^{-U} \qquad \min \begin{pmatrix} 2 & 1.5 \end{pmatrix} \begin{pmatrix} x_{1\alpha}^{-U} \\ x_{2\alpha}^{-U} \end{pmatrix}$$

Subject to :

$$\begin{pmatrix} -3 & -4 \\ -4 & -3 \\ 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x_{1\alpha}^{-U} \\ x_{2\alpha}^{-U} \end{pmatrix} \leq \begin{pmatrix} -5 \\ -4 \\ 0.1667 \\ 1.1667 \end{pmatrix}.$$

Follows the upper approximation (rough) solution of problem P_U^{+U} $x_{1\alpha}^{-U} = 0.1429$, $x_{2\alpha}^{-U} = 1.1429$, then for $w_1 = w_2 = 0.5$ and $\alpha = 0.5$ the rough fuzzy optimal solution of $IRFLP_2(w)$ is

$$x^R = (x_1^R, x_2^R) = [[0.1429, 0.333] : [0.1667, 0.25]], [[1.1429, 1.333] : [1.1667, 1.25]]$$

with rough fuzzy objective value is

$$f^R = [[2, 6] : [2.750, 4.625]]$$

6 Conclusion

In this paper, we have obtained an the efficient solutions of multi-objective linear programming problem with inexact rough interval fuzzy coefficients in both objective and constraints. The problem IRFMOLP is transformed into the single objective linear programming problem with an inexact rough interval fuzzy coefficients by using the weighting method IRFLP(w). The new arithmetic operations based convert each an inexact rough fuzzy problem to two problems corresponding to the upper and lower approximation fuzzy set, respectively. Solve all the problems and incorporate the solution of all problems to the obtain optimal solution as follows rough interval.

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On Solving of Rough Interval Multi-objective Transportation Problems

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Abstract

A new method namely, separation method [15], we apply this concept for rough interval based on upper upper interval (UUIT), upper lower interval (ULIT),lower upper interval (LUIT) and lower lower interval (LLIT), are proposed On Solving of Rough Interval Multi-objective Transportation Problems (RITP) where transportation cost, supply and demand are rough intervals. The separation method can be served as an important tool for the decision makers when they are handling various types of logistic problems having rough interval parameters of transportation problems. The solutions of four problems (UUIT, ULIT. LUIT, LLIT) constrict to the rough solution set of our (RITP) and also the optimal transportation cost. The solution procedure is illustrated with a numerical example.

Keywords

Rough Interval Transportation problem, Multi-objective transportation problems, efficient solution, Weighting method .

1 Introduction

The theory of rough sets proposed by Pawlak [16] can be regarded as an effective mathematical vehicle for dealing with imprecise and ambiguous data analyses, which can be subsequently applied to pattern recognition, machine learning, and knowledge discovery [1, 2, 4]. The equivalence relation is a key notion in Pawlaks rough set model. All equivalence classes form a partition of a universe of discourse. Using equivalence classes, an arbitrary subset can be approximated by two subsets called the lower approximation and the upper approximation. However, the equivalence relation is a stringent condition that may limit the applications of rough sets in practical problems. Hence, various extensions of Pawlaks rough set were developed from an equivalence relation into a more general mathematical concept, e.g., binary relations [2], neighborhood systems and Boolean algebras [20], and coverings of the universe of discourse [21]. Moreover, generalizations of rough sets to the fuzzy environment have become a rapidly progressing research area receiving much attention in recent years.
Dubois and Prade [7] first proposed the notions of rough fuzzy sets and fuzzy rough sets. Subsequently, various definitions of fuzzy rough sets with reference to different fuzzy logical operators and binary relations have been presented in the literature [9]. Through constructive and axiomatic approaches, Wu and Zhang [19] generalized fuzzy rough set models by an arbitrary fuzzy relation and presented a systematic study on the four fuzzy rough approximation operators. Also they studied the transportation various efficient methods were developed for solving transportation problems with the assumption of precise source, destination parameter, and the penalty factors. In real life problems, these conditions may not be satisfied always.
To deal with inexact coefficients in transportation problems, many researchers [3-6, 11-14, 18] have proposed Rough interval programming techniques for solving them. Das et al. [6] proposed a method, called fuzzy technique to solve interval transportation problem by considering the right bound and the midpoint of the 1820 P. Pandian and G. Natarajan interval. Sengupta and Pal [18] proposed a new fuzzy orientated method to solve interval transportation problems by considering the midpoint and width of the interval

in the objective function.
In this paper, we propose a new method namely, separation method to find an efficient solution for transportation problems where transportation cost, supply and demand are rough intervals. The proposed method is based lower lower interval, upper lower interval, lower upper interval and upper upper interval [9]. The new method can be served as an important tool for the decision makers when they are handling various types of logistic problems having rough interval parameters. The solutions of four problems (UUIT, ULIT. LUIT, LLIT) constrict to the rough solution set of our (RITP) and also the optimal transportation cost. The solution procedure is illustrated with a numerical example.

2 Preliminaries

In this section, we introduce some notions related to the considered problems. the following concepts can be found in [10], that is a^R *and* b^R . are the rough interval following as : $a^R = [[a^{-U}, a^{+U}] : [a^{-L}, ^{+L}]]$ is a rough interval such that $[a^{-L}, a^{+L}] \subseteq [a^{-U}, a^{+U}]$

Definition 1
Let R^R denote a set of rough intervals. A rough interval A^R is tuple of rough intervals, and a rough interval matrix A^R_{ij} a matrix of rough interval whose element are rough intervals:
$A^R = \{a^R_i = [[a_i^{-U}, a_i^{+U}] : [a_i^{-L}, a_i^{+L}]], \quad \forall i\}, A^R \in R^{R(1\times n)}$
$A^R_{ij} = \{a^R_{ij} = [[a_{ij}^{-U}, a_{ij}^{+U}] : [a_{ij}^{-L}, a_{ij}^{+L}]], \quad \forall i, j\}, A^R \in R^{R(m\times n)}$

Definition 2
Suppose that

$$a^R = [[a^{-U}, a^{+U}] : [a^{-L}, a^{+L}]] \ , \ b^R = [[b^{-U}, b^{+U}] : [b^{-L}, b^{+L}]]$$

, we define

1. $a^R + b^R = \quad [[a^{-U} + b^{-U}, \ a^{+U} + b^{+U}] : [a^{-L} + b^{-L}, a^{+L} + b^{+L}]]$
2. $a^R - b^R = \quad [[a^{-U} - b^{+U}, \ a^{+U} - b^{-U}] : [a^{-L} - b^{+L}, a^{+L} - b^{-L}]]$

3. $a^R \times b^R = \left[[a^{-U} \times b^{-U}] : [a^{+U} \times b^{+U}] : [a^{-L} \times b^{-L}] : [a^{+L} \times b^{+L}]\right]$

Definition 3

The order relation $\leq$ for a rough interval a^R and b^R is defined by
$a^R \leq b^R$ *iff* $a^{+L} \leq b^{+L}$ *and* $a^{-U} \leq b^{-U}$

Definition 4

$\{ \ [[a_i^{-U}, a_i^{+U}] : [a_i^{-L}, a_i^{+L}]], \quad i \in I \ \}$ *I is the index set, then*

1.

$$\wedge_{i\in I} \ [[a_i^{-U}, a_i^{+U}] : [a_i^{-L}, a_i^{+L}]] = [[\wedge_{i\in I} \ a_i^{-U}, \wedge_{i\in \ I} \ a_i^{+U}] : [\wedge_{i\in I} \ a_i^{-L}, \wedge_{i\in I} \ a_i^{+L}]]$$

if $\wedge_{i\in \ I} \, a_{i\in I} \geq \ 0$

2.

$$\vee_{i\in I} \ [[a_i^{-U}, a_i^{+U}] : [a_i^{-L}, a_i^{+L}]] = [[\vee_{i\in I} \ a_i^{-U}, \vee_{i\in \ I} \ a_i^{+U}] : [\vee_{i\in I} \ a_i^{-L}, \vee_{i\in I} \ a_i^{+L}]]$$

if $\vee_{i\in \ I} \, a_{i\in I} \geq \ 0$

3 Problem formulation

Consider the following rough interval multi objective transportation problem $(RIMTP)$:

$$Min(Z^R)^r = \sum_{i=1}^{m} \sum_{j=1}^{n} [[(c_{ij}^{-U})^r, (c_{ij}^{+U})^r] : [(c_{ij}^{-L})^r, (c_{ij}^{+L})^r]] \ x_{ij}^R \quad r = 1, 2, ..., k$$

such that

$$\sum_{j=1}^{m} x_{ij}^R = [[a_i^{-U}, a_i^{+U}] : [a_i^{-L}, a_i^{+L}]], \quad i = 1, 2, ..., m \tag{1}$$

$$\sum_{i=1}^{n} x_{ij}^R = [[b_j^{-U}, b_j^{+U}] : [b_j^{-L}, b_j^{+L}]], j = 1, 2, ..., n \tag{2}$$

$$\sum_{i=1}^{n} [[a_i^{-U}, a_i^{+U}] : [a_i^{-L}, a_i^{+L}]] = \sum_{j=1}^{m} [[b_j^{-U}, b_j^{+U}] : [b_j^{-L}, b_j^{+L}]], \tag{3}$$

$$[[x_{ij}^{-U}, x_{ij}^{+U}] : [x_{ij}^{-L}, x_{ij}^{+L}]] \geq 0 \tag{4}$$

$i = 1, 2, ..., m$ *and* $j = 1, 2, ..., n$, *are integers.* where $(Z^R)^r = [[(Z_{ij}^{-U})^r, (Z_{ij}^{+U})^r] : [(Z_{ij}^{-L})^r, (Z_{ij}^{+L})^r]]$, $[[c_{ij}^{-U}, c_{ij}^{+U}] : [c_{ij}^{-L}, c_{ij}^{+}]] \geq 0$, $[[a_{ij}^{-U}, a_{ij}^{+U}] : [a_{ij}^{-L}, a_{ij}^{+}]] \geq 0$ *and* $[[b_{ij}^{-U}, b_{ij}^{+U}] : [b_{ij}^{-L}, b_{ij}^{+}]] \geq 0$ are positive for all i and j .

Definition 5 *(Rough interval efficient solution)*
The rough interval vector x^{*R} which satisfies the condition in $(RIMTP)$ 1 ,2, 3 and 4 is called a rough interval efficient solution of $(RIMTP)$ if and only if there does not exist another x^R such that

$$(Z^R)^r(x) \leq (Z^R)^r(x^*)$$

for all r and $(Z^R)^r(x) \neq (Z^R)^r(x^*)$ for at least one $r = 1, 2, ..., k$.

Problem $RIMTP$ will be treated using the weighting method (i.e by considering the following problem with scalar objective $RITP(w)$, as in the following:)

$$RITP(w) \qquad MinZ_W^R = \sum_{r=1}^{k}\sum_{i=1}^{m}\sum_{j=1}^{n} w_r[[(c_{ij}^{-U})^r, (c_{ij}^{+U})^r] : [(c_{ij}^{-L})^r, (c_{ij}^{+L})^r]]\ x_{ij}^R$$

such that

$$\sum_{j=1}^{m} x_{ij}^R = [[a_i^{-U}, a_i^{+U}] : [a_i^{-L}, a_i^{+L}]], \quad i = 1, 2, ..., m \tag{5}$$

$$\sum_{i=1}^{n} x_{ij}^R = [[b_j^{-U}, b_j^{+U}] : [b_j^{-L}, b_j^{+L}]], \quad j = 1, 2, ..., n \tag{6}$$

$$\sum_{i=1}^{n}[[a_i^{-U}, a_i^{+U}] : [a_i^{-L}, a_i^{+L}]] = \sum_{j=1}^{m}[[b_j^{-U}, b_j^{+U}] : [b_j^{-L}, b_j^{+L}]] \tag{7}$$

$$[[x_{ij}^{-U}, x_{ij}^{+U}] : [x_{ij}^{-L}, x_{ij}^{+L}]] \geq 0 \tag{8}$$

$i = 1, 2, ..., m$ *and* $j = 1, 2, ..., n$ and are integer $\Omega = \{w_r > 0,\ r = 1, ..., k,\ \sum_{r=1}^{k} w_r = 1\}$
where $[[c_{ij}^{-U}, c_{ij}^{+U}] : [c_{ij}^{-L}, c_{ij}^{+L}]] \geq 0$, $[[a_{ij}^{-U}, a_{ij}^{+U}] : [a_{ij}^{-L}, a_{ij}^{+}]] \geq 0$ *and* $[[b_{ij}^{-U}, b_{ij}^{+U}] : [b_{ij}^{-L}, b_{ij}^{+}]] \geq 0$ are positive for all i and j.

Definition 6

For given $w^* \in \Omega$, *the set* $[[x_{ij}^{-U}, x_{ij}^{+U}] : [x_{ij}^{-L}, x_{ij}^{+L}]]$, *for all* $i = 1, 2, ..., m$ *and* $j = 1, 2, ..., n$ *is said to be a rough feasible solution of* $(RITP(w^*))$ *if they satisfy the equations 5, 6, 7 and 8.*

Definition 7

For given $w^* \in \Omega$ *a feasible solution* x_{ij}^{*R}, *for all* $i = 1, 2, ..., m$ *and* $j = 1, 2, ..., n$ *of the problem* $(RITP(w^*))$ *is said to be an rough optimal solution of* $(RITP(w^*))$ *if*

$$\sum_{r=1}^{k}\sum_{i=1}^{m}\sum_{j=1}^{n} w_r^*[[c_{ij}^{-U}, c_{ij}^{U}] : [c_{ij}^{-L}, c_{ij}^{+L}]]\ (x_{ij}^*)^R \leq \sum_{r=1}^{k}\sum_{i=1}^{m}\sum_{j=1}^{n} w_r^*[[c_{ij}^{-U}, c_{ij}^{U}] : [c_{ij}^{-L}, c_{ij}^{+L}]]\ x_{ij}^R$$

for i = 1,2,...,m and j = 1,2,...,n and for all feasible $[[x_{ij}^{-U}, x_{ij}^{+U}] : [x_{ij}^{-L} : x_{ij}^{+L}]]$ for i = 1,2,...,m and j = 1,2,...,n .

Now, we prove the following theorem illustrates the relation between optimal solutions of a fully rough interval transportation problem and upper upper interval, upper lower interval, lower lower interval and lower upper interval of induced transportation problems and also, is used in the proposed method.

Theorem 1 *The rough optimal solution of the problem* $(RITP(w^*))$ *can be characterized by the solution of the following models:*

- *Model 1*

$$(UUITP(w^*)) : \quad MinZ_{ij}^{+U} = \sum_{r=1}^{k}\sum_{i=1}^{m}\sum_{j=1}^{n} w_r^* c_{ij}^{+U} x_{ij}^{+U}$$

subject to

$$\sum_{j=1}^{m} x_{ij}^{+U} = a_i^{+U}, \quad i = 1, 2, ..., m \tag{9}$$

$$\sum_{i=1}^{n} x_{ij}^{+U} = b_j^{+U}, j = 1, 2, , ..., n \tag{10}$$

$$\sum_{i=1}^{n} a_i^{+U} = \sum_{j=1}^{m} b_j^{+U} \tag{11}$$

$$x_{ij}^{+U} \geq 0 \tag{12}$$

$i = 1,2,...,m$ *and* $j = 1,2,...,n$ *and are integers and the set* $(x^*_{ij})^{+U}$ *for all* i *and* j *is an optimal solution of the upper upper bounded transportation problem* $(UUITP(w^*))$ *of* $(RITP(w^*))$ *where*

- *Model 2*

$$(ULITP(w^*)): \quad MinZ_{ij}^{+L} = \sum_{r=1}^{k}\sum_{i=1}^{m}\sum_{j=1}^{n} w^*_{r=1} c_{ij}^{+L} x_{ij}^{+L}$$

subject to

$$\sum_{j=1}^{m} x_{ij}^{+L} = a_i^{+L}, \quad i = 1,2,...,m \tag{13}$$

$$\sum_{i=1}^{n} x_{ij}^{+L} = b_j^{+L}, j = 1,2,,...,n \tag{14}$$

$$\sum_{i=1}^{n} a_i^{+L} = \sum_{j=1}^{m} b_j^{+L} \tag{15}$$

$$x_{ij}^{+L} \geq 0 \tag{16}$$

$i = 1,2,...,m$ *and* $j = 1,2,...,n$ *and are integers and the set* $(x^*_{ij})^{+L}$ *for all* i *and* j *is an optimal solution of the upper lower bounded transportation problem* $(ULITP)$ *of* $(RITP)$ *provided* $x_{ij}^{+L} \leq x_{ij}^{+U}$ *for all i,j where*

- *Model 3*

$$(LLITP(w^*)): \quad MinZ_{ij}^{-L} = \sum_{r=1}^{k}\sum_{i=1}^{m}\sum_{j=1}^{n} w^*_{r=1} c_{ij}^{-L} x_{ij}^{-L}$$

subject to

$$\sum_{j=1}^{m} x_{ij}^{-L} = a_i^{-L}, \quad i = 1,2,...,m \tag{17}$$

$$\sum_{i=1}^{n} x_{ij}^{-L} = b_j^{-L}, j = 1,2,...,n \tag{18}$$

$$\sum_{i=1}^{n} a_i^{-L} = \sum_{j=1}^{m} b_j^{-L} \tag{19}$$

$$x_{ij}^{-L} \geq 0 \tag{20}$$

$i = 1, 2, ..., m$ *and* $j = 1, 2, ..., n$ *and are integers and the set* $(x_{ij}^{*})^{-L}$ *for all* i *and* j *is an optimal solution of the lower lower bounded transportation problem* $(LLITP)$ *of* $(RITP)$ *provided* $x_{ij}^{-L} \leq x_{ij}^{+L}$ *for all* i, j *where*

- *Model 4*

$$(LUITP(w^{*})): \qquad MinZ_{ij}^{-U} = \sum_{r=1}^{k} \sum_{i=1}^{m} \sum_{j=1}^{n} w_{r=1}^{*} c_{ij}^{-U} x_{ij}^{-U}$$

subject to

$$\sum_{j=1}^{m} x_{ij}^{-U} = a_{i}^{-U}, \quad i = 1, 2, 3 \tag{21}$$

$$\sum_{i=1}^{n} x_{ij}^{-U} = b_{j}^{-U}, j = 1, 2, 3, 4 \tag{22}$$

$$\sum_{i=1}^{n} a_{i}^{-U} = \sum_{j=1}^{m} b_{j}^{-U} \tag{23}$$

$$x_{ij}^{-U} \geq 0 \tag{24}$$

$i = 1, 2, ..., m$ *and* $j = 1, 2, ..., n$ *and are integers*

and the set x_{ij}^{-U} *for all* i *and* j *is an optimal solution of the lower upper bounded transportation problem* $(LUITP(w^{*}))$ *of* $(RITP(w^{*}))$ *.*

proof:

Let for $w^{*} \in \Omega$ $\{ [[x_{ij}^{-U}, x_{ij}^{+U}] : [x_{ij}^{-L}, x_{ij}^{+L}]]$, for all i and j $\}$ be a rough feasible solution of the problem $(RITP(w^{*}))$.

Therefore, $\{ x_{ij}^{+U}$, for all i and j $\}$, $\{ x_{ij}^{+L}$, for all i and j $\}$, $\{ x_{ij}^{-L}$, for all i and j $\}$ and $\{ x_{ij}^{-U}$, for all i and j $\}$ are feasible solutions of the problems $(UUITP(w^{*}))$, $(ULITP(w^{*}))$, $(LLITP(w^{*}))$ and $(LUITP(w^{*}))$.

Now, since $\{ (x_{ij}^{*})^{+U}$, for all i and j $\}$, $\{ (x_{ij}^{*})^{+L}$, for all i and j $\}$, $\{ (x_{ij}^{*})^{-L}$, for all i and j $\}$ and $\{ (x_{ij}^{*})^{-U}$, for all i and j $\}$ are optimal solutions of $(UUITP(w^{*}))$, $(ULITP(w^{*}))$, $(LLITP(w^{*}))$ and $(LUITP(w^{*}))$, we have

$$\sum_{r=1}^{k} \sum_{i=1}^{m} \sum_{j=1}^{n} w_{r}^{*} c_{ij}^{+U} (x_{ij}^{*})^{+U} \leq \sum_{r=1}^{k} \sum_{i=1}^{m} \sum_{j=1}^{n} w_{r}^{*} c_{ij}^{+U} x_{ij}^{+U} \; i = 1, 2, ..., m \; and \; j = 1, 2, ..., n$$

$$\sum_{r=1}^{k}\sum_{i=1}^{m}\sum_{j=1}^{n} w_r^* c_{ij}^{+L} (x_{ij}^*)^{+L} \leq \sum_{r=1}^{k}\sum_{i=1}^{m}\sum_{j=1}^{n} w_r^* c_{ij}^{+L} x_{ij}^{+L}\; i = 1,2,...,m\; and\; j = 1,2,...,n$$

and $(x_{ij}^*)^{+L} \leq (x_{ij}^*)^{+U} i = 1,2,...,m\ and\ j = 1,2,...,n$

$$\sum_{r=1}^{k}\sum_{i=1}^{m}\sum_{j=1}^{n} w_r^* c_{ij}^{-L} (x_{ij}^*)^{-L} \leq \sum_{r=1}^{k}\sum_{i=1}^{m}\sum_{j=1}^{n} w_r^* c_{ij}^{-L} x_{ij}^{-L}\; i = 1,2,...,m\; and\; j = 1,2,...,n$$

and $(x_{ij}^*)^{-L} \leq (x_{ij}^*)^{+L} i = 1,2,...,m\ and\ j = 1,2,...,n$

$$\sum_{r=1}^{k}\sum_{i=1}^{m}\sum_{j=1}^{n} w_r^* c_{ij}^{-U} (x_{ij}^*)^{-U} \leq \sum_{r=1}^{k}\sum_{i=1}^{m}\sum_{j=1}^{n} w_r^* c_{ij}^{-U} x_{ij}^{-U}\; i = 1,2,...,m\; and\; j = 1,2,...,n$$

and $(x_{ij}^*)^{-U} \leq (x_{ij}^*)^{-L} i = 1,2,...,m\ and\ j = 1,2,...,n$
This implies that,

$$[[\sum_{r=1}^{k}\sum_{i=1}^{m}\sum_{j=1}^{n} w_r^* c_{ij}^{-U} (x_{ij}^*)^{-U}, \sum_{r=1}^{k}\sum_{i=1}^{m}\sum_{j=1}^{n} w_r^* c_{ij}^{+U} (x_{ij}^*)^{+U}] : [\sum_{r=1}^{k}\sum_{i=1}^{m}\sum_{j=1}^{n} w_r^* c_{ij}^{-L} (x_{ij}^*)^{-L},$$
$$\sum_{r=1}^{k}\sum_{i=1}^{m}\sum_{j=1}^{n} w_r^* c_{ij}^{+L} (x_{ij}^*)^{+L}]] \leq [[\sum_{r=1}^{k}\sum_{i=1}^{m}\sum_{j=1}^{n} w_r^* c_{ij}^{-U} x_{ij}^{-U}, \sum_{r=1}^{k}\sum_{i=1}^{m}\sum_{j=1}^{n} w_r^* c_{ij}^{+U} x_{ij}^{+U}] :$$
$$[\sum_{r=1}^{k}\sum_{i=1}^{m}\sum_{j=1}^{n} w_r^* c_{ij}^{-L} x_{ij}^{-L}, \sum_{r=1}^{k}\sum_{i=1}^{m}\sum_{j=1}^{n} w_r^* c_{ij}^{+L} c_{ij}^{+L}]]$$

That is,

$$\sum_{r=1}^{k}\sum_{i=1}^{m}\sum_{j=1}^{n} w_r^* [[c_{ij}^{-U}, c_{ij}^{U}] : [c_{ij}^{-L}, c_{ij}^{+L}]]\ (x_{ij}^*)^R \leq \sum_{r=1}^{k}\sum_{i=1}^{m}\sum_{j=1}^{n} w_r^* [[c_{ij}^{-U}, c_{ij}^{U}] : [c_{ij}^{-L}, c_{ij}^{+L}]]\ x_{ij}^R$$

Now, since, { $(x_{ij}^*)^{+U}$, for all i and j }, { $(x_{ij}^*)^{+L}$, for all i and j }, { $(x_{ij}^*)^{-L}$, for all i and j } and { $(x_{ij}^*)^{-U}$, for all i and j } satisfies 9 to 24 and $[x_{ij}^{-U}, x_{ij}^{+U}] \subseteq [x_{ij}^{-L}, x_{ij}^{+L}]$ for all i and j we can conclude that the set $[[(x_{ji}^*)^{-U}, (x_{ji}^*)^{+U}] : [(x_{ji}^*)^{-L}, (x_{ji}^*)^{+L}]]\ for\ all\ i\ and\ j$ is a feasible solution of $(RITP(w^*))$.
Thus, the set of rough intervals $[[(x_{ji}^*)^{-U}, (x_{ji}^*)^{+U}] : [(x_{ji}^*)^{-L}, (x_{ji}^*)^{+L}]]$ for all i and j is an rough optimal solution of the problem $(RITP(w^*))$. Hence the theorem.

4 Separation method

We, now introduce a new algorithm namely based on separation method [15] for finding an optimal solution for a rough interval transportation problem. The separation method proceeds as follows.

1. Consider the rough interval multi objective transportation problem $(RITR)$ and given its secularization weighting problem $(RIMTP)$.
2. Construct the $(UUITP(w^*))$ of the given $(RITP(w^*))$.
3. Solve the $(UUITP(w^*))$ using the simplex method. Let $(x^*_{ij})^{+U}$, for all i and j, an optimal solution of the $(UUITP(w^*)$.
4. Construct the $ULITP(w^*)$ of the given $(RITP(w^*))$.
5. Solve the $(ULITP(w^*))$ with the upper bound constraints $x^{+L}_{ij} \leq (x^*_{ij})^{+U}$ using the simplex method . Let $(x^*_{ij})^{+L}$, for all i and j be the optimal solution of $(ULITP(w^*))$.
6. Construct the $(LLITP(w^*))$ of the given $(RITP(w^*))$.
7. Solve the $LLITP(w^*)$ with the upper bound constraints $x^{-L}_{ij} \leq (x^*_{ij})^{+L}$ using the simplex method . Let $(x^*_{ij})^{-L}$, for all i and j be the optimal solution of $(LLITP(w^*))$.
8. Construct the $(LUITP(w^*))$ of the given $(RITP(w^*))$.
9. Solve the $(LUITP(w^*))$ with the upper bound constraints $x^{-U}_{ij} \leq (x^*_{ij})^{-L}$ using the simplex method. Let $(x^*_{ij})^{-U}$, for all i and j be the optimal solution of $(ULITP(w^*))$.
10. The optimal solution of the given $(RITP(w^*))$ is

$$x^{*R} = [[(x^*_{ij})^{-U}, (x^*_{ij})^{+U}] : [(x^*_{ij})^{-L}, (x^{ast}_{ij})^{+L}]]$$

and $[(x^*_{ij})^{-L}, (x^*_{ij})^{+L}] \subseteq [(x^*_{ij})^{-U}, (x^*_{ij})^{+U}]$ for all i and j .

$$f^{*R} = [[(f^*_{ij})^{-U}, (f^*_{ij})^{+U}] : [(f^*_{ij})^{-L}, (f^*_{ij})^{+L}]]$$

and $[(f^*_{ij})^{-L}, (f^*_{ij})^{+L}] \subseteq [(f^*_{ij})^{-U}, (f^*_{ij})^{+U}]$ for all i and j .

The proposed algorithm is illustrated by the following example.

Example 1

$$MinZ^R = \left(\sum_{i=1}^{3} \sum_{j=1}^{4} c_{ij}^{1R} x_{ij}^R, \quad \sum_{i=1}^{3} \sum_{j=1}^{4} c_{ij}^{2R} x_{ij}^R \right)$$

such that

$$\sum_{j=1}^{4} x_{ij}^R = a_i^R \qquad i = 1, 2, 3$$

$$\sum_{i=1}^{3} x_{ij}^R = b_j^R \quad j = 1, 2, 3, 4$$

$$\sum_{i=1}^{m} a_i^R = \sum_{j=1}^{n} b_j^R \ \ and \ \ x_{ij}^R \geq 0$$

assume that the rough interval characterized as:

1. Rough interval coefficients (the first one C^{1R})

$$C^{1R} = \begin{pmatrix} [[0,3]:[1,2]] & [[0,4]:[1,3]] & [[4,10]:[5,9]] & [[3,9]:[4,8]] \\ [[0,3]:[1,2]] & [[6,11]:[7,10]] & [[1,7]:[2,6]] & [[2,6]:[3,5]] \\ [[6,10]:[7,9]] & [[6,12]:[7,11]] & [[2,6]:[3,5]] & [[4,8]:[5,7]] \end{pmatrix}$$

(the second one C^{2R})

$$C_{2R} = \begin{pmatrix} [[2,5]:[3,4]] & [[2,6]:[3,5]] & [[6,12]:[7,11]] & [[5,11]:[6,10]] \\ [[2,5]:[3,4]] & [[8,13]:[9,12]] & [[3,9]:[4,8]] & [[4,8]:[5,7]] \\ [[8,12]:[9,11]] & [[8,14]:[9,13]] & [[4,8]:[5,7]] & [[6,10]:[7,9]] \end{pmatrix}$$

2. Supplies

$$a_1 = [[6,10]:[7,9]], \quad a_2 = [[16,22]:[17,21]], \quad a_3 = [[15,19]:[16,18]]$$

3. Demand

$$b_1 = [[9,13]:[10,12]], \quad b_2 = [[1,5]:[2,4]], \quad b_3 = [[12,16]:[13,15]],$$

$$b_4 = [[15,17]:[15,17]]$$

Then the Problem $RITP(w)$ takes the form

$$RITP(w) \qquad MinZ_W^R = w_1 \sum_{i=1}^{3}\sum_{j=1}^{4} c_{ij}^{1R} x_{ij}^R + w_2 \sum_{i=1}^{3}\sum_{j=1}^{4} c_{ij}^{2R}\ x_{ij}^R = \sum_{i=1}^{3}\sum_{j=1}^{4} c_{ij}^{*R} x_{ij}^R$$

such that

$$\sum_{j=1}^{4} x_{ij}^R = a_i^R, \qquad i = 1,2,3$$

$$\sum_{i=1}^{3} x_{ij}^R = b_j^R, \qquad i = 1,2,3,4$$

$$\sum_{i=1}^{3} \dot{a}_i^R = \sum_{j}^{4} b_j^R$$

At $(w_1^*, w_2^*) = (0.5, 0.5)$, then the coefficient matrices $C^{*R} = w_1 C^{1R} + w_2 C^{2R}$

$$C^{*R} = \begin{pmatrix} [[1,4]:[2,3]] & [[1,5]:[2,4]] & [[5,11]:[6,10]] & [[4,10]:[5,9]] \\ [[1,4]:[2,3]] & [[7,12]:[8,11]] & [[2,8]:[3,7]] & [[3,7]:[4,6]] \\ [[7,11]:[8,10]] & [[7,13]:[8,12]] & [[3,7]:[4,6]] & [[5,9]:[6,8]] \end{pmatrix}$$

$$a_1 = [[6,10]:[7,9]], \quad a_2 = [[16,22]:[17,21]], \quad a_3 = [[15,19]:[16,18]]$$

$$b_1 = [[9,13]:[10,12]], \quad b_2 = [[1,5]:[2,4]], \quad b_3 = [[12,16]:[13,15]],$$

$$b_4 = [[15,17]:[15,17]]$$

Now, we consider the following variables $[[x_{ij}^{-U}, x_{ij}^{+U}] : [x_{ij}^{-L}, x_{ij}^{+L}]]$ for all i and j corresponding to the given $RITP$.

Now, the $UURITP$ of the RITP is given below.

					Supply
	4	5	11	10	10
	4	12	8	7	22
	11	13	7	9	19
Demand	13	5	16	17	51

Now, the optimal solution to the $UURITP$ is

$$(x_{11}^*)^{+U} = 5, \quad (x_{12}^*)^{+U} = 5, \quad (x_{13}^*)^{+U} = 0, \quad (x_{14}^*)^{+U} = 0$$

$$(x_{21}^*)^{+U} = 8, \quad (x_{22}^*)^{+U} = 0, \quad (x_{23}^*)^{+U} = 0, \quad (x_{24}^*)^{+U} = 14$$

$$(x_{31}^*)^{+U} = 0, \quad (x_{32}^*)^{+U} = 0, \quad (x_{33}^*)^{+U} = 16, \quad (x_{34}^*)^{+U} = 3$$

Now, the $ULRITP$ of the $RITP$ with the bounded constraints is given below.

					Supply
	3	4	10	9	9
	3	11	7	6	21
	10	12	6	8	18
Demand	12	4	15	17	4 8

and also, $x_{ij}^{+L} \leq x_{ij}^{+U}$, $i = 1, 2, ..., m$ and $j = 1, 2, ..., n$ and are integers.

$$(x_{11}^*)^{+L} = 5, \quad (x_{12}^*)^{+L} = 4, \quad (x_{13}^*)^{+L} = 0, \quad (x_{14}^*)^{+L} = 0$$

$$(x_{21}^*)^{+L} = 7, \quad (x_{22}^*)^{+L} = 0, \quad (x_{23}^*)^{+L} = 0, \quad (x_{24}^*)^{+L} = 14$$

$$(x_{31}^*)^{+L} = 0, \quad (x_{32}^*)^{+L} = 0, \quad (x_{33}^*)^{+L} = 15, \quad (x_{34}^*)^{+L} = 3$$

Now, the $LLRITP$ of the $RITP$ with the bounded constraints is given below.

					Supply
	2	2	6	5	7
	2	8	3	4	17
	8	8	4	6	16
Demand	10	2	13	15	4 0

and also, $x_{ij}^{-L} \leq x_{ij}^{+L}$, $i = 1, 2, ..., m$ and $j = 1, 2, ..., n$ and are integers.

$$(x_{11}^*)^{-L} = 5, \quad (x_{12}^*)^{-L} = 2, \quad (x_{13}^*)^{-L} = 0, \quad (x_{14}^*)^{-L} = 0$$

$$(x_{21}^*)^{-L} = 5, \quad (x_{22}^*)^{-L} = 0, \quad (x_{23}^*)^{-L} = 0, \quad (x_{24}^*)^{-L} = 12$$

$$(x_{31}^*)^{-L} = 0, \quad (x_{32}^*)^{-L} = 0, \quad (x_{33}^*)^{-L} = 13, \quad (x_{34}^*)^{-L} = 3$$

Now, the LURITP of the RITP with the bounded constraints is given below.

					Supply
	1	1	5	4	6
	1	7	2	3	16
	7	7	3	5	15
Demand	9	1	12	15	37

and also, $x_{ij}^{-U} \leq x_{ij}^{-L}$, $i = 1, 2, ..., m$ and $j = 1, 2, ..., n$ and are integers. Now, the optimal solution to the $LURTP$ with bounded constraints is

$$(x_{11}^*)^{-U} = 5, \quad (x_{12}^*)^{-U} = 1, \quad (x_{13}^*)^{-U} = 0, \quad (x_{14}^*)^{-U} = 0$$
$$(x_{21}^*)^{-U} = 4, \quad (x_{22}^*)^{-U} = 0, \quad (x_{23}^*)^{-U} = 0, \quad (x_{24}^*)^{-U} = 12$$
$$(x_{31}^*)^{-U} = 0, \quad (x_{32}^*)^{-U} = 0, \quad (x_{33}^*)^{-U} = 12, \quad (x_{34}^*)^{-U} = 3$$

Thus, an optimal solution to the $RITP$ is

$$x_{1i}^{*R} = ([[5,5] : [5,5]], \ [[1,5] : [2,4]], \ [[0,0] : [0,0]], \ [[0,0] : [0,0]])$$
$$x_{2i}^{*R} = ([[4,8] : [5,7]], \ [[0,0] : [0,0]], \ [[0,0] : [0,0]], \ [[12,14] : [12,14]])$$
$$x_{3i}^{*R} = ([[0,0] : [0,0]], \ [[0,0] : [0,0]], [[12,16] : [13,15]], \ [[3,3] : [3,3]])$$

$i = 1, 2, 3, 4$

and also, the minimum transportation cost is .

$$(f^*)^R = [[(f^*)^{-U}, (f^*)^{+U}] : [(f^*)^{-L}, (f^*)^{+L}]] = [[97, 314] : [142, 250]]$$

5 Conclusion

The separation method provides an optimal value of the objective function for the rough interval transportation problem. This method is a systematic procedure, both easy to understand and to apply . The proposed method provides more options and can be served an important tool for the decision makers when they are handling various types of logistic problems having rough interval parameters.

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On the Complete Stability Set of the First Kind for Parametric Multi- Objective Linear Programming Problems (Parameters in the Objective Functions)

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Abstract

This paper uses parametric study for providing essential information about the problem's behavior to the decision maker. Two algorithms are presented in this work. The first algorithm is obtained to find the complete stability set of the first kind for parametric multi-objective linear programming problems. It is based on the weighting method for scalarizing the multi- objective linear programming problems and Kuhn- Tucker conditions for mathematical programming in general.

The second algorithm presents a technique to decompose of the parametric space in parametric linear programming problem according to the complete stability set of the first kind.

Two numerical examples are introduced to clarify the obtained results.

Key words: Multi- objective linear programming, efficient solution, solvability set, stability set of the first kind, and multicriteria simplex method.

1- Introduction

In early work, the notions of the solvability set and the stability of the first kind are analyzed for parametric convex programming problems [6, 8].

The relation to the importance of the results in multi- objective convex programming problems can be seen by the fundamental role which parametric techniques play in multi- objective programming.

In this paper, the basic concepts are defined and analyzed quantitatively for parametric multi- objective linear programming problems.

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Also, the connectedness nature of the efficient extreme solutions of such problems is utilized together with the fruitful relation that the Kuhn-Tucker conditions play in solving such problems.

This paper is structured as follows: the parametric multi- objective linear programming problem is introduced in the second section. The third section covers the definitions with characterization of the basic notions of solvability, stability set of the first kind, and the complete stability set of the first kind for such problems. Also, an algorithm for determining the complete stability set of the first kind for a given parameter is presented. The fourth section is devoted to present an algorithm for decomposing parametric space according to the complete stability set of the first kind. Two examples are given which clarify the developed theory.and algorithms. Finally, conclusion is reported in the fifth section.

2- Parametric multi- objective linear programming problem

Consider the following parametric multi-objective linear programming (MOL) problem with parameters in the objective functions:

(MOL):

Min $F(\lambda , X) = (\sum_{j=1}^{n} C_{1j} (\lambda_1) x_j, \sum_{j=1}^{n} C_{2j} (\lambda_2) x_j, \ldots\ldots, \sum_{j=1}^{n} C_{kj} (\lambda_k) x_j)$,

Subject to

$$G = \{X \in R^n \mid \sum_{j=1}^{n} a_{ij} x_j \leq b_i , i=1, 2, \ldots, m\}$$

Where:

$C_{ij} (\lambda_i)$, $i=1, 2, \ldots,k$, $j=1, 2, \ldots., n$ are continuous functions in λ_i,

$\lambda_i \in R^{l_k}$, $i= 1, 2, \ldots., k$ are real valued parameters,

$\lambda = (\lambda_1, \lambda_2, \ldots\ldots. ,\lambda_k) \in R^{l_1} \otimes R^{l_2} \otimes \ldots, \otimes R^{l_k}$

It must be noted that the nonnegativity constraints $x_j \geq 0, j=1, 2, \ldots., n$ are included in the feasible region "G".

Using the weighting method to solve (MOL) problem, we get the following single objective parametric linear programming problem "P(W)" as follows:

P(W): Min $\sum_{i=1}^{k} \sum_{j=1}^{n} w_i c_{ij} (\lambda_i) x_j$,

Subject to

$$G = \{ X \in R^n \mid \sum_{j=1}^{n} a_{ij} x_j \leq b_i , x_j \geq 0, i=1, 2, \ldots, m, \; j=1, 2, \ldots., n \} \text{ [7]},$$

$\sum_{i=1}^{k} w_i = 1,\ w_i \geq 0,\ i = 1, 2, \ldots, k,$

$W = (w_1, w_2, \ldots, w_k) \in R^k.$

It is well known [3, 5] that; if $\bar{X}$ is an efficient solution of (MOL) for $\lambda = \bar{\lambda}$, then there exists $\bar{W} \geq 0$, such that $\bar{X}$ solves P(W) for $\lambda = \bar{\lambda}$, and if $\bar{X}$ solves P(W) uniquely for $\lambda = \bar{\lambda}$, $\bar{W} \geq 0$, or if $\bar{X}$ solves P(W) for $\lambda = \bar{\lambda}$, $\bar{W} > 0$ then $\bar{X}$ is an efficient solution of (MOL) for $\lambda = \bar{\lambda}$.

Moreover, if we take $\lambda = \bar{\lambda}$, then all the efficient solutions of (MOL) could be generated from P(W) by taking only positive values of W.

3- The complete stability set of the first kind

For the parametric multi- objective linear programming problem, the following basic notions can be defined:

Definition (The solvability set)

The solvability set of problem (MOL) which is denoted by B, is defined by: $B = \{ \lambda \in R^{l_1} \otimes R^{l_2} \otimes \ldots, \otimes R^{l_k}$ | there exists an efficient solution of problem (MOL) for the given $\lambda\}$.

It is clear that if G is bounded then $B = R^{l_1} \otimes R^{l_2} \otimes \ldots, \otimes R^{l_k}$.

Definition (The stability set of the first kind)

Assume that for $\bar{\lambda} \in B$, $\bar{X}$ is an efficient solution of problem (MOL) corresponding to $\bar{\lambda}$, then the stability set of the first kind of problem (MOL) corresponding to $\bar{X}$ which is denoted by $S(\bar{X})$ is defined by:

$S(\bar{X}) = \{ \lambda \in B \mid \bar{X}$ is an efficient solution of the problem (MOL) corresponding to $\lambda\}$.

Definition (The complete stability set of the first kind)

Assume that for $\bar{\lambda} \in B$, all the corresponding efficient solutions of the problem (MOL) are the points X_i, $i \in L_1$ then the complete stability set of the first kind of problem (MOL) denoted by $S_C(\bar{\lambda})$ is defined by:

$$S_C(\bar{\lambda}) = \bigcap_{i \in L_1} S(X_i).$$

It must be noted that the efficient solutions of problem (MOL) for any $\bar{\lambda} \in B$ is connected [1, 9], then $S_C(\bar{\lambda})$ can be obtained by:

$$S_C(\bar{\lambda}) = \bigcap_{i \in L} S(X_i),$$

Where L is a finite set, X_i, i= 1, 2, ..., L are the efficient extreme solutions of (MOL) corresponding to $\lambda = \bar{\lambda}$.

Assume that $\bar{X}$ is an efficient solution of problem (MOL) for $\bar{\lambda} \in B$, and $I = \{ i \mid \sum_{j=1}^{n} a_{ij}\ x_j = b_i , i=1, 2, \ldots, m\}$.

i.e., i denote the set of all indices corresponding to the active constraints of G at $\bar{X}$, with cardinality **r**.

Then the corresponding Kuhn- Tucker (K.T.) conditions [2, 4] of the P(W) for $\lambda \in B$ takes the form:

$$\left.\begin{array}{l} \sum_{i=1}^{k} w_i\, c_{ij}(\lambda_i) + \sum_{i \in I} a_{ij}\, u_i = 0, \quad j=1, 2, \ldots, n, \\ u_i \geq 0,\ i \in I, \end{array}\right\} \textbf{K.T. problem}$$

where

u_i , $i \in I$ are the corresponding multipliers of the K.T. problem.

The equations in K.T. problem is a linear system of equations in the multipliers u_i , $i \in I$ which could be solved by any suitable technique from which either one of the following three cases can arise assuming the linear independence of the equations:

1) r = n

In this case u_i , $i \in I$ could be obtained from the equations in **K.T. problem**, substituting in $u_i \geq 0$, we get **r** conditions on w_i , λ_i , i =1, 2, ..., k.

2) r < n

In this case, we use **r** of the **n** equations to get u_i, $i \in I$ and substituting in the rest of the equations and in the inequalities $u_i \geq 0$, we get **n** conditions on w_i , λ_i , i=1, 2, ... , k.

3) r > n

In this case, **n** of multipliers u_i, $i \in I$ are obtained from the equations, and substituting in the inequalities $u_i \geq 0$, $i \in I$ we get **r** conditions on w_i , λ_i , i= 1, 2, ..., k.

Let $A(\bar{X}) = \{(W, \lambda) \in R^k \times B \mid (W, \lambda)$ solves **K.T. problem**$\}$, then the stability set of the first kind $S(\bar{X})$ can be obtained by:

$$S(\bar{X}) = \{ \lambda \in B \mid (W, \lambda) \in A(\bar{X}) \}.$$

In the following, the algorithm ALg.(I) is presented to obtain the complete stability set of the first kind of problem (MOL) for a given $\lambda \in B$.

ALg. (I)

1. Start with any $\bar{\lambda} \in B$.
2. Solve problem (MOL) for $\lambda = \bar{\lambda}$ using the multicriteria simplex method [9] to get all the efficient extreme points of the problem "X_i", $i \in L$.
3. Put i = 1, get I_1 such that
   $I_1 = \{ i \mid \sum_{j=1}^{n} a_{ij} x_{1j} = b_i, i=1, 2, \ldots, m\}$.
4. Solve K.T. problem to get $A(X_1)$ from which we obtain $S(X_1)$.
5. Go to step (3) and put i= 2, repeat step (4) to get the set $S(X_2)$.
6. The process is repeated till all the efficient extreme points X_i, $i \in L$ are exhausted.
7. The complete stability set of the first kind is obtained from the relation:

   $$S_C(\bar{\lambda}) = \bigcap_{i \in I} S(X_i).$$

Example 1:

Let us consider the following bicriterion parametric linear programming problem with two parameters λ_1, λ_2 in the first objective function f_1 and one parameter λ_3 in the second objective function f_2. $\lambda_1, \lambda_2, \lambda_3 \in R$.

$$\textbf{Min } F = (f_1 = \lambda_1 x_1 + \lambda_2 x_2, \quad f_2 = -3 x_1 + \lambda_3 x_2),$$

Subject to

$$G = \{ X \in R^2 \mid x_1 + x_2 \le 4,$$
$$x_1 + 2 x_2 \le 6,$$
$$- x_1 \le 0,$$
$$- x_2 \le 0\}.$$

Since G is bounded then $B = R^3$. If we take $\lambda = (\lambda_1, \lambda_2, \lambda_3) = (1, 1, 1)$, the problem will have two efficient extreme solutions $X_1 = (0, 0)$, $X_2 = (4, 0)$, (see Fig. (1)).

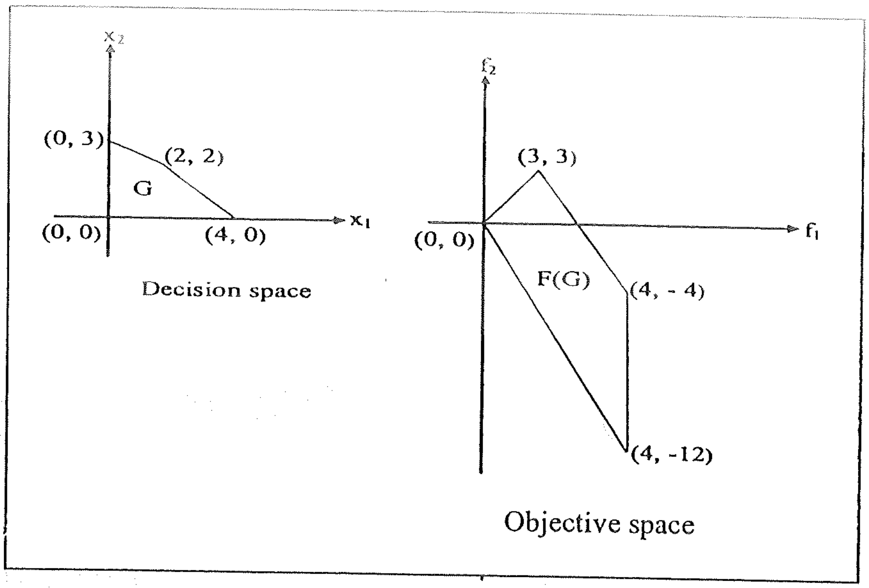

Fig. (1) G and F(G) when ($\lambda_1, \lambda_2, \lambda_3$) = (1, 1, 1).

(1) For $X_1 = (0, 0)$

It is clear that $I(X_1) = \{3, 4\}$.

The Kuhn- Tucker conditions at X_1 for the corresponding weighting problem take the form:

$$\begin{aligned} & w\,\lambda_1 - 3(1-w) - u_3 = 0, \\ & w\,\lambda_2 + (1-w)\,\lambda_3 - u_4 = 0, \\ & u_3 \geq 0, \\ & u_4 \geq 0. \end{aligned}$$

From which, we get:

$A(X_1) = \{(w, \lambda) \in R^4 \mid w(\lambda_1 + 3) \geq 3,\ w(\lambda_3 - \lambda_2) \leq \lambda_3,\ 0 < w < 1\}$.

Therefore,

$S(X_1) = \{\lambda \in R^3 \mid (w, \lambda) \in A(X_1)\}$.

(2) For $X_2 = (4, 0)$

It is clear that $I(X_2) = \{1, 4\}$.

The Kuhn- Tucker conditions at x_2 for the corresponding weighting problem take the form:

$$\begin{aligned} & w\,\lambda_1 - 3(1-w) + u_1 = 0, \\ & w\,\lambda_2 + (1-w)\,\lambda_3 + u_1 - u_4 = 0, \\ & u_1 \geq 0, \\ & u_4 \geq 0. \end{aligned}$$

From which, we get:

$A(X_2) = \{(w, \lambda) \in R^4 \mid w(\lambda_1 + 3) \leq 3,\ w(3 + \lambda_1 - \lambda_2 + \lambda_3) \leq \lambda_1 + 3,$
$0 < w < 1\}$.

Therefore,
$S(X_2) = \{ \lambda \in R^3 \mid (w, \lambda) \in A(X_2) \}$.

The complete stability set of the first kind for the given problem for $\lambda = (1, 1, 1)$ is given as $S_C(1, 1, 1) = S(X_1) \cap S(X_2)$.

4- Decomposition of the parametric space

It is clear that the parametric space is of dimension $\sum_{i=1}^{k} \lambda_i$, the solvability set B which is a subset of this space could be decomposed according to the complete stability sets of first kind as well be clear from the following algorithm which is denoted to ALg. (II):

ALg. (II)

1. Start with any $\lambda_1 \in B$.
2. Use ALg. (I) to get $S_C(\lambda_1)$.
3. Choose $\lambda_2 \in B - S_C(\lambda_1)$, and use ALg. (1) to get $S_C(\lambda_2)$.
4. The process is repeated by taking $\lambda_{i+1} \in B - S_C(\lambda_i)$, i=1, 2,…., τ.
5. The algorithm is terminated when $B - S_C(\lambda_\tau) = \varphi$.

It must be noted that, the complexity of determining the complete stability set of the first kind depends mainly on the structure of $C_{ij}(\lambda_i)$, i=1, 2, …, k and j=1, 2, ….., n.

Remark 1:

If for any $\bar{\lambda} \in B$, $\bar{X}$ is the unique efficient extreme point of problem (MOL), then it must be noted that there is a difference between the sets $S(\bar{X})$ and $S_C(\bar{X})$ since $S(\bar{X})$ gives the set of $\lambda\underline{s}$ that maintains $\bar{X}$ as an efficient extreme point while $S_C(\bar{\lambda})$ gives the set of $\lambda\underline{s}$ that maintains $\bar{X}$ as the unique efficient extreme point. In this case $S_C(\bar{\lambda}) = \{\bar{\lambda}\}$.

Remark 2:

If at step**1** of ALg. (II) for $\lambda_1 \in B$, we get unique efficient extreme point, then choose $\lambda_2 \in B$, $\lambda_2 \neq \lambda_1$, and go to step**1**.

Example 2:

Let us consider the following bicriterion parametric linear programming problem with the same parameter $\lambda \in R$ in the two objective functions f_1, and f_2.

$$\text{Min } F = (f_1 = \lambda x_1 + 3x_2, \quad f_2 = 5x_1 - \lambda x_2),$$

Subject to

$$G = \{X \in R^2 \mid x_1 + x_2 \le 4,\ x_1 + 2x_2 \le 6,\ -x_1 \le 0,\ -x_2 \le 0\}.$$

Since G is bounded, then $B \in R$.

(1) If we take $\lambda = 0$, the problem will have one efficient extreme solution $X_1 = (0, 0)$, (see Fig. (2)).

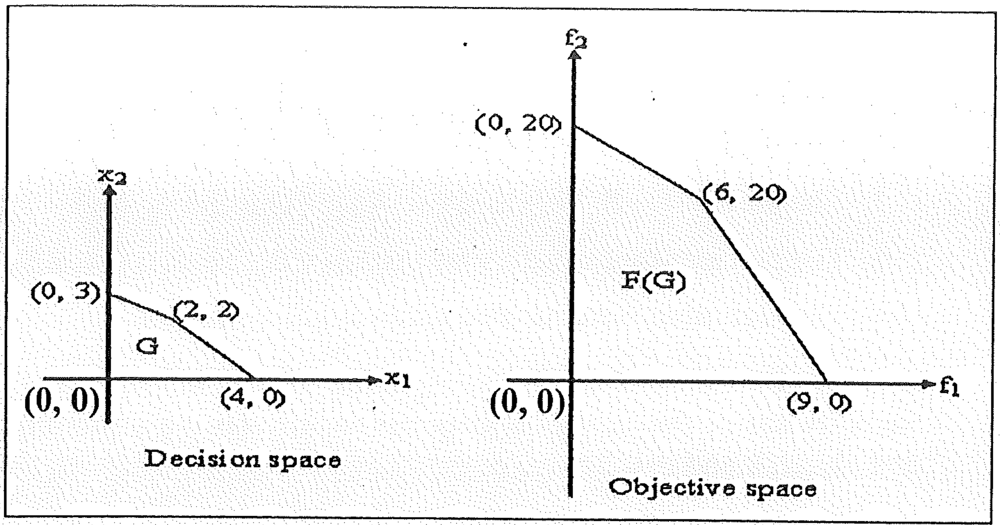

Fig. (2) G and F(G) when $\lambda = 0$.

It is clear that $I(X_1) = \{3, 4\}$. The Kuhn- Tucker conditions at X_1 for the corresponding weighting problem take the form:

$$w\lambda + 5(1-w) - u_3 = 0,$$
$$3w - \lambda(1-w) - u_4 = 0,$$
$$u_3 \ge 0,$$
$$u_4 \ge 0.$$

From which, we get:

$$A(X_1) = \{(w, \lambda) \in R^2 \mid \lambda \ge \frac{-5(1-w)}{(w)}, \lambda \le \frac{(3w)}{(1-w)}, \ 0 < w < 1\}.$$

Therefore,

$$S(X_1) = \{\lambda \in R \mid (w, \lambda) \in A(X_1)\}.$$

From Fig. (3), it is clear that $S(X_1) \equiv R$.
Then from **remark 1**, it follows that $S_C(0) = \{0\}$.

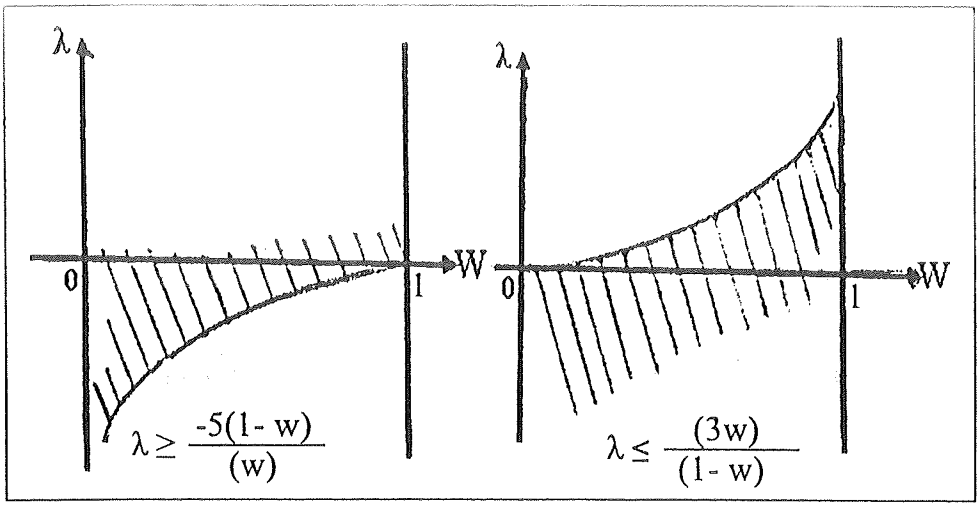

Fig. (3) The relation between λ and W when X_1= (0, 0).

(2) Take $\lambda = 1$, the problem will have two efficient extreme solutions $X_1 = (0, 0)$, $X_2 = (0, 3)$, (see Fig. (4)).

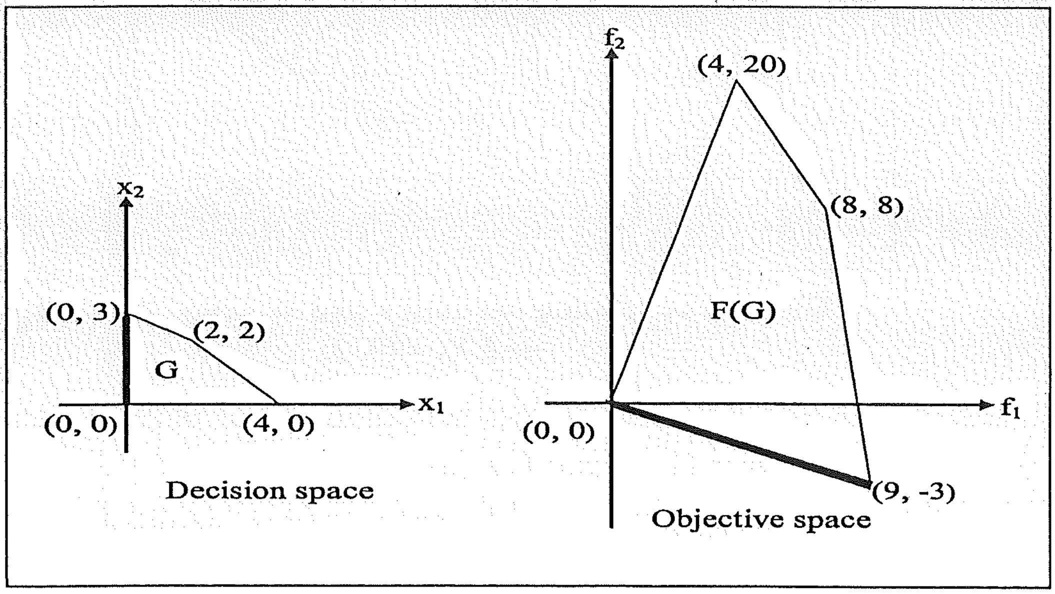

Fig. (4) G and F(G) when $\lambda = 1$.

It is clear that $I(X_2) = \{2, 3\}$.

The Kuhn- Tucker conditions at X_2 for the corresponding weighting problem take the form:

$$\begin{aligned} &w\lambda + 5(1-w) + u_2 - u_3 = 0, \\ &3w - \lambda(1-w) + u_2 = 0, \\ &\qquad u_2 \geq 0, \\ &\qquad u_3 \geq 0. \end{aligned}$$

From which, we get:

$$A(X_2) = \{(w, \lambda) \in R^2 \mid \lambda \geq \frac{(3w)}{(1-w)}, \ \lambda \geq 8w-5, \ 0 < w < 1\}.$$

Therefore,

$S(X_2) = \{\lambda \in R \mid (w, \lambda) \in A(X_2)\}.$
From Fig. (5), it is clear that
$S(X_2) = \{\lambda \in R \mid \lambda > 0\}.$
Then $S_C(1) = S(X_1) \cap S(X_2) = \{\lambda \in R \mid \lambda > 0\}.$

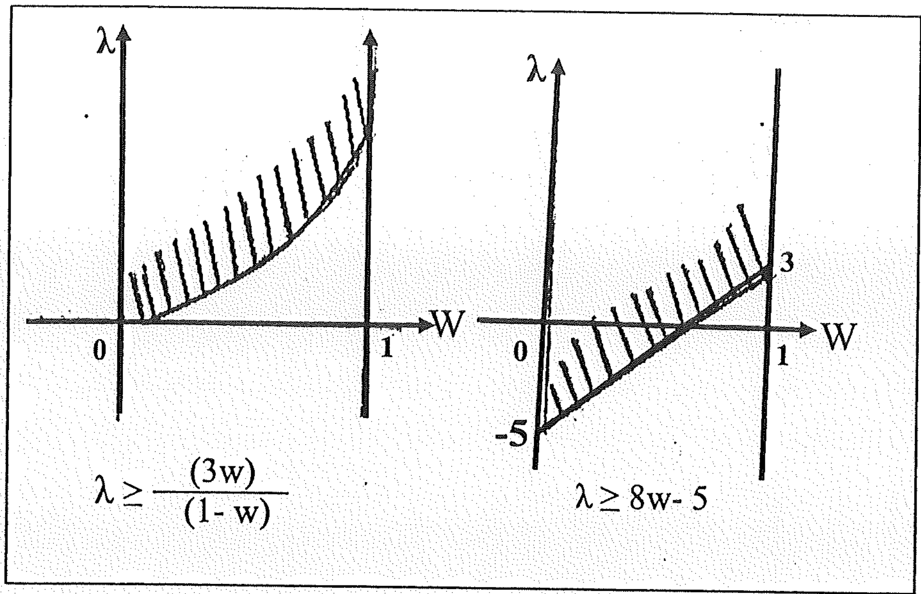

Fig. (5) The relation between λ and W when X_1= (0, 0), and X_2 = (0, 3).

3) Take $\lambda = -1$, the problem will have two efficient extreme points $X_1 = (0, 0)$, $X_3 = (4, 0)$, (see Fig. (6)).

It is clear that $I(X_3) = \{1, 4\}$. The Kuhn- Tucker conditions at X_3 for the corresponding weighting problem take the form:

$$\begin{aligned} w\lambda + 5(1-w) + u_1 &= 0, \\ 3w - \lambda(1-w) + u_1 - u_4 &= 0, \\ u_1 &\geq 0, \\ u_4 &\geq 0. \end{aligned}$$

From which, we get:

$$A(X_3) = \{(w, \lambda) \in R^2 \mid \lambda \leq \frac{-5(1-w)}{(w)}, \ \lambda \leq 8w-5, 0 < w < 1\}.$$

Therefore,

$S(X_3) = \{\lambda \in R \mid (w, \lambda) \in A(X_3)\}$.

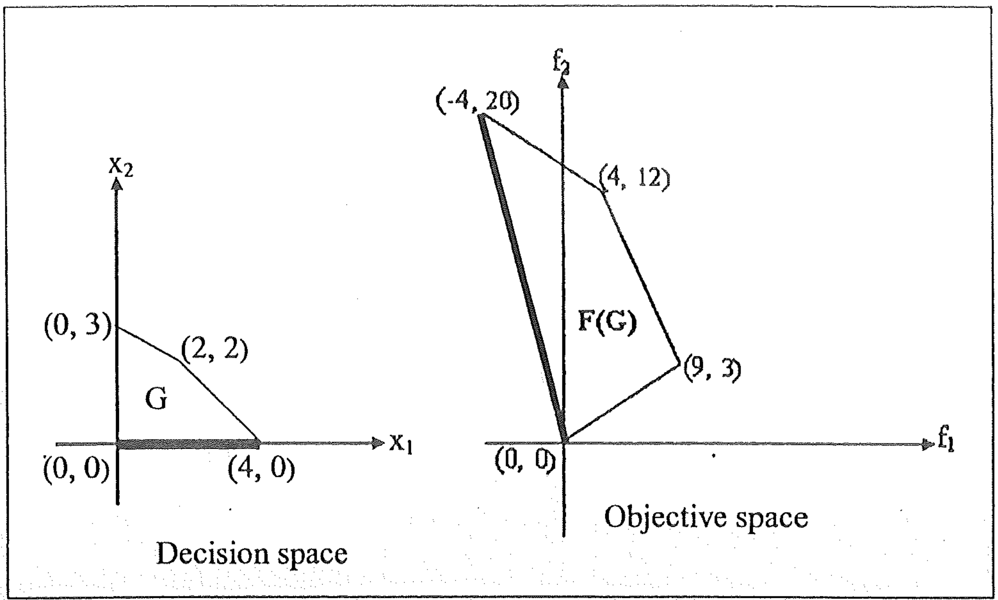

Fig. (6) G and F(G) when $\lambda = -1$.

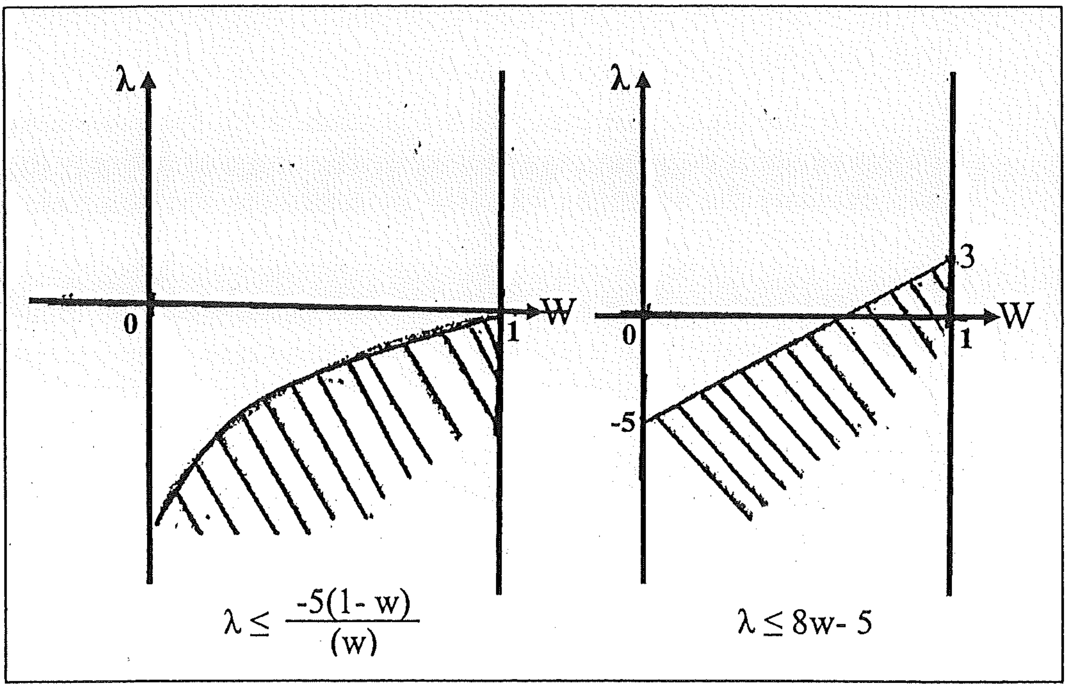

Fig. (7) The relation between λ and W when $X_1 = (0, 0)$, and $X_2 = (4, 0)$.

From Fig. (7) , it is clear that
$S(X_3) = \{\lambda \in R \mid \lambda < 0\}$.
Then $S_C(-1) = S(X_1) \cap S(X_3) = \{\lambda \in R, \lambda < 0\}$.

A summary of these results are shown in the following Fig. (8).

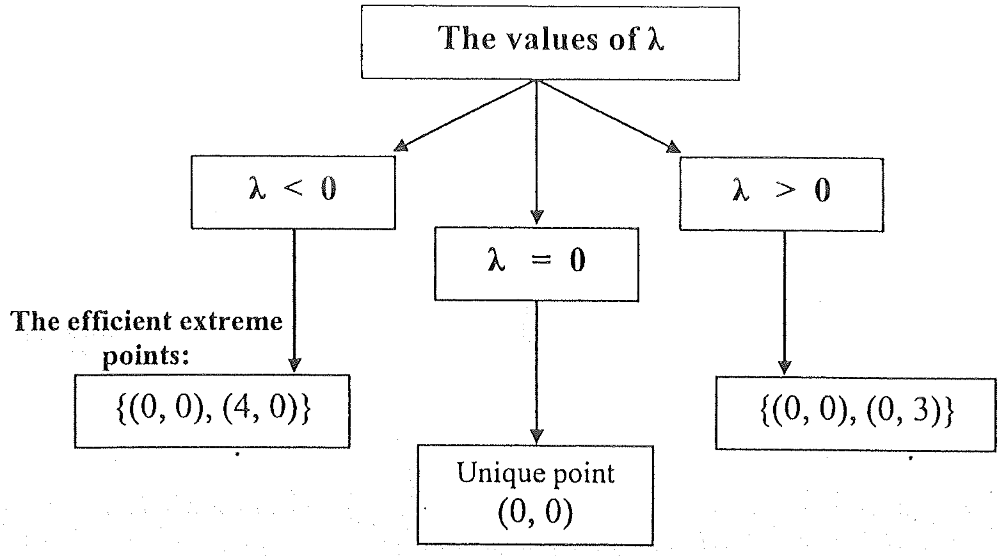

Fig.(8) The decomposition of the parametric space of the example.

5- Conclusion

In this paper, the efficient set is studied for the parametric multi-objective linear programming problems. The aim was not only to measure and evaluate the solutions but also to determine the complete stability efficient sets of the first kind for such problems.

The problem was treated for general parameters and many special cases can be derived from this general structure. The complexity of determining the complete stability efficient set of the first kind of this problem is depended on the structure nature of the parameters.

This work could be generalized for parametric multi- objective linear programming problems with parameters in constraints, and quadratic programming problems.

Finally, this study is essentially a new trend towards a good decision for multi- objective linear programming problems.

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A Modified Bat Clustering Algorithm for Solving the Capacitated Vehicle Routing Problem

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Abstract

The capacitated vehicle routing problem (CVRP) is a class of vehicle routing problems (VRPs). In CVRP a set of fixed capacity vehicles is required to fulfill customers' demands for a single commodity. The main objective in this respect is to minimize the total cost or distance while satisfying a number of constraints, such as: the capacity constraint imposed on each vehicle, logical flow constraints, etc. The cluster-first route-second method is a technique employed in VRP based on grouping of customers into a number of clusters, where each cluster is served by one vehicle. Once clusters are formed, a route determining the best sequence of customers is established in each cluster. The bio-inspired bat algorithm (BA), introduced in 2010, has proven to be efficient in solving combinatorial optimization problems. It is integrated to the traditional Kmeans clustering algorithm to generate the bat clustering algorithm known as 'C-bat' algorithm. In this study, we modified the aforementioned algorithm in order to incorporate a capacity constraint into the original algorithm. The resulting algorithm is utilized in the clustering phase of the cluster-first route-second method to solve the CVR problem. Experimental results show that the algorithm is efficient when tested on a number of benchmark problems.

Keywords: CVRP, cluster-first route-second, clustering, Kmeans, bat algorithm.

1. Introduction

The Capacitated Vehicle Routing Problem (CVRP), one of the variants of the classical Vehicle Routing Problem (VRP), was first introduced by Dantzig and Ramser[1]. In the VRP, a fleet of homogeneous vehicles with fixed capacity, departing from a central point (depot), must fulfil the demands of a number of spatially separated customers at a minimum cost. This must be done without exceeding vehicle's capacity. The cost is estimated either by the travelled distance or travelling time.

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Several methods for solving CVRP have been reported in the literature. These algorithms are generally based on exact methods, heuristics, metaheuristics or a combination of them. For a detailed review of CVRP solution methods, interested readers are referred to Takes [2]. Cluster-first route-second method is one of the heuristics employed to solve CVRP, where clustering represents the core step within the heuristic [3]. In cluster-first route-second method, customers are first grouped (cluster first) into feasible clusters that maintain the capacity constraint. A vehicle is responsible for serving each cluster, and so the clustering phase can be viewed as a capacitated cluster problem (CCP). Efficient routes are then established (route second) by conducting a permutation between customers of each cluster, specifying their best order in the route.

Bat algorithm (BA) is a bio-inspired metaheuristic that is based on the echolocation behavior of bats. It was first introduced by Yang [4] in 2010. In BA, a population of bats (solutions) uses both: a 'frequency-tuning' technique to explore the search space and an 'automatic zooming' to exploit it. Thus, the algorithm creates a balance between exploration and exploitation, which represents a key issue in any metaheuristic. BA proved to be efficient in solving optimization problems, and accordingly was extensively studied by many researchers. Since it was first proposed, BA was subject to modifications to enhance its performance. These modifications lead to a number of variants applied in diverse areas. For example, the authors in [7] incorporate a hybrid BA for solving CVRP. In their study, routes are incrementally built using bat algorithm, then further improved using local operators. Reference [5] provides a review of variants and applications of BA.

The Kmeans clustering algorithm is one of the widely used traditional partitioning approaches. It has a major drawback of falling into local minima. To overcome this drawback, a technique to hybridize Kmeans with an optimization algorithm is widely used. The merits of combining a number of bio-inspired algorithms, including BA to Kmeans have been studied in [6]. Experimental validation showed the benefits of such integration. Results showed that clustering with bats, known as C-bat, is highly ranked among its metaheuristic rivals. Therefore, we were encouraged to use it in our proposed work to solve the CVRP.

In the current study, modifications of the C-bat clustering algorithm are conducted in order to include capacity constraints. This renders the C-bat algorithm appropriate for solving capacitated clustering problems (CCP).

The proposed modified C-bat algorithm will be further used in the clustering phase of the cluster-first route-second technique to solve the CVRP. To the best of our knowledge, this is the first time to incorporate capacity conditions with C-bat algorithm.
The rest of this work is organized as follows: in section 2 we introduce the problem definition. Details of the solution methodology are given in section 3, where bat algorithm in first introduced followed by the C-bat algorithm and finally our modified C-bat algorithm. Experimental computations are presented in section 4. Finally the paper is concluded in section 5.

2. Problem definition

The vehicle routing problem (VRP) is considered as one of the most analysed combinatorial optimization problems [8]. The classical VRP consists of determining the optimal routes used by a fleet of vehicles to satisfy the demands of a number of spatially separated customers. Many variants of VRP exist in practical applications. These variants usually depend on the type of constraint(s) imposed on constructing the route. The CVRP considered in this work consists of distributing a single commodity from a central depot to a known set of customers, using a given number of identical vehicles of fixed capacity. All the customers have predetermined deterministic demands. Demands may not be split; hence, each customer should be visited exactly once. The total demand assigned to each vehicle must not exceed its capacity. The objective is to minimize the total distance traveled all by the vehicles. Once each vehicle is assigned the associated group of customers, a minimum route is to be established. This route starts at the depot, passing through all the assigned customers and back to the depot.
According to [9] VRP is an NP-Hard problem. Heuristics and metaheuristics are therefore considered to solve the problem and its associated variances [10].

3. Proposed solution methodology

Our approach to tackle the CVR is based on following the cluster-first route-second method. In the clustering phase, customers are grouped into feasible clusters. Feasible clustering means that the total demand of the customers belonging to a cluster does not exceed the capacity of a single vehicle. Then, in the routing the phase, customers are suitably ordered within each cluster, which amounts this phase to solving a TSP [11].

In the present work, clustering will be performed using a modified C-bat algorithm. Details of the proposed algorithm are presented below.

3.1 Bat algorithm

The original bat algorithm (BA), inspired by the echolocation features of microbats, was first introduced in [4]. In BA, a population of N bats (solutions), flying in a D-dimensional search space, update their positions x_i^t and velocities v_i^t at any generation t according to the following three equations [4];

$$f_i = f_{min} + (f_{max} - f_{min})\beta, \quad (1)$$
$$v_i^t = v_i^{t-1} + (x_i^t - x^*)f_i, \quad (2)$$
$$x_i^t = x_i^{t-1} + v_i^t, \quad (3)$$

Where, $\beta \in [0, 1]$ is a random vector drawn from a uniform distribution. A fitness function $F(x_i)$ is associated with each bat, and x^* represents the current global best solution which is determined after comparing all the solutions. Each bat has its own frequency $f_i \in [f_{min}, f_{max}]$, where the values of f_{min} and f_{max} depend on the domain of each problem, its pulse emission rate r_i^t, and its loudness A_i^t.
A local search around the current best is performed with probability r_i^t. Thus a new solution is locally obtained for each bat using random walk

$$x_{new} = x^* + \epsilon A^t, \quad (4)$$

Where $\epsilon \in [-1,1]$, and A^t is the average loudness of all the bats at current generation t. As the loudness usually decreases once a bat has found its prey (i.e. better $F(x_i)$), while the rate of pulse emission increases, A_i^t and r_i^t have to be updated accordingly as generations proceed

$$A_i^{t+1} = \propto \times A_i^t, \; r_i^{t+1} = r_i^0 \times [1 - \exp(-\gamma \times t)], \quad (5)$$

Where $\propto$ and γ are constants. This update is done, if and only if the solution improves or not too loud, i.e. $F(x_{new}) < F(x_i)$ & $rand < A_i^t$.
The balance between: 1) exploration which is depicted by randomly flying bats over the search space, and 2) exploitation represented by the local search in promising locations, can be considered one of the BA's points of strength.
Integrating the BA with Kmeans algorithm to overcome the latter's drawbacks is discussed in the next section.

3.2C-bat algorithm

The Kmeans clustering algorithm assigns points to k clusters based on their proximity to the cluster's centroid. Initially centroids are selected at random and then iteratively reallocated until a predetermined stopping criterion is met. One can broadly divide Kmeans algorithm into: an initialization phase, a cluster assignment and centroids update phase, and finally an exploration and evaluation phase [5]. Kmeans algorithm suffers from two main drawbacks, namely: its dependency on initial centroid values, and its tendency to fall into local optima. Its integration with bio-inspired optimization algorithms to overcome the aforementioned drawbacks was studied in [12, 6].

In the C-bat algorithm introduced in [6], bats represent solutions, each of which holds a set of centroids. A population of bats collaboratively searches for the best possible configuration of the clusters. The best configuration of clusters is reflected by the optimal positions of the centroids. These centroids are subsequently used to grow clusters following the basic Kmeans principle: points are assigned to the cluster with nearest centroid. The algorithm aims at minimizing an objective function defined as

$$F = \sum_{j=1}^{k} \sum_{i=1}^{N} \left\| x_{i,j} - cen_j \right\|^2, \qquad (6)$$

Where, k is the number of clusters, N is the number of points to be clustered, $x_{i,j}$ is the i^{th} data point belonging to the j^{th} cluster, and cen_j is the centroid of cluster j. Bats use equations from (1) to (3) for updating their frequencies, velocities and locations identically as BA. For performing local search, equation (4) is redefined as follows

$$x_{new} = x^* + 0.01 \times rand(0,1), \qquad (7)$$

Where, $rand(0,1)$ being a uniform random number and the value 0.01 acts as a static step for the bat.

Modifications on the C-bat algorithm, to incorporate a capacity constraint in order to use it for solving capacitated clustering problems, are discussed in the next section.

3.3Modified C-bat algorithm

Two main modifications were performed on the C-bat algorithm. The first modification concerns the objective function. The second modification consists of improving exploitation by means of a dynamic local search operation. Figure 1 gives a Pseudo code of the Modified C-bat algorithm.

As mentioned earlier, the capacitated clustering problem CCP consists of imposing a capacity constraint on each cluster. Mainly, there exist three approaches to handle constraints: direct approach, Lagrange multipliers, and penalty method [13]. The penalty approach is selected in this proposed method, since it is the simplest to implement in practice, without reducing the quality of the results. In our approach, constraints violations are incorporated in the objective function defining a penalized function. This function penalizes infeasible solutions relative to the amount of constraint's violation [14]. Accordingly, the new fitness function can be defined as

$$F_P(X) = F(X) + p \times (\Delta\, Capacity)^c, \tag{8}$$

Where: $F_P(X)$ is the penalized fitness function, $F(X)$ is the original fitness function, and $\Delta\, Capacity$ is the amount of capacity violation. While p and c are user defined penalty parameters. In the current study, these parameters are determined experimentally. Incorporating the penalized fitness function to the C-bat algorithm yields a modified C-bat algorithm appropriate for CCP.

The second modification deals with improving local search performed around the best solution. Our proposed modification involves using an adaptive local search technique in order to improve exploitation. The new solution is obtained by modifying equation (4) as follows

$$x_{new} = x^* + S \times N(0,1), \tag{9}$$

$$S = R/5 \times (G - t)^2/G^2, \text{ where } S \to 0 \text{ as } t \to G \tag{10}$$

Where, S represents a 'step size' for the bat, R is a user defined range, G is the maximum number of generations, t is the current generation number, and $N(0,1)$ is a random number generated from a normal distribution with zero mean and standard deviation 1. As can be seen in equation (10), the 'step size' diminishes as generations proceed. Letting the bat reduce its step as it approaches final iterations is a more realistic representation. It illustrates what should actually happen as generations evolve. It is considered fine tuning, since the bat narrows its search around the best as iterations proceed.

The modified C-bat algorithm is used in the clustering phase of the cluster-first route-second technique to solve the CVRP.

In the next section, experimental computations are carried out in order to investigate the efficiency of these modifications.

Modified C-bat algorithm

Determine the number of clusters k
Initialize the bat population x_i $(i = 1,2, \ldots, n)$ *and velocity* v_i
Define pulse frequency range $[f_{min}, f_{max}]$ *and initialize* f_i
Initialize pulse rates r_i *and the loudness* A_i
For each bat randomly select k objects from S data objects as initial centroids and evaluate corresponding fitness $F_P(x_i)$ *[equation (8)]*
while $(t < Max\ number\ of\ iterations)$
for *each bat* x_i *in population*
Generate a new solution by adjusting frequency, and updating velocity and location/solution [equations (1) to (3)]
if $rand > r_i$
Generate a new solution by performing an adaptive local search around the best solution x^*
[equations (9) and (10)]
end if
Evaluate the new solution $F_P(x_{i\,new})$ *and reassign the clusters following the basic k-means principle*
if $rand < A_i$ & $F_P(x_{i\,new}) < F_P(x_i)$
Accept the new solution
Increase r_i *and reduce* A_i
end if
Rank the bats and find the current best x^*
end while

Figure 1: Pseudo code of the Modified C-bat algorithm

4. Computational results and analysis

In order to verify the efficiency of the proposed capacitated C-bat algorithm, a set of experiments are carried out. The modified algorithm is tested using benchmark problems downloaded from the web http://www.branchandcut.org/. In this work, 10 instances from three different sets are selected. These sets are known in the literature as set A, set E, and set P. Set A and set P are obtained from Augerat et al [16], while set E is obtained from Christofides and Eilon[17]. For the instances in set A and set E, customers' locations and demands are randomly generated. Instances in set P are modified instances from the literature. Each test instance has the following nomenclature: the first letter refers to the set name followed by the number of customers and finally the minimum number of required clusters. For each instance, the depot and customers' coordinates, as well as customers' demands, are given.

Computational experiments are coded using MATLAB 2009a, on a PC with Pentium Dual Core T3200 Processor 2.0GHz and 2.0 GB memory. In the experiments, we run each instance for 10 independent runs and the best result is reported. The stopping criterion is the number of bat generations.

The parameters of the proposed modified C-bat algorithm were set experimentally after a number of test runs. Chosen values of the parameters that are as follows: the bat population size is 10; the maximum number of generation is 400. The loudness A_i and the pulse rates $r_i$of bat i are uniformly generated $A_i \in [0.4,0.6]$ and $r_i \in [0.5,0.8]$. The minimum and maximum frequencies are set as: $f_{min} = 0$ and $f_{max} = 100$. The parameters $\propto$ and γ are given values of 0.9 and 0.98 respectively.

Results of these runs are summarized in Table 1, which includes the following information: instance name, vehicle capacity, tightness (total demand/total capacity), "optimum" cost, and modified C-bat cost. The solution reported is "optimum" according to http://www.branchandcut.org/. The "optimum" routes and the corresponding total cost are provided in the website. The reported cost is the sum of Euclidean distances travelled by all vehicles starting at the central depot, visiting the assigned customers, and travelling back to the depot. The values in Table 1, either the optimum or the modified C-bat, are those of the total distance travelled by all vehicles.

As can be seen, the proposed modified C-bat algorithm was able to obtain very promising results. Optimum values were obtained in 3 instances, namely; E-n22-k4, P-n20-k2 and P-n22-k2. While a deviation of less than 1% from optimum, in terms of cost values, was recorded in 3 instances namely; E-n23-k3, E-n33-k4 and E-n51-k5. The deviation for the remaining 4 instances was less than 4%. The maximum recorded deviation from optimum was 3.9%, while the average deviation was 1.13%. Deviation from optimum was calculated using equation (11)

$$Deviation\% = \frac{optimum - modified}{optimum}\% \tag{11}$$

Table 1: Comparison of results for benchmark instances

No.	Instance	Capacity	Tightness	Opt	modified C-bat	Deviation %
1	A-n33-k5	100	0.89	661	687	3.9334
2	A-n33-k6	100	0.9	742	751	1.2129
3	A-n38-k5	100	0.96	730	751	2.8767
4	E-n23-k3	4500	0.75	569	570	0.1757
5	E-n22-k4	6000	0.94	375	375	0
6	E-n33-k4	8000	0.92	835	843	0.9581
7	E-n51-k5	160	0.97	521	525	0.7678
8	P-n20-k2	160	0.97	217	217	0
9	P-n22-k2	160	0.96	216	216	0
10	P-n51-k10	80	0.97	741	752	1.4845

Among traditional arguments used to analyze the difficulty of a VRP instance one can cite: the depot position, the number of customers, the number of vehicles and the tightness ratio [17]. For the 10 instances tested in this study, the depot was positioned almost in the middle of all customers. For the tightness ratio, all of the above tested instances were for tightly packed vehicles, where the tightness ratio was close to 1. Regarding the number of customers, all the 10 instances ranged from 20 to 51 customers. This might be categorized as medium size problems. In analysing the results in Table 1 above, we notice that the quality of the solutions obtained using the proposed algorithm was problem size independent. Concerning the number of vehicles as an argument to analyze the difficulty of the VRP, instance P-n51-k10 was generated based on instance E-n51-k5 by increasing the number of vehicles employed, while decreasing their capacity. This modification resulted in categorizing test instance P-n51-k10 more difficult than instance E-n51-k5 as stated in [17]. Comparing the results of these two instances, it was noticed that the solution recorded for instance E-n51-k5 is superior to that of instance P-n51-k10. This was reflected in the higher deviation percentage from optimal recorded for instance P-n51-k10. Thus, an increased number of vehicles with decreased capacity resulted in a more difficult VRP problem, and hence, the problem was harder for the algorithm to obtain optimum results.

The computational time for the 10 test instances was in the order of a few seconds.

5. Conclusion

In this paper, a modified C-bat clustering algorithm is developed and used to solve the CVRP. Two main modifications are adopted; the first one involves using a penalized objective function to account for a capacity constraint. The second modification improves the local search of bats by employing an adaptive local search technique. Experimental analysis shows the ability of the algorithm to improve on obtaining optimum and near optimum solutions for benchmark problems. Applied to 10 benchmark instances, the algorithm was able to obtain optimum for 3 instances and near optimum for the other seven instances. The maximum recorded deviation from optimum was 3.9% and it occurred only once, while the remaining deviations from optimum were less than 3%. The average deviation from optimum for the 10 test instances was 1.13%. To sum, the main contribution of this work is incorporating clustering algorithms with BA to obtain a new modified C-Bat algorithm. To further enhance the performance of the proposed modified C-Bat algorithm we introduced a new adaptive local search technique. The proposed algorithm was applied to CVRP and promising results were recorded.

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InformationTechnology Investments Decisions in Egypt: State of the Art of InformationTechnology Evaluation Methodologies[I]

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Abstract

This Paper aims to explore IT investment decisions evaluation practices in Egypt. IT evaluation literature was reviewed concerning IT investment evaluation models and methodologies. Traditional models include Economic Models (Productivity, and Profitability and financial analysis). Emerging trends in IT Evaluation include, but not limited to, Risk and Portfolio Management Approaches (Value-at-risk, Real Options), as well as Composite Balanced Scorecards (IT BSC).

To recognize state of the art of IT investment decisions evaluation in Egypt, an exploratory study is conducted based on Questionnaires distributed into three categories of respondents (User of the system – Decision maker in investments in the system – Decision maker who has authority to use system) who worked in a number of listed companies operating in Egypt.

The results of the exploratory study are in general so far consistent with findings of related literature. Results indicate that Net Present Value (NPV) and Return on Investment (ROI) are of great interest to decision makers in IT investment decision in Egypt. However, emerging IT evaluation trends (IT BSC and Value-at-risk) also gained their concern. One can interpret this as among financial measures, NPV and ROI are still widespread measures of IT value. Also, both Decision makers and Users are aware of recent trends of IT evaluation methodologies.

Keywords: Information Technology investment decisions; Information Technology evaluation; Information Technology Balanced Scorecard; Value-at-risk; Real options.

1. Introduction

While organizations sought to meet the growing needs of information, especially in the modern business environment, which is characterized by risk and uncertainty, they adopt information technology to enable them to meet their information needs. Nowadays, the information technology is not a competitive advantage in organization anymore [2].

As pointed out by [3], IT Investment decision making framework differed dramatically within last years, because IT does not still support office automation only. IT is considered to be a strategic necessity that is difficult to manage [4]. Furthermore, IT investment decision is a strategic one. This decision is of increasing importance in case of investing in IT based information system, because management information systems are the most important source of information within any organization. Recent surveys indicate that issues such as 'measuring the value of IT' and 'evaluating IS

I Based on exploratory study conducted within a master dissertation (in Arabic) in Accounting (Management Accounting Information Systems) [1]

II Mohammad Ibraheem Ahmad, is an Information Systems PhD research student, MSc. Management Accounting Information System, Email: mohammad.ibraheem.ahmad@gmail.com performance' are of great importance to managers [5]. In spite of this, it is difficult to ascertain the benefits of developing software applications [6].

As indicated in [7], developing Information Systems is time-consuming, in addition to high cost of application and configuration. However, these systems do not provide the benefits expected from them, which occur only after a period of applications. As shown in [8], strategic systems projects involve a huge amount of risk, thus IT investment decision is made in an uncertain environment. In addition, the more increased strategic importance of information technology, the more risk and difficult both direction and timing of allocation decision of resources to that technology.

In general, Information Technology does not meet the needs of one party only, but is also multi-parties stakeholders investment [7]. IT Investments Stakeholders are identified by [7] in several parties which are: organization (management), users, application and implementation team, technical support personnel, and other stakeholders who do not bring benefit to the project, or does not affect it (trading partners).

Risks inherent in investment in IT projects are reviewed by [8] which are; Technical Risk (incompatibility of system capabilities with current technology), Functional Risk (system failure in achieving the desired benefits of IT), Project Risk (technology capabilities are beyond staff's technical skills of Organization), Internal Political Risk (IT investment may be involved in conflicting interests which impede its implementation as desired), External Environment Risk (Unexpected response from competitors or regulatory bodies), and Systemic Risk (Technology may dramatically change environment so that achievement of expected benefits is hindered). Project risk and functional risk are prevalent. They affect each other, as the procedures that will reduce one of them, will increase the other risk at the same time [8].

On the other hand, there is seldom evidence about presence of direct relationship between IT investment and firm performance. Actually, it is not as ease to conduct quantitative analysis for evaluation purposes, because it not easy to quantify IT benefits. While many studies tried to identify the relationship between IT investment decisions and firm performance, results about IT returns were confusing. Some others tried to interpret these findings but merely few of them make sense (see [9]).

In summary, the nature and characteristics of IT projects investments impose methods of evaluation and criteria that suit this special nature [1, 10].

This paper is organized as follows. Section 2 represents an overview of IT Investments and IT value. Section 3 shows an overview of IT evaluation. Section 4 literature reviews IT evaluation methodologies. Section 5 reviews popular IT evaluation methods. Section 6 shows Research Methodology. Section 7 summarizes results. Finally, some concluding remarks are reported in section 8.

2. Overview of IT investments and IT value

In terms of information systems (IS) one can distinguish between the worth of the product (the IS), and the worth of the process (the activities involved in producing the IS). The worth of the IS development process is normally evaluated in terms of an assessment of some features of the worth of the product [11].

As claimed by [12], the debate over IT value is related to the controversy research question to be answered, and what the appropriate (null) hypothesis should be. In this regard, [12] argued that IT value research question may be linked to either Productivity (IT enable firms to produce more "output" for a given quantity of inputs) or Profitability (IT enable firms to gain a competitive advantage and thus earn higher profits than earned otherwise) or Consumer surplus (value or substantial benefits delivered to the consumer). While Productivity addresses value creation perspective, Profitability and Consumer surplus address value distribution perspective. Underlying theory for each of these differs. Productivity is derived from Production Theory, Profitability is derived from theory of Competitive Strategies, and Consumer surplus is derived from Economic Welfare and Consumer Theory [12]. Thus, Productivity is linked to Economic marginal benefits of IT, Consumer Surplus is linked to benefits that are delivered to the consumer and on the contrary, Profitability is linked to benefits generated from IT competitive advantage that are delivered to shareholders (firm owners) [12].

In this regard, [4] declares that Potential benefits of IT are dependent on a number of related business functions and external parties. The success of many IT projects is a complex mixture of financial, technical, organizational and social effects, both within the organization and external: competitive reactions, influence on the value system, technology acceptance behavior [4].

In the same context [13] identified and discussed five factors of IT investments value delivery which are financial (the monetary values of investment which accounts for money in and out of the organization), operational (the improvement in the business functionality and processes in the organizations facilitated by the IT investment), organizational (organizational resources that include human, assets, infrastructure that are benefited from the IT investment), strategic (corporate benefits through management, and planning accomplished through IT investments), and service (improvements of treatments, information, products and services to the stakeholders especially the customers).

As shown in [14], two perspectives have been predominantly used as the conceptual bases to study the strategic value of information technologies, resource-centered (consider IT itself to be a strategic resource that can directly influence organizational performance when properly combined with other strategic resources) and contingency-based (strategic value of IT must be understood in conjunction with a firm's strategy and stresses the importance of a "good fit" between business strategy and IT strategy).

According to [14], two research streams draw on the resource-centered perspective as their main theoretical basis for studying the strategic value of IT; the production function view and the resource-based view (focuses on the importance of the scope of resources). On the other hand, the Contingency Perspective of the Strategic Value of IT deals with the strategic alignment of IT (which refers to the degree to which the priorities, goals, and objectives of the IS strategy are aligned with the priorities, goals, and objectives of the firm's business strategy) [14]. Obviously, managing the congruence between stakeholders' expectations and project goals is very important [4]. As mentioned earlier, issues such as 'measuring the value of IT' and 'evaluating IS performance' are of great importance to managers [5], although being unable to quantify the expected return of the project, the decision is based on gut feeling [4]. Thus [15] raised a need to adopt Benefits Realization Methodologies in order to ensure that expected benefits are realized by planning how and when benefits will be realized and deciding who will be responsible for achieving benefits as well as actually overseeing the realization of benefits.

3. Overview of IT evaluation

The IS/IT investment evaluation methodology improves the decision-making process for IS/IT investments while the benefit realization methodology ensures that the benefits claimed for those investments are realized [15].

In [16], it was argued that IT evaluation should be used for measurements ex-ante (to support decision making), during the project (for controlling purposes) and ex-post for post implementation reviews.

In this regard, a preliminary model that makes a distinction between four types of IS evaluation activity is proposed in [11], primarily in terms of when they take place in a standard linear model of the IS life cycle. These types of IS evaluation can be summarized as follows [11] :

1. *Strategic evaluation*, which is sometimes referred to as pre-implementation evaluation, primarily involves assessing or appraising an IS/IT investment in terms of its potential for delivering benefits against estimated costs.
2. *Formative evaluation,* which involves assessing the shape of an IS whilst in the development process itself. Formative evaluation may be used to make crucial changes to the design of an IS or to make critical decisions concerning the degree of project abandonment.
3. *Summative evaluation,* sometimes referred to as post-implementation evaluation, occurs after an IS has been implemented. Traditional approaches to summative evaluation involve signing off some system against its specification. More recently, there has emerged an emphasis

on usability testing. Summative evaluation may also produce ideas for new systems and/or components.

4. Post mortem analysis is a variant of summative evaluation particularly directed at organizations learning from their partially or totally abandoned projects.

In this regard, [4] showed that resource-based view and the real-option view can be used for ex-ante evaluation on project level, while microeconomics-based view can be used as an ex-post justification on firm level [4] (these views will be discussed in section IV).

In summary, evaluating every investment, before and after being made, is very crucial for organizations in determining successful decision that has or will be set [13].

In this context, vast majority of IT evaluation studies contributed to IT investment evaluation literature in project selection stage. Numerous analytical techniques ranging from simple weighted scoring to complex mathematical programming approaches have been proposed to solve effective project evaluation and selection problems [17].

Optimization and decision-making problems for complex systems are traditionally solved using either the deterministic numerical approach (which provides an approximate solution that totally ignores uncertainty) or the probabilistic approach (assumes that any uncertainty can be represented as a probability distribution). Both approaches only partly take into account the factor of uncertainty which actually exists in the form of known possibility distributions [18].

Early attempts focused on constrained optimization methods were given a set of candidate projects; the goal is to select a subset of projects to maximize some objective function without violating the constraints [17]. Some prior studies have considered the interaction among projects in deterministic environments [17]. Others have dealt with stochastic environments but have not considered project interdependencies [17].

The investment decision-making process is influenced by various uncertainty factors as well as by the need to formalize and process fuzzy, insufficient and, mostly, expert data. Fuzzy statistics, possibility approaches and methods of fuzzy optimization are widely used when solving problems by modeling optimal decisions and especially when constructing decision support systems [18].

4. IT evaluation methodologies

Numerous studies contributed to IT evaluation literature. IT evaluation methodologies can be categorized from different points of view. Follows brief discussion of these classifications. [1]

IT investment evaluation methods are diversified and classified into four

[1] Appendix II summarizes classification of IT Evaluation methods and underling Methodologies by number of IT evaluation studies

main categories in [13]: financial methods, Operation method/management science methods, Technique specifically designed for particular IT and/or organizations, and other methods (i.e. Balanced Scorecard, Critical Success Factors, and Value Chain Analysis).

In [19], four basic approaches of IT investment evaluation are discerned: Financial approach, Multi criteria approach, Ratio approach, and Portfolio approach.

Based on a classification of IT evaluation techniques in [20] into four groups (Economic, Strategic, Analytic, and Integrated approaches), [21] further proposed a sub classification of two of these groups. On one hand, [21] discriminate between three types of economic approaches; Ratio-based, Discounting techniques, and Future value techniques. On the other hand, [21] discriminate between two types of analytic approaches; Portfolio analytic approaches, and analytic approaches other than Portfolio approaches which include Risk and Value Analysis.

Similarly, [22] identified major categories of IT/IS projects evaluation and justification Tools and Techniques which are: Economic approaches, strategic approaches, Analytical Hierarchy Process (AHP), Data Envelopment Analysis (DEA), expert systems, goal programming, multi-attribute utility theory (MAUT), simulation, and scoring models [22].

While some approaches can be clearly assigned to one particular dimension, others combine characteristics of several dimensions (e.g., the BSC method offers multi-criteria measures to support strategic planning) [23]. Thus, most frequently used IT evaluation methods are clustered by [23] in three major dimensions; Financial, Multi-criteria, and Strategic. Some others lie in-between these dimensions.

All of these mentioned above methodologies deal with IT evaluation in practice. None of them reached theoretical level of classification. Upon classifying IT investment evaluation methodologies, from a theoretical point of view, [4] addressed three major categories in IT investment evaluation literature. First category, The microeconomics-based view (searches to show whether IT investments create excess return over other types of capital investment or other production factors) and the process-based view (is based on the idea that IT investments can create value by improving operational efficiency which can lead to better firm-level performance). Second category, the resource-based view (the fact that resources, per se, do not create value; value is created by an organization's ability to utilize and mobilize those resources) and the real-option view (is based on the real-options approach and values the inherent flexibility of many IT projects). Third category, the pragmatic-justification view (uses a wider definition of value and takes into account the organizational and political context, with the need to conform to social norms of acceptable behavior and legitimacy).

According to [4], the microeconomics-based view includes three approaches; Productivity approach, financial performance approach, and firm value approach. The process-based view studies the process of value creation; i.e. tries to understand the way the value is created [4]. On the other hand, the resource-based view and composite scorecards includes two approaches; Information Economics, and Balanced Scorecard (BSC).

In the next section IT evaluation methods will be discussed in brief.

5. IT evaluation methods and techniques

Methods and techniques have been suggested over the years to evaluate IT and IS investments. Many studies addressed IT evaluation techniques. As [4] claims, at least 200 techniques are documented (mostly composite techniques) in IT evaluation literature. Early investments evaluation methods emphasize the use of accounting indicators such as payback period and Discounted Cash Flow (DCF) methods, whereas later methods link the decision making process more strategically [15]. These traditional methods focus on well-known financial measures [2], [5]. DCF approaches are the most cited financial approaches [23]. Most Popular IT evaluation methods are addressed below.

a. Productivity

Productivity is the ratio of output value to its related input value [24]. Productivity is measured in various ways, some studies have used a single production function to try to find a direct correlation between IT spending and firm performance [4]. In their model [24], consider three measures of productivity, which they term output productivity, firm productivity, and social productivity. Output productivity is defined as the ratio of the number of products produced to its related input value. Firm productivity is defined as the ratio of output value to the firm to its related input value. Finally, social productivity is defined as the ratio of social output value (i.e., the sum of firm revenues and consumer surplus) to its related input value [24]. This view of IT value is that IT can be treated as an input in the production function of a firm and that there is a substitution effect between IT and other production factors [4].

b. Payback Period (PP)

Payback Period projects are judged on the period needed to compensate the initial investment [25].

c. Net Present Value (NPV)

Net Present Value calculates the present value of the investment's money flows, using a discount rate [25].

d. Internal Rate of Return (IRR)

Internal Rate of Return takes the time value of money into consideration by introducing a discount factor, but risks are not accounted for in mutual exclusive investment projects [25].

e. Return On Investment (ROI)

Return On Investment is a well-known representative of financial measures ratios, which are often used for comparisons or benchmarking [23]. In ROI, Total lifecycle of investment is taken into account. While time value of money is not taken into consideration, Risk can be entered into the appraisal to a certain extent [25]. A further development addressed by [26] is the Social Return On Investment (SROI) which includes three types of return: Economic returns (all literally financial returns that a certain project or organization creates); Socio-economic returns (which are savings of the state (society) realized through avoidance of public transfers (e.g. to jobless people) as well as the associated increase in personal income and tax revenues through consumption); and Social returns (which are less-tangible effects such as an increased sense of self-esteem and personal independence as well as the enhancement of knowledge and skill levels) [26].

f. The Return on Management (ROM)

The Return on Management presupposes that in today's information economy management has become the scarce resource. In the ROM method the value added by management is related to the costs of management [19].

g. Information Economics

Information Economics is sometimes called the enhanced return on investment (ROI). The ROI not only looks at cash flows, arising from cost reduction and cost avoidance, but also provides some additional techniques to estimate incoming cash flows: value linking; value acceleration; value restructuring; innovation valuation [19]. Cumulative scores are calculated by multiplying the weighted criteria. Adding value from other departments, improved operations, increased productivity, and innovative investment aspects extend cash flows [23].

h. SIESTA 'Strategic Investment Evaluation and Selection Tool Amsterdam'

The evaluation criteria are deduced from a model, in which a distinction is made between the business and the technological domain and three levels of decision-making are discerned. Benefit and risk criteria are deduced from the extent into which the different elements of the model fit [19]. This method is a multi-criteria method, using extensive questionnaires [23].

i. IT assessment

IT assessment deals with the evaluation of information technology (IT) effectiveness from a strategic point of view. An important part of the method focuses on the analysis of financial and non-financial ratio's [19].

j. Bedell's method

The central premise of Bedell's method is that a balance is needed between quality and importance [19], [23]. The prioritization of investment proposals is carried out by calculating the contribution of each information system. The contribution of an IS is defined as the importance of the system multiplied with the improvement of quality after development [19].

k. Investment Portfolio

The portfolio serves as a framework to make the preferences of the different stakeholders explicit and debatable [19], where a defined set of criteria is used to appraise IT in multi-criteria approaches, whereas single criteria are rated for quantification [23]. This method evaluates IS investment proposals on three criteria simultaneously, which are [19]:

- The contribution to the business domain;
- The contribution to the technology domain; and
- The financial consequences, by means of net present value (NPV) calculation.

l. Investment Mapping

Investment proposals are plotted against two main evaluation criteria: the investment orientation and the benefits of the investment. The investment orientation is broken up into infrastructure, business operations and market influencing. The benefits are broken up into enhancing productivity, risk minimization and business expansion [19].

m. Decision Tree Analysis

Decision Tree Analysis can be used for decision-making in conditions of uncertainty. It enables decision makers to break down a large, complex decision-making problem into several smaller issues [23].

n. SWOT analysis

SWOT analysis is used to find the strengths and weaknesses of the organization, as well as the opportunities and threats present in the (future) environment [23].

o. Data Envelopment Analysis (DEA)

Data Envelopment Analysis is a widely used mathematical programming approach for comparing the inputs and outputs of a set of homogenous Decision Making Units (DMUs) by evaluating their relative efficiency [17]. A fuzzy variable is a variable with an imprecise value, as opposed to a variable with an exact value. Fuzzy Data Envelopment Analysis (FDEA) is a method for measuring and comparing the attractiveness of a set of alternatives under condition of uncertainty [17].

p. Analytical Hierarchy Process (AHP)

Analytical Hierarchy Process is a multi-objective, multi criteria decision-making approach that employs a pair-wise comparison procedure to arrive at a scale of preferences among a set of alternatives. AHP is based on a hierarchical structuring of the criteria that are involved in a decision problem. The hierarchy incorporates the knowledge, the experience and the intuition of the decision-maker for the specific issue [27].

q. Multi-criteria method

Multi-criteria method solves the problem of comparing different consequences on an equal basis by creating one single measure for each investment [19]. Before using a multi-criteria method, a number of goals or decision criteria have to be designed. Subsequently scores have to be assigned to each criterion for each alternative considered. Also the relative importance of each alternative should be established, by means of weights. The final score of an alternative is calculated by multiplying the scores on the different decision criteria with the assigned weights [19].

r. Real Options

An option is the right to do something, without the obligation to do it [4]. A Real option is used in IT evaluation as the initial phase of an investment project is implicitly equivalent to buying an option [8], [28] where the implementation of new architectures can be handled in a conditional way by establishing flexible growth paths to take advantage of increasing returns (in case of wide adoption) and network effects [4].

s. Value-at-risk

Value-at-risk indicates the greatest potential loss of a position or a portfolio, which can be verified with a certain probability, in a defined time horizon [29].

t. Balanced Scorecard

Balanced Scorecard (IT BSC) measures the performance of the firm from four different dimensions. (1) User Orientation: How do Users see us? (2) Business Value: How do we look to shareholders? (3) Internal Processes: What must we excel at? (4) Future Readiness: Can we continue to improve and create value? [30], [5]. Balanced Scorecard uses in IT field are not limited to IT evaluation, It is used to IT Management and IT Governance as well [31].

Based on the foregoing, the exploratory study methodology will be discussed in the next section.

6. Methodology

An exploratory study was conducted based on questionnaire includes questions to be answered by a number of listed corporate employees working in Egypt as follows:

(i) Methods explored

This paper explores IT investment evaluation practices in Egypt using most popular IT evaluation methods represented by the following methods:

Balanced scorecard

Balanced scorecard (IT BSC) includes four different dimensions or perspectives; User Orientation, Business Value, Internal Processes, and Future Readiness.

Financial perspective of IT BSC

Financial dimension involve number of traditional financial metrics. These metrics include Net present Value (NPV), Pay Back Method (PB), Internal Rate of Return (IRR), and Return On Investment (ROI). For evaluation purposes, organizations choose one or even a combination of those measures into their evaluation metrics.

Non financial dimensions of IT BSC include:

1. User Orientation Perspective
2. Internal Process Perspective
3. Learning and Innovation (Future Readiness) Perspective

Non traditional methods include:

1. Value-at-Risk Approach
2. Real Option Approach

(ii)Study population

Study population consists of three sub-samples (IT investment Decision maker - System User - Decision maker who has authority to use system) which take into account the demographic characteristics of participants as possible. Each sub-sample consists of a number of observations (individual participant), both from senior management (Decision maker who has authority to use system - Decision maker) or Finance departments and financial sector staff (System User) within a number of companies (private and public sector) operating in Egypt listed in the Egyptian Stock Exchange. The researcher selected companies listed in the Stock Exchange for many reasons, the most important one is those companies are subject to SEC control in terms of binding standards for registration in the stock market. Also these companies are characterized by, relatively large size, which helps on the existence of an integrated organizational structure. Furthermore, these companies are managed by professional staff and specialists, both academically and practically qualified.

(iii)Data collection:

Based on five ranks Likert scale, an ordinal scales questionnaire was developed to perform data collection. This questionnaire includes three categories of questions; the first includes questions to investigate how to make an IT investment decision in Egypt (Actual Settings). The second includes questions to investigate the subjective opinions of respondents in general about how best to make such decisions (Self-reported Settings). This category represents respondents' point of view by giving weights for the investigated items. This examines the extent to which these elements are taken into consideration when making such decisions. Finally, third category includes general questions about respondents, representing individual and demographic characteristics.

(iv) Data analysis:

To determine the possibility of generalization of results, Researcher used three levels of data analysis to analyze Responses of respondents to questionnaire items. The first level depends on methods and measures of descriptive statistics to identify data distribution in question. Perhaps the most important finding of this level of analysis is to identify frequencies of individuals' responses to each item. The second level of analysis depends on statistical inference methods to determine the extent to which findings of first level of analysis can be generalized. Finally, the third level of analysis is a further analysis (between groups) to determine the differences between sub-samples, in addition to analyzing the differences between the Actual (What applied in companies in reality) and Self-reported (Respondent's subjective personal views) responses.

Second and third levels of analysis rely on nonparametric statistical inference methods for a number of reasons. Most important of them is that [32] showed that many studies warned against application of parametric tests in empirical research in IS and IT field (AI). Further more, Reference [33] explained that, Kruskal-Wallis nonparametric test is most useful tests available as statistical hypothesis testing for behavioral and social research to test differences between at least three sub-samples. Thus current study uses nonparametric statistical inference methods.

7. Results

The following results had emerged from the statistical tests of the exploratory study [I]:

- The results of descriptive statistics of actual settings for Decision maker who has authority to use the system sample (Cs) show that traditional IT BSC dimensions gained highest frequencies followed by NPV. However, NPV gained highest frequencies for Decision Makers sample (Ds) followed by Value-at-Risk. On the other hand, ROI gained highest frequencies for User sample (Us) followed by other IT BSC dimensions. One can interpret this as NPV is still a widespread measure among financial measures. Also, both Decision makers and Users are aware of recent trends of IT evaluation.
- The results of descriptive statistics of Self-reported settings show that traditional IT BSC dimensions gained highest frequencies followed by ROI for Both User sample (Us) and Decision maker who has authority to use the system sample (Cs). However, NPV and IRR gained equal frequencies for Decision Makers sample (Ds). Again, one can interpret this as Both User sample (Us) and Decision maker who has authority to

[I] Appendix I summarizes the results of statistical tests of actual and Subjective (personal self-reported) opinion among each of the three samples.

use the system sample (Cs) are aware of recent trends of IT evaluation compared to Decision Makers sample (Ds).

- The results of statistical inference show possibility of generalization of findings for different elements. This applies for Decision maker who has authority to use the system sample (Cs), User sample (Us). This doesn't apply for Decision Makers sample (Ds) where frequencies of many of elements were not significant.
- There is no significant differences between the three samples in Actual Settings (what companies apply in practice), however if there are differences they are not significant. The possible reason for these differences is that frequencies of Decision Makers Sample (Ds) are the lowest for most variables among other samples. This result can be explained with respect to Decision Makers Sample, as they are representatives of the decision-making process in companies that they work for. This confirms the conclusion of existence of significant differences between the subjective opinions of the three sub-samples. Again, the reason for these differences is that frequencies of Decision Makers Sample (Ds) are the lowest for most variables among other samples in Self-reported settings.
- There are significant differences between what companies actually apply in practice (Real settings) and what should be applied according to individuals' opinion (Self-reported settings) for Decision Makers sample (Ds). The reason for these differences is that frequencies of Decision Makers Sample (Ds) are the greatest for most variables in the Actual settings compared to Self-reported settings. This applies to Decision Maker who has authority to use the system sample (Cs), but the differences between actual and Self-reported settings are nonsignificant. The researcher believes that the interpretation of this result may be accounted for the conservative nature of the decision makers (both samples include decision makers), particularly most of them are risk-averse according to descriptive statistics results.

In summary, the overall results were consistent to a great extent with what proposed by literature in this regard. The reasons for obtaining different results from expected, on one hand Sample-related reasons; that is small sample size of decision makers, as well as composition of each sample. On the other hand, Environment related reasons; that is, convergence of cultural values within the same society (exploratory study applied in Egypt).

8. Concluding remarks

To recognize state of the art of IT investment decisions evaluation in Egypt, This paper explores IT investment evaluation practices in Egypt using most popular IT evaluation methods.

IT evaluation literature was reviewed. An exploratory study was conducted based on Questionnaires distributed into three categories of respondents (User of the system – Decision maker in investments in the system – Decision maker who has authority to use system) who worked in a number of listed companies operating in Egypt.

Empirical findings are in general so far consistent with findings of related literature. Results indicate that among financial measures, NPV and ROI are still widespread measures of IT value. Also, both Decision makers and Users are aware of recent trends of IT evaluation methodologies.

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APPENDIX I

STATISTICAL ANALYSIS RESULTS [a]

Constructs	Cs Actual Settings	Ds Actual Settings	Us Actual Settings	Cs Self-reported Settings	Ds Self-reported Settings	Us Self-reported Settings
A. IT BSC Financial Dimensions						
Net present Value (NPV)	81.82 %	100.00 %	88.37 %	24.49 %	33.33 %	22.22 %
Bay Back Method (BP)	70.74 %	66.67 % [b]	71.80 %	18.37 %		17.78 %
Internal Rate of Return (IRR)	47.21 % [b]	100.00 % [b]	78.38 %	2.04 %	33.33 %	2.22 %
Return On Investment (ROI)	79.54 %	75.00 % [b]	74.12 %	32.65 %		35.56 %
B. IT BSC Non Financial Dimensions						
User Perspective	83.02 %	80.00 % [b]	91.22 %	70.59 %	50.00 % [b]	78.94 %
Learning and Innovation Perspective	79.25 %	25.00 % [b]	87.72 %	74.51 %	60.00 % [b]	82.46 %
Internal Process Perspective	94.00 %	75.00 % [b]	94.74 %	83.33 %	50.00 % [b]	73.21 %
C. Other IT Evaluation Approaches						
Value-at-Risk Approach	62.00 %	100.00 %	78.95 %	60.00 %	80.00 %	73.22 %
Real Option Approach	81.14 %	50.00 % [b]	75.00 %	80.77 %	50.00 % [b]	69.64 %

[a] Columns 2-6 represent Descriptive Statistics Results (respond percentage for a given construct).
[b] Results are Nonsignificant at P-value = 0.05 according to Wilcoxon Signed Rank test.

APPENDIX II

IT INVESTMENT EVALUATION METHODS AND METHODOLOGIES [a]

IT Evaluation Methods	Financial Approaches	Multi-criteria Approaches	Ratio Approaches	Portfolio Approaches	Strategic Approaches	Strategic/ Multi-criteria	Strategic/ Financial	Financial/ Multi-criteria	Economic (Ratio based)	Economic (Discounting)	Economic (Future Value)	Analytic Approach	Integrated Approaches
Discounted Cash Flow (DCF) [B]	[19], [23]									[21]			
Bay Back Method (BP)	[19]								[21]				
Return On Investment (ROI)	[23]								[21]				
Return On Management (ROM)			[19]					[23]					
Information Economics(IE)		[19]						[23]					[21]
SIESTA		[19], [23]											
IT Assessment			[19]										
Bedell's method (B's)				[19]				[23]					
Investment Portfolio				[19]				[23]					
Investment Mapping				[19]				[23]					
Critical Success Factors (CSF)					[21], [23]								
SWOT					[23]								
Scenario Planning					[23]								[21]
Total Cost of Ownership							[23]						
Cost-Benefit Analysis							[23]		[21]				
Decision Trees							[23]						
Real Option Approach							[23]				[21]		
Balanced Scorecard (BSC)						[23]							[21]
Analytic Hierarchy Process												[21]	
Multi-Attribute Utility													[21]
Fuzzy Logic												[21]	

[a] Rows represent IT investment evaluation methods while Columns represent IT investment evaluation categories. Numbers in brackets represent reference

[b] Net present Value (NPV) and Internal Rate of Return (IRR) are Discounted Cash Flow (DCF) methods.

An Artificial Immune System for Solving the Assembly Line Balancing Problem

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Abstract

An assembly line is a flow-oriented production system in which the productive units performing the operations are aligned in a serial manner. The simple assembly line balancing problem is a fundamental version of the general problem which has attracted attention of researchers and practitioners of operations research for almost half a century. In this paper an artificial immune system is developed to minimize the number of stations with a given cycle time, which is the type-1 of the simple assembly line balancing problem. To validate the output, the results are compared with 99 selected sample problems from a well-known benchmark. After comparing the results with the output of Scholl and Klein (1999) branch and bound approach and the bi-directional based on CPM heuristic approach which produced by (Yeh and Kao, 2009), only 12 solutions are worse than Scholl and Klein (1999) approach and 3 solutions are worse than Yeh and Kao (2009) approach. There are also 4 solutions better than Yeh and Kao (2009) approach.

Keywords: Assembly line balancing problem, Convergence, Artificial immune

1. Introduction

Assembly lines used extensively in mass production systems to increase systems' efficiency and speed, to decrease per-unit cost, and to make the manufacture and control easier. An assembly line is a collection of sequential stations typically connected by a continuous material-handling system. The work pieces visit stations successively as they carried along the line usually by some kind of transportation system (Mozdgir et al., 2013). Scholl and Becker (2006) define the assembly line balancing problem (ALBP) as the decision problem of optimally partitioning the assembly work among the stations with respect to some objectives. Numerous approaches developed for solving the ALBP which have proven useful in various assembly industries such as manufacturing, automobiles, consumer electronics, etc. In the traditional ALBP, a set of tasks is given along with their processing times and precedence relationships that define the permissible ordering of tasks. The objective of the ALBP is to assign the set of tasks to successive stations in order to meet specified production requirement (Yeh and Kao, 2009).The objective of this paper is to develop an artificial immune system (AIS) for solving the simple assembly line balancing problem (SALBP). The paper organized as follows; Section 2 assigned to describe and formulate the problem of assembly line balancing. In Section 3, we discussed some literature that solves computational problems by using AIS and demonstrated some literature that presents algorithms for AIS. The proposed immune system presented in section 4; followed by section 5, which presents the computational results compared with

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solutions of some problems taken from a well-known benchmark; eventually the conclusion and future research addressed in last section.

2. The Assembly Line Balancing Problem

According to the underlying concept of any SALB formulation, an assembly line consists of $k = \{1, \ldots, m\}$ stations arranged along a conveyor belt or a similar mechanical material handling device. The jobs consecutively launched down the line and hence moved on from station to station until they reach the end of the line. A certain set of operations performed repeatedly on any job that enters a station, whereby the time span between two entries referred to as cycle time. In general, the line-balancing problem consists of optimally partitioning the assembly work among all stations with respect to some objective. For this purpose, the total amount of work necessary to assemble a job is split up into a set $V = \{1, \ldots, n\}$ of elementary operations named tasks. Tasks are indivisible units of work and thus each task j is associated with a processing time t_j also referred to as task time. Due to technological and/or organizational requirements, tasks cannot be carrying out in an arbitrary sequence, but are subject to precedence constraints (Boysen et al., 2007).

Type 1 simple assembly line balancing problem (SALBP-1), which is addressed in this work, may be stated in the following terms: given a set of tasks, each has to be assigned to exactly one station without exceeding a given cycle time, so that the number of used stations is minimized, while satisfying the precedence constraints among tasks (Fleszar and Hindi, 2003).The problem formulated as follows (Wei and Chao, 2011):

Notations:

n	Number of tasks ($i = 1, \ldots, n$)
m	Number of stations ($j = 1, \ldots, m$)
t_i	Operation time of task i
ct	Cycle time
p	Subset of task (i, k), given the direct precedence relations
m^*	Minimal number of stations

Decision variable:

$x_{ij} \in \{0,1\}$ 1 if task i is assigned to station j; 0 otherwise

The original SALBP-1 formulation is as follow (Wei and Chao, 2011):

$$\text{Minimize } Z = m^* \qquad (1)$$

Such that

$$\sum_{j=1}^{m^*} x_{ij} = 1, \forall i = 1, \dots, n \qquad (2)$$

$$\sum_{i=1}^{n} t_i \, . \, x_{ij} \le ct \,, \forall j = 1, \dots, m^* \qquad (3)$$

$$\sum_{j}^{m^*} j \, . \, x_{ij} \le \sum_{j}^{m^*} j \, . \, x_{kj} \,, \forall (i,k) \in p \qquad (4)$$

$$x_{ij} \in \{0,1\} \qquad (5)$$

Constraints (2) simply say that all tasks have to be assigned, and each task has to be assigned to one and only one station. Constraints (3) require that for any station, the total processing time of tasks assigned must not exceed the cycle time, while constraints (4) ensure that precedence relationships between tasks must be maintained. The objective is resolving the issue of task assignment by using the minimum number of stations.

3. The Artificial Immune System

AIS are a field of study focusing on the development of computational models based on the principles of the biological immune system. It is an emerging field that studies and uses different immunological mechanisms to solve computational problems (Al-Anzi and Allahverdi, 2013).AIS covers huge areas of complex computational and engineering problems such as pattern recognition, fault and anomaly detection, data mining and classification, scheduling, machine learning, autonomous navigation, robot navigation, clustering/classification, bio-informatics, image processing, numeric function optimization, and computer security (Diabat et al., 2013).

There are different algorithms for AIS. Forrest et al. (1994) proposed a negative selection algorithm. Hunt et al. (1996, 2002) developed the theory of cloning selection. Chun et al. (1997) presented an immune genetic algorithm (IGA), which is an integration of a genetic algorithm and an immune algorithm that increases the diversity of the chromosomes. Castro and Zuben (2000, 2002) suggested a cloning selection algorithm based on an antibody cloning selection mechanism. Castro and Zuben (2001) presented an artificial immune network for data analysis.

3.1. The Clonal Selection

Clonal selection is a widely accepted theory for modeling the immune system responses to infection. The theory of clonal selection was proposed by Burnet (1959). It is based on the concept of cloning and *affinity maturation*. B and T lymphocytes are selected to destruct specific antigens invading the body. When the body is exposed to an exogenous antigen, those B cells that best bind the antigen proliferate by cloning. The B cells are programmed to clone a specific type of antibodies (Ab) that will bind to a specific antigen (Ag). The binding between Ag and Ab is governed by how well a paratope on Ab matches an epitope of Ag (Figure1). The closer is this match, the stronger is the bind. This property is called *affinity*. Some of the cloned cells are differentiated into *plasma cells* that are antibody secretors. The other cloned cells become *memory cells*. A high-rate *somatic mutation* (hypermutation) is applied to the cloned cells that promote a genetic diversity. Moreover, a selection pressure is performed, which implies a survival of the cells with higher affinity. Let us notice that clonal selection is a kind of an evolutionary process. Clonal selection immune algorithms may be viewed as a class of evolutionary algorithms that are inspired by the immune system.

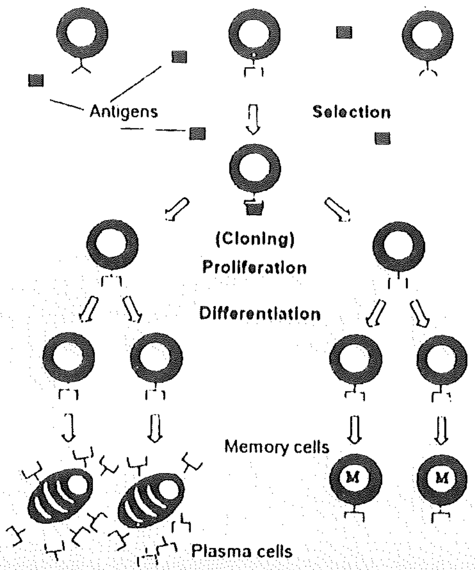

Figure 1: The clonal selection principle (Castro and Zuben, 2000).

4. The Proposed Model

In this section according to ALBP and its constraints, we will discuss how to construct the initial population of antibodies, the memory of the search, the affinity function, the cloning and hyper-mutation, the replacement of the worst member of antibodies, the proposed algorithm, and illustrative example.

4.1 The Initial Population

The proposed seed of the population of antibodies is based on generating an initial solution by using the bi-directional procedure which is mentioned by Sholl and Voβ (1996), but we use the positional weight priority role which is mentioned by Ponnambalam et al. (1999) rather than using the dynamic priority role. The remaining antibodies of the initial population will be generated by using a random number of task swapping and shifting. The boundaries of swapping tasks depend on determining the boundaries of shifting procedure. Pape (2015) refers to these boundaries as the following:

1- The last station to which any task of the immediate predecessors of a selected task j is assigned to and the earliest station to which any task of immediate successors of j is assigned to set the boundaries within which task j can be shifted.
2- Two tasks can be swapped if each is within the shifting boundaries of the other one.

The seed will be mutated by shifting or swapping a random number of tasks n times to produce the next antibody in the population which is also considered as the next seed. The process will continue producing new antibodies until the population size is reached.

4.2 The Memory Of The Search

The memory of the search helped to discard the antibodies which are previously selected for the cloning process from being selected again. So each antibody included in the population will be tested if it is included in the memory of the search or not. If the antibody isn't included in the memory of the search, it will be ready for the affinity calculation.

4.3 The Affinity Function

The calculations of the affinity function (*AF*) between each two antibodies are applied to extract the more diversified antibodies from the population to be prepared for the cloning and hyper-mutation process. Two criteria are used to determine the similarity or the affinity between antibodies. The first criterion is the smoothness index (*SI*) of the antibody and the second one is based on the solution structure. The smoothness index shows how the solution is balanced with respect to the idle time. We choose smoothness index criterion because most the solutions of the population have the same number of stations, but the smoothness index mostly differs. The smoothness index formula found in (Mozdgir et al., 2013) as the following equation:

$$SI = \sqrt{\sum_{k}^{m} (ct - ts_k)^2} \text{ , where } ts_k \text{ represents station time} \qquad (6)$$

The solution structure can be represented as the set of all ordered pairs of tasks and their stations. If we refer to the disputed ordered pairs, which is the number of the ordered pairs of the relative complement of the solution structure *i* in the solution structure *j*, as ((DO_{ij})), the proposed affinity function for antibody *i*can be formulated as follows:

$$AF_i = \frac{1}{SI_i} + \max_{j \neq i}(DO_{ij}) \qquad (7)$$

The affinity function for each antibody that is not included in the memory of the search has to be calculated and the higher affinity antibodies will be selected for the cloning and hyper-mutation process.

4.4 The Cloning and Hyper-mutation

Each antibody is cloned number of times, determined by the Cloning Number (N_c). The clones are then mutated to get new antibodies that are different from their parents. Castro and Zuben (2002) illustrated the formula which calculates the amount of clones generated for all *n* selected antibodies as the following:

$$N_c = \sum_{i=1}^{n} round\left(\frac{\beta . N}{i}\right) \qquad (8)$$

Where N_c is the total amount of clones generated for each of the antigens, β is a multiplying factor, N is the total amount of antibodies and round(.) is the operator that rounds its argument towards the closest integer. Each term of this sum corresponds to the clone size of each selected antibody, e.g., for N = 100 and β = 1, the highest affinity antibody (i = 1) will produce 100 clones, while the second highest affinity antibody produces 50 clones, and so on.

Each cloned antibody will be mutated by using either shifting tasks from stations or shifting tasks to stations or swapping.

4.5 The Replacement of the Worst Member of Antibodies

After the cloning and hyper-mutation process has been completed, the bad antibodies, which have higher number of stations and higher smoothness index, should be replaced by new random antibodies. The proposed seed of the new antibodies after removing the worst ones is the antibody which has the highest smoothness index of the cloned antibodies.The remaining antibodies of the population after removing the worst ones will be generated as the same manner of generating antibodies of the initial population.

4.6 The Proposed Algorithm

Step 1: Generate (P) initial population which has n antibodies.
Step 2: Create the memory set (M) = Φ.
Step 3: Create the elitism set (E) = Φ.
Step 4: Initialize loop counter (k) = 1 and the maximum number of iterations ($k_{\max}$) = m.
Step 5: Create a set of antibodies (P_n) which are not included in M.
Step 6: Calculate AF_i for each antibody in P_n.
Step 7: Select N_c antibodies of P_n which have the highest affinity to create the set (P_c) and Store the solution structure of each element in P_c in M.
Step 8: Clone and hyper-mutate each antibody in P_c to produce (C) cloned antibodies and append the antibodies (C*), which have the minimum number of stations and the lower smoothness index, to P.
Step 9: Remove r antibodies which are the worst antibodies in P.
Step 10: Select E by comparing the evaluation of E with the evaluation of the best antibody in P and E will be the best one of them.
Step 11: Select the antibody (A_s) which has the highest smoothness index to be the seed of the remaining antibodies to complete P for reaching n antibodies by using various swapping and shifting procedures.
Step 12: If $k = k_{\max}$ then go to Step 14.
Step 13: $k = k + 1$ and go to Step 5.
Step 14: The best solution found is E.

Figure 2 shows the flowchart of the proposed algorithm.

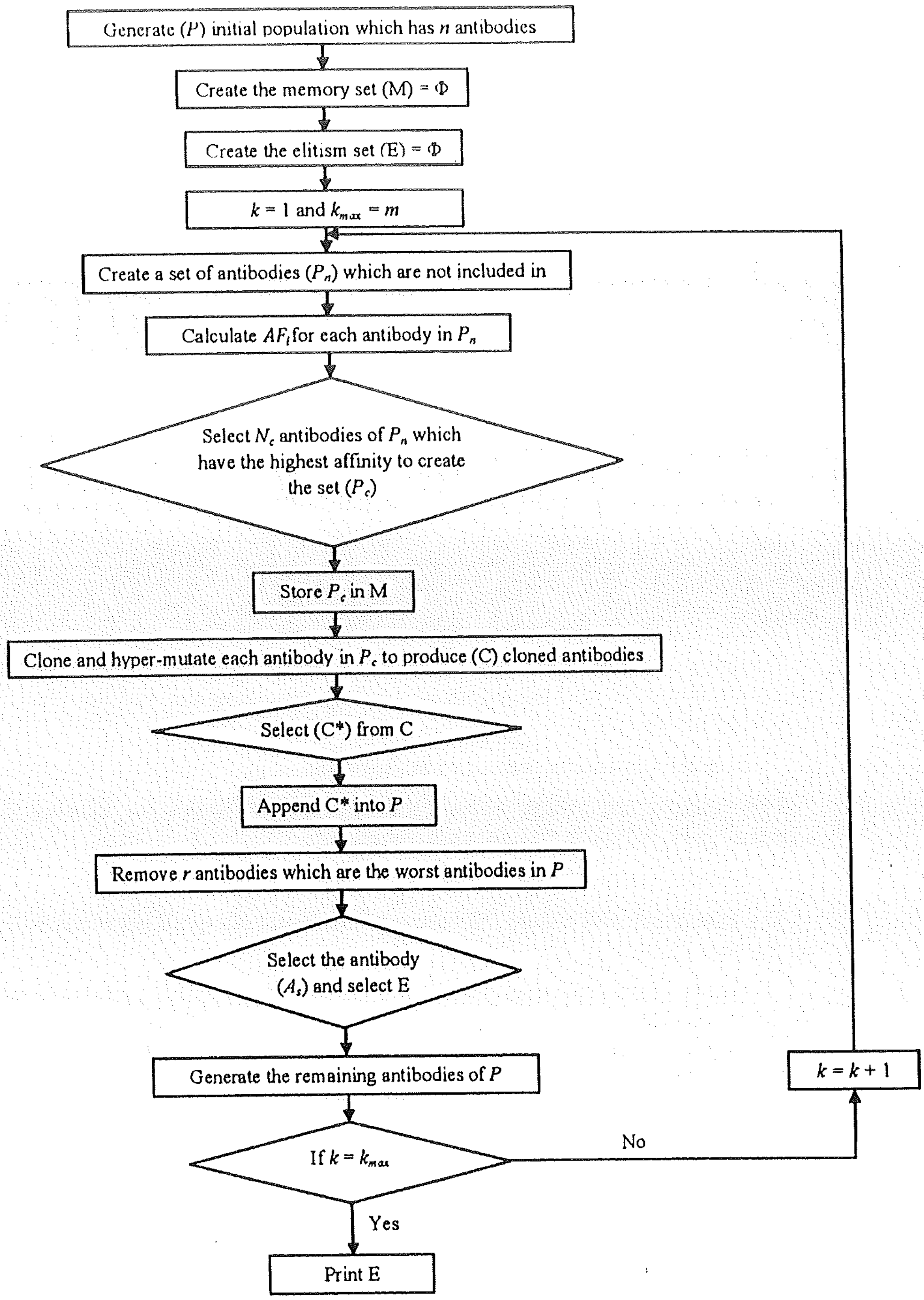

Figure 2 the flowchart of the proposed AIS algorithm

4.7 Illustrative example

We select the illustrative example which is found in Yeh and Kao (2009) to solve it by the proposed AIS approach. The cycle time = 14 and the precedence graph of such example can be summarized as the following table:

Table 1 initial data

Task	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
Processing time	4	3	9	5	9	4	8	7	5	1	3	1	5	3	5	3	13	5	2	3	7
Pre.1		1	1	3	4	5	5	6	8	9	9	9	9	9	10	15	13	13	14	17	2
Pre.2								7							11		16	15	18		4
Pre.															12						

Step1: Generate (P) initial population which has n antibodies.

As we mentioned about the initial seed, we apply the bi-directional procedure based on the positional weight priority rule and the solution as in Table 2:

Table 2 the solution found by bi-directional procedure based on positional weight

Task	1	3	4	5	7	6	8	9	2	11	12	21	16	15	13	10	17	20	19	18	14
Processing time	4	9	5	9	8	4	7	5	3	3	1	7	3	5	5	1	13	3	2	5	3
Station	1	1	2	2	3	3	4	4	5	6	6	6	7	7	7	7	8	9	9	9	9

The proposed population size n here is 10 antibodies. After generating the initial antibody, another 9 antibodies will be generated by using swapping and shifting mutations to generate the remaining antibodies of P.

Step2: Create the memory set (M) = Φ.

The memory set, which discard the selected antibodies for cloning process from being selected again, will be initialized as an empty set.

Step3: Create the elitism set (E) = Φ.

The elitism set which store the best solution found along all iterations will be initialized as an empty set.

Step4: **Initialize loop counter (k) = 1 and the maximum number of iterations (k_{max}) =** $m = 2$.

Step5: Create a set of antibodies P_n which are not included in M.

P_n set will be initialized to have all the antibodies included in the initial population and in the next iteration it will be varied depend on the included antibodies inside the memory set M.

Step6: Calculate AF_i for each antibody in P_n.

Table 3 illustrates the output of the affinity function of the 10 antibodies sorted in descending order according to their affinity value:

Table 3 Calculate the affinity function for each antibody in the initial population

Antibody	2	4	5	6	7	8	9	10	1	3
Disputed ordered pairs $max(DO_{ij})$	7	7	7	7	7	7	6	6	6	5
Smoothness index (SI_i)	9.85	10.54	10.63	10.63	11.7	11.87	11.87	12.04	9.22	9.85
Affinity function $AF_i = \frac{1}{SI_i} + max(DO_{ij})$	7.101	7.095	7.094	7.094	7.085	7.084	6.1085	6.084	6.083	5.101

Step7: Select N_c antibodies of P_n which have the highest affinity to create the set (P_c) and Store the solution structure of each element in P_c in M.

Step 7 is to select the first 3 antibodies in Table 3,$N_c = 3$, to compose $P_c$to be ready for the cloning and hyper-mutation process. The 3 antibodies will be stored in the memory M to discard them from being cloned again in the next iterations. The included antibodies' structures in M at the first iteration are:

Antibody (1) =
{(1,1),(2,6),(3,1),(4,2),(5,2),(6,3),(7,3),(8,4),(9,5),(10,5),(11,7),(12,5),(13,5),(14,9),(15,7),(16,7),(17,8),(18,9),(19,9),(20,9),(21,6)}

Antibody (3) =
{(1,1),(2,6),(3,1),(4,2),(5,2),(6,3),(7,3),(8,4),(9,5),(10,6),(11,5),(12,5),(13,7),(14,6),(15,7),(16,7),(17,8),(18,9),(19,9),(20,9),(21,6)}

Antibody (2) =
{(1,1),(2,5),(3,1),(4,2),(5,2),(6,3),(7,3),(8,4),(9,4),(10,5),(11,5),(12,4),(13,5),(14,9),(15,7),(16,7),(17,8),(18,9),(19,9),(20,9),(21,6)}

Step8: Clone and hyper-mutate each antibody in P_c to produce (C) cloned antibodies and append the antibodies C*, which have the minimum number of stations and the lower smoothness index, to P.

The number of cloned antibodies C which produced from each antibody in P_c is 18, 8, and 3 respectively. These numbers are calculated according to the formula (8). After applying hyper-mutation on the cloned antibodies, the evaluation for each cloned antibody has to be done. Table 4 illustrates the evaluation of the best 14 antibodies included in C sorted in ascending order:

Table 4 the evaluation of the best 14 cloned antibodies

Antibody	6	7	2	14	5	4	9	10	1	13	3	12	8	11
No. of stations	8	8	9	9	9	9	9	9	9	9	9	9	9	9
Smoothness index	3.3	3.3	8.1	8.6	9	9.1	9.3	9.3	9.4	9.4	9.5	9.5	10.0	12.1

After evaluating the cloned antibodies, we select the first 7 best antibodies, C*, in Table 4 and append them to P. Table 5shows the evaluation of the current antibodies in P sorted in ascending order:

Table 5 the evaluation of the population after appending the best cloned antibodies

Antibody	11	12	17	13	14	15	1	16	2	3	4	5	6	7	8	9	10
No. of stations	8	8	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9
Smoothness index	3.3	3.3	8.1	8.6	9	9.1	9.2	9.3	9.8	9.8	10.5	10.6	10.6	11.7	11.8	11.8	12

Step9: Remove r antibodies which are the worst antibodies in P.

The current step is to remove the worst antibodies from P. From Table 5 the best number of stations is 8. So the antibodies which their number of stations is equal to 8 will be kept in P and the other will be removed. If it occasionally happened that all antibodies have the same number of stations, the best 5 antibodies according to the smoothness index will be kept in P and the other will be removed. Table 6 illustrates the evaluation of the current population after removing the worst antibodies.

Table 6 the evaluation of population after removing the worst antibodies

Antibody	11	12
No. of stations	8	8
Smoothness index	3.3	3.3

Step10: Select E by comparing the evaluation of E with the evaluation of the best antibody in P and E will be the best one of them.

In the first iteration E = Φ and Accidently the current antibodies have the same number of stations and smoothness index. So we choose the first antibody as in Table 6 to be the new E and its structure is:

E =
{(10,6),(12,5),(19,8),(20,8),(14,8),(16,6),(11,5),(2,5),(1,1),(6,3),(4,2),(18,8),(15,6),(13,6),(9,4),(8,4),(21,5),(7,3),(3,1),(5,2),(17,7)}

Step11: Select the antibody (A_s) which has the highest smoothness index to be the seed of the remaining antibodies to complete P for reaching n antibodies by using various swapping and shifting procedures.

The remaining antibodies of the population will be generated by using a new seed. Our proposed seed is the antibody A_s which has the highest smoothness index. After completing the population to reach 10 antibodies by using various swapping and shifting procedure, a new population will be ready for a new iteration.

Step12: If $k = k_{max}$ then go to step 14.

In the first iteration $k = 1$ and our proposed $k_{max} = 2$. So step 13 is still active.

Step13: $k = k + 1$ and go to step 5.

Instep 13 k incremented by 1 and a repetition of the steps from 1 to 12 has to be done until achieving the condition in step 12.

Step14: The best solution found is E.

The final best antibody which has 8 stations and its smoothness index equal 3 as in Table 7:

Table 7 the final antibody after 2 iterations

Tasks	1	3	4	5	7	6	8	12	9	11	21	2	16	15	13	10	17	20	19	18	14
Processing time	4	9	5	9	8	4	7	1	5	3	7	3	3	5	5	1	13	3	2	5	3
Station	1	1	2	2	3	3	4	4	4	5	5	5	6	6	6	6	7	8	8	8	8

5. Computational Results

In this paper, we select 99 sample problems from: http://www.assembly-line-balancing.de and Scholl and Klein (1997), ranging from small-size to large-size problems with more than 100 tasks. The selected sample problems and the related data are listed in Table 8. We apply AIS, as well as the bidirectional CPM heuristic by Yeh and Kao (2009) and the bidirectional branch and bound approach by Scholl and Klein, to solve the selected sample problems. The code of our proposed AIS for solving SALBP-1 has been written by R programming and the population size was 10 antibodies.

Table 8 Data set and results for the tested sample problem

Sample problem	Total processing time	No. of tasks	Cycle time	Solution found by Scholl and Klein B&B	Solution found by Bidirectional CPM heuristic	Solution found by Proposed AIS
Arcus1	75,707	83	4206	19	19	19
			4732	17	17	17
			5408	15	15	15
			6309	13	13	13
			7571	11	11	11
Arcus2	150,399	111	5785	27	27	27
			6016	26	26	26
			7520	21	21	21
			8847	18	18	18
			10,027	16	16	16
Barthold2	4234	148	85	51	51	52
			89	48	49	50
			91	47	48	48
			118	36	37	37
			142	30	31	31

Bowman	75	8	20	5	5	5
Buxey	324	29	27	13	13	13
			33	11	11	11
			36	10	10	10
			41	8	8	8
			54	7	7	7
Gunther et al.	483	35	41	14	15	14
			44	12	12	12
			49	11	11	11
			61	9	9	9
			81	7	7	7
Hahn	14,026	53	2004	8	9	8
			2338	7	8	7
			2806	6	6	6
			3507	5	5	5
			4676	4	4	4
Heskiaoff	1024	28	138	8	8	8
			205	5	5	5
			216	5	5	5
			256	4	4	4
			342	3	3	3
Jackson	46	11	7	8	8	8
			9	6	6	6
			10	5	5	5
			14	4	4	4
			21	3	3	3
Jaeschke	37	9	6	8	8	8
			7	7	7	7
			8	6	6	6
			10	4	4	4
			18	3	3	3
Kilbridge and Wester	552	45	57	10	10	10

			92	6	6	7
			110	6	6	6
			138	5	5	5
			184	3	3	3
Lutz1	14,140	32	1414	11	11	11
			1768	9	9	9
			2020	8	8	8
			2357	7	7	7
			2828	6	6	6
Lutz3	1644	89	75	23	24	24
			87	20	21	21
			103	17	17	17
			127	14	14	14
			150	12	12	12
Mansoor	185	11	48	4	4	4
			62	3	3	3
			94	2	2	2
Mertens	29	7	6	6	6	6
			7	5	5	5
			8	5	5	5
			10	3	3	3
			18	2	2	2
Mitchell	105	21	14	8	8	8
			21	5	5	5
			26	5	5	5
			35	3	3	3
			39	3	3	3
Mukherjee and Basu	4208	94	176	25	25	25
			201	22	23	23
			222	20	21	20
			263	17	17	17
			351	13	13	13
Rosenberg	125	25	14	10	10	10

and Ziegler						
			16	8	9	9
			18	8	8	8
			21	6	6	6
			25	6	6	6
Sawyer	324	30	25	14	14	14
			27	13	13	13
			30	12	12	12
			36	10	10	10
			54	7	7	7
Tonge	3510	70	176	21	21	21
			364	10	10	10
			410	9	9	9
			468	8	8	8
			527	7	7	7
Wee and magazine	1499	75	49	32	32	32
			50	32	32	32
			52	31	32	32
			54	31	31	31
			56	30	31	31

The first approach to compare with, which is the branch and bound approach which is produced by Scholl and Klein (1999), considered one of the exact methods to solve the problem by using an intelligent fathoming technique, but it still an enumerative approach and it requires some higher computational complexity to solve the problem. The bidirectional based on CPM approach, which is the other approach to compare with, is one of the heuristic approaches which can produce a fast effective solution. The need to use one of the metaheuristic approaches helps to improve any heuristic solution. In this paper, a bidirectional heuristic based on positional weight used to be the seed of the population of the proposed AIS and the results are quite efficient as will be seen in the conclusion and future research section.

6. Conclusion and Future Research

In this paper, we propose the AIS to resolve the problem of SALBP-1. Yeh and Kao (2009) illustrated that the algorithm of Scholl and Klein (1999) aimed at finding optimal solution, so we compare the results of our proposed AIS with Scholl and Klein (1999) results to measure the efficiency of our algorithm. After solving 99 selected sample problems, we found that the solutions of only 12 problems are worse than its corresponding Scholl and Klein branch and bound algorithm's solutions. We also found only 3 solutions worse than its corresponding Bi-directional CPM (Yeh and Kao, 2009) and 4 solutions are better than the results of its corresponding solutions of the same approach. Computational results of the sample problems show that the proposed approach is quite effective in comparison with the optimal solution. Future research may include modification of the proposed AIS to further improve the effectiveness, and extension of AIS to solve various types of ALBP, including U-type, parallel and resource-constrained.

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معهد الدراسات والبحوث الإحصائية

المؤتمر السنوى التاسع والأربعون للإحصاء
وعلوم الحاسب وبحوث العمليات

مجلـــد

بحـــوث عمـــليات

٢٢-٢٥ديسمبر ٢٠١٤